CS 172: Computability and Complexity, Spring 2008 **S. A. Seshia & A. Kulkarni**

HW 5: Turing Machines and Variants

Assigned: March 6, 2008 *Due in class: March 13, 2008*

Note: Take time to write clear and concise solutions. Confused and long-winded answers may be penalized. Consult the course webpage for course policies on collaboration.

1. (6 points)

   (a) Let $L = \{0^n 1^n 2^n \mid n \geq 0\}$.
   Design a Turing Machine that decides L. (Give a high-level description of the Turing Machine as Sipser does.)

   (b) Now consider the language $L_k = \{0^n 1^n 2^n ... k^n \mid n \geq 0\}$ for some integer $k > 0$. Describe how you will generalize the TM you designed in part (a) to decide L_k.

2. (6 points) Prove that if languages L_1 and L_2 are Turing-recognizable, then so is $L_1 \circ L_2$. (Recall that $\circ$ denotes concatenation.)

3. (8 points) Recall the *FIFO Automaton* from Sample Midterm 1 – a FIFO automaton is just like a pushdown automaton except that the unbounded stack is replaced by an unbounded queue. In this question we will be concerned with *deterministic* FIFO automata.

   As usual, we can view the input to a FIFO automaton as being on a read-only tape with clearly marked end-of-input (say with blank symbols). The queue can be viewed as being on a separate tape with the restriction that symbols can be only written on the right end (a "push") and only read from the left end (a "pop").

   Further, we will define the FIFO automaton to accept by entering a designated q_{accept} state *at any time*.

   Prove that a language is Turing-recognizable if and only if it can be recognized by a deterministic FIFO automaton.

4. (10 points) Not all Turing-machine variants are as expressive as the standard TM!

   Define a *write-restricted Turing machine* (WR-TM) as a one-tape Turing machine that cannot write on the portion of the tape containing the input string, but which can write on the remainder of the tape.

   Show that this Turing machine can recognize only regular languages.

   It is thus strictly less expressive than the standard Turing machine model.