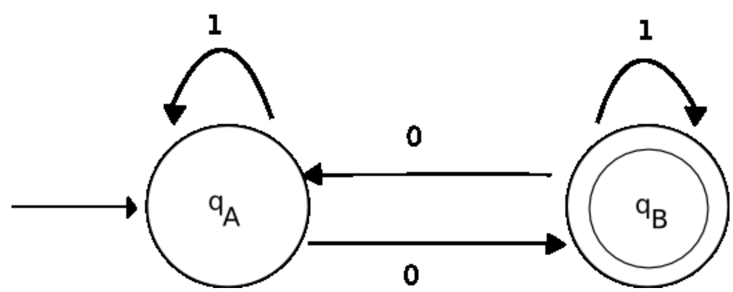
CS 172: Computability and Complexity, Spring 2008 **S. A. Seshia & A. Kulkarni**

HW 3: Regexps, Non-Regular Languages and Minimization

Assigned: February 7, 2008 *Due: February 14, 2008*

Note: Take time to write clear and concise solutions. Confused and long-winded answers may be penalized. Consult the course webpage for course policies on collaboration.

1. (4 points) Use the procedure discussed in class to convert the following DFA into a regular expression. First eliminate state q_A, then eliminate state q_B when simplifying the GNFA. Include all steps in your work.



2. (6 points) Prove that the converse of Homework 2 Problem 2 is false. In other words, it is possible that $L^{1/2}$ is regular but L is not.

3. (8 points) Let k be an arbitrary integer greater than zero.

   Give a language L such that L can be recognized by a DFA with exactly k states, but that no DFA with fewer than k states can recognize this language, and prove that your language L satisfies these properties.
   Hint: Think unary. Once you have a language, generate distinguishing strings to count the number of equivalence classes, then use the Myhill-Nerode theorem.

4. (6 points)

   Let k be a fixed non-negative integer ≥ 2 and let $L = \{1^{k^n} \mid n \geq 0\}$. Here, 1^{k^n} means a string of k^n ones. Show that L is not regular.

   (You may be interested to know that this result generalizes: if some function f grows faster than linearly, then $L_f = \{$all the strings of 1s whose length is $f(n)\}$ is not regular.)

5. (6 points) In this homework problem, you will use a method other than the pumping lemma to prove that a language is non-regular.

   Recall the definition of $\sim_L$ from the lecture on the Myhill-Nerode theorem and DFA Minimization:

   $$\text{For } x, y \in \Sigma^*, x \sim_L y \quad \text{iff} \quad \forall z \in \Sigma^*, xz \in L \Leftrightarrow yz \in L.$$

   Recall also that if L is regular, $\sim_L$ has only finitely many equivalence classes.

   Prove that the language $L = \{0^n 1^n \mid n \geq 0\}$ is non-regular by showing that $\sim_L$ has infinitely many equivalence classes. (Precisely describe what these equivalence classes are.)