CS172 Midterm 2 Practice Questions

You may assume without proof any result that was proved in class or on a Homework, but state your assumptions clearly. Descriptions of Turing machines can be in the form of Sipser's "high-level descriptions".

1. State True or False. If your answer is "true", give a short proof. If "false", give a simple counterexample.

   (a) If L_1 and L_2 are Turing-recognizable, then $L_2 \setminus L_1$ is also Turing-recognizable.

   (b) If $L_1 \leq_m L_2$, then $L_2 \leq_m L_1$.

   (c) Let $PAL = \{w \mid w \in \{0,1\}^*$ and w is a palindrome $\}$. Then $PAL \in$ NP.

2. Let $L = \{\langle M \rangle \mid M$ is a TM that accepts string w^R whenever it accepts $w\}$. Prove that L is undecidable.

   [Hint: Use a mapping reduction from A_{TM}.]

3. A *useless state* in a PDA is a state that is never entered on any input string. Let $L = \{\langle P \rangle \mid P$ is a PDA that has useless states $\}$. Prove that L is decidable.

   [Hint: Use a reduction to E_{CFG}.]

4. Recall the problem of *linear programming* from CS 170. This question is concerned with the *integer linear programming* problem.

   We define an integer linear program (ILP) as a system of m inequalities in variables $x_1, x_2, \ldots, x_n$, as follows:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n &\geq b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n &\geq b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n &\geq b_m \end{aligned}$$

   where the a_{ij}'s and b_i's are all integral.

   We say that an ILP is *feasible* iff there are integral values of the variables $x_1, x_2, \ldots, x_n$ such that each of the m inequalities evaluates to true (i.e., the ILP has an integral solution).

   Define $FIP = \{\langle P \rangle \mid P$ is a feasible ILP $\}$.

   Prove that FIP is NP-hard, using a reduction from 3SAT.