

HW 2: Equivalence of DFAs & NFAs; Regular Expressions*Assigned: January 31, 2008**Due in 283 Soda box, 5pm: February 7, 2006*

Note: Take time to write clear and concise solutions. Confused and long-winded answers may be penalized. Consult the course webpage for course policies on collaboration.

1. (8 points)
 - (a) Give an NFA recognizing all binary strings with a 1 in the third position from the end and a 0 in the fourth position from the end.
 - (b) Following the equivalence proof, convert this to a DFA.
 - (c) Give a regular expression for this language. You are not required to use the equivalence proof.
 - (d) Simplify the DFA given in part (b) so that every state is reachable by some possible computation of the machine.
2. (6 points) If $\underline{w} = w_1w_2 \dots w_n$ is a string, define $\underline{w}^{1/2}$ to be the first half of \underline{w} . I.e. $\underline{w}^{1/2} = w_1w_2w_3 \dots w_{\lfloor \frac{n}{2} \rfloor}$. For any language L , define $L^{1/2} = \{\underline{w}^{1/2} : \underline{w} \in L\}$.

Prove that if L is regular, then $L^{1/2}$ is regular.

Hint: If a DFA recognizing L has state space Q , construct an NFA with state space $Q \times Q$ where the first coordinate goes forward and the second coordinate goes backward.

3. (4 points)
 - (a) Prove that every NFA can be converted to an equivalent DFA that has a single accept state.
 - (b) Give an example to support the falsity of the following statement: every DFA can be converted into an equivalent DFA with a single accept state.
 - (c) (no points, optional) Give an NFA with a single accept state that recognizes $L = \emptyset$. Also, prove formally that the statement in part (b) is false.

4. (3 points) Explain in words what languages the following regular expressions represent.

Let $\Sigma = \{a, b\}$.

- (a) $(a \cup ba \cup bb)\Sigma^*$
(b) $\Sigma^*a\Sigma^*b\Sigma^*a\Sigma^*$
5. (4 points) Let L be a language comprising all strings w over $\{0, 1\}$ such that w contains either:
- an even number of 1s, no occurrences of the sub-string 1101, and total length greater than five, OR
 - a 0 in the third position and odd length.

Write down a regular expression that generates L . Justify your answer.

6. (5 points)

- (a) Let A be a regular expression. Prove that $A \circ \varepsilon = A$.
- (b) In the C programming language, comments appear between delimiters such as $/*$ and $*/$. Let L denote the language of all valid delimited comment strings. A member of L must begin with $/*$ and end with $*/$ but have no intervening $*/$. For simplicity, assume that the comments themselves are written only with the symbols 0 and 1, so the alphabet Σ is $\{0, 1, *, /\}$.
Give a regular expression that generates L .