CS 172: Computability and Complexity, Spring 2010 **S. A. Seshia & O. Etesami**

HW 10: L & NL; Log-Space Reductions; Hierarchy Theorems

Assigned: April 28, 2010 *Due: May 5, 2010*

Note: Take time to write clear and concise solutions. Confused and long-winded answers may be penalized. Consult the course webpage for course policies on collaboration.

1. (10 points) [Sipser 8.29]
   Recall that $A_{NFA} = \{\langle B, w\rangle | B$ is an NFA that accepts input string $w\}$.

   In this problem, we will show that A_{NFA} is NL-complete.

   (a) (5 points) Show $A_{NFA} \in$ NL.

   (b) (5 points) Prove that PATH $\leq_L A_{NFA}$.

2. (10 points) The Space Hierarchy theorem we proved in class was for *deterministic* TMs. Here we consider a version of this theorem for *non-deterministic space*, i.e., for non-deterministic TMs.

   (a) (5 points)
   Let f be any space constructible function such that $f(n) \geq (\log n)^2$.
   Prove that there exists a language that is in NSPACE($f(n)$) but not in NSPACE($g(n)$) for any $g(n) = o(\sqrt{f(n)})$.
   You may assume $g(n) \geq \log n$.
   [Hint: Use Savitch's Theorem and the Space Hierarchy Theorem we proved in class.]

   (b) (5 points)
   The proof of the Space Hierarchy Theorem in Sipser will not work for non-deterministic space. Identify the part of the construction that breaks down if we have non-deterministic TMs M that must be simulated by D.
   [Hint: there is one key proof idea we used in class that breaks down.]

3. (10 points) [Sipser 8.19] The game of *Nim* is played with a collection of piles of sticks. In one move a player may remove any nonzero number of sticks from a single pile. The two players alternately take turns making moves. The player who removes the very last stick wins. Say that we have a game position in Nim with k piles containing $s_1, \ldots, s_k$ sticks. Call the position *balanced* if, when each of the numbers s_i is written in binary and the binary numbers are written as rows of a matrix aligned at the low order bits, each column of bits contains an even number of 1s. Prove the following two facts:

(a) (4 points) Starting in an unbalanced position, a single move exists that changes the position into a balanced one.

(a) (2 points) Starting in a balanced position, every single move changes the position into an unbalanced one.

Let $NIM = \{\langle s_1, \ldots, s_k \rangle |$ each s_i is a binary number and Player I has a winning strategy in the Nim game starting at this position $\}$.

(4 points) Use the preceding facts about balanced positions to show that $NIM \in L$.