

Midterm 2*November 15, 2006*YOUR NAME:

Instructions:

This exam is *closed-book, open-notes*. Please turn off electronic devices: cell phones, laptops, PDAs, etc.

You have a total of 70 minutes. There are 4 questions worth a total of 100 points. The questions vary in difficulty, so if you get stuck on any question, it might help to leave it for a while and try another one.

Answer each question in the space provided below the question. If you need more space, you can use the reverse side of that page.

<i>Do not turn this page until the instructor tells you to do so!</i>

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

Problem 1: [True or False, with justification] (30 points)

For each of the following questions, state TRUE or FALSE. If TRUE, give a short proof. If FALSE, give a simple counterexample.

(a) The sentence

$$\forall x, y [(x \times y > 0) \implies (x + y \geq 2)]$$

is in $\text{Th}(\mathbb{N}, +, \times)$ but not in $\text{Th}(\mathbb{N}, +)$. (Recall that \mathbb{N} denotes the set of natural numbers: $\{0, 1, 2, 3, \dots\}$.)

(b) For every language L , it is true that $L \leq_m \bar{L}$.

(c) The following language L is decidable:

$$L = \{ \langle M \rangle \mid M \text{ is a TM and, for all inputs } w, M \text{ halts on } w \text{ within 42 steps} \}$$

Problem 2: (25 points)

A *jumpy Turing machine* (JTM) is just like the standard, single-tape, deterministic TM except for its transition function. On a transition, a JTM can move its head a finite, but arbitrary distance from its current location. Formally, the transition function is

$$\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times (\{L, R\} \times \mathbb{N})$$

For example, $\delta(q_i, a) = (q_j, b, (L, n))$ will cause the JTM's head to move n places to the left (stopping at the left-most cell if the jump would cause the head to move off the tape).

Prove that every JTM has an equivalent standard, single-tape, deterministic TM. (Include all steps in your proof.)

Problem 3: (20 points)

Recall the *one way Turing machine* (OTM) from Homework 5. An OTM is a single-tape, deterministic Turing machine that has a “stay put” move S in place of the regular “move left” L . In other words, the transition function is of the form

$$\delta : Q \times \Gamma \longrightarrow Q \times \Gamma \times \{R, S\}$$

Answer the *three* questions (a), (b), (c) below (remember to turn this page over!). You may use without proof results from previous homeworks and theorems covered in class and in Sipser’s book.

- (a) Let the language class $C = \{L \mid L \text{ is a language recognized by a OTM}\}$. Among the languages classes you have studied in this class (regular, context-free, decidable, Turing-recognizable, etc.), which one is C equal to? Briefly justify your answer (no need to give a complete proof).

- (b) Prove that the following language is undecidable:
 $L_2 = \{\langle M \rangle \mid M \text{ is a TM and } L(M) \in C\}$

- (c) Prove that the following language is decidable:
 $L_1 = \{\langle M \rangle \mid M \text{ is an OTM and } L(M) = \emptyset\}$

Problem 4: (25 points)

Recall that a *clique* in a (undirected) graph G is a subset of vertices of G such that every two vertices in that subset are connected by an edge.

We discussed the following CLIQUE problem in class:

$$\text{CLIQUE} = \{\langle G, k \rangle \mid G \text{ is an undirected graph with a clique of size } k\}$$

Two variants of the CLIQUE problem are given below. For each language, state whether it is in P or is NP-hard. If P, give a polynomial-time algorithm to decide it. If NP-hard, give a reduction from CLIQUE or one of the NP-hard problems discussed in class.

(a) $11\text{-CLIQUE} = \{\langle G \rangle \mid G \text{ is an undirected graph with a clique of size } 11\}$

(b) $\text{SpecialCLIQUE} = \{\langle G, v, k \rangle \mid G \text{ is an undirected graph with a clique of size } k \text{ that includes vertex } v\}$