CS 172: Computability and Complexity, Fall 2006 **S. A. Seshia**

Midterm 1

October 4, 2006

YOUR NAME:

Instructions:

This exam is closed-book, closed-notes, except for the one page of notes that you are permitted. Please turn off electronic devices: cell phones, laptops, etc.

You have a total of 60 minutes. There are 4 questions worth a total of 100 points. The questions vary in difficulty, so if you get stuck on any question, it might help to leave it and try another one.

Answer each question in the space provided below the question. If you need more space, you can use the reverse side of that page.

Do not turn this page until the instructor tells you to do so!

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

Problem 1: [True or False, with justification] (30 points)

For each of the following questions, state TRUE or FALSE. Justify your answer in brief, indicating only the "proof idea" or counterexample, drawing an annotated picture if needed.

(a) The regular expression $0^*(100 \cup 010 \cup 001)0^*$ generates the language $\{w \mid w \in \{0,1\}^*$ and w contains at least 2 $0s$ and at most one $1\}$.

(b) The following grammar is unambiguous: $(\{S, A\}, \{0, 1\}, R, S)$ where R has the rules $S \to 0A1$ and $A \to \epsilon \mid 0A1 \mid AA$

(c) Let b_i denote the standard binary encoding of non-negative integer i, without leading 0s. The language $\{b_i \mid i$ is a multiple of $3\}$ is regular.

Problem 2: (20 points)

Let $L = \left\{ 1^{n^2} \mid n \text{ a positive integer} \right\}$. Show that L is *not* context free. Include all steps in your proof.

Problem 3: (25 points)

Useful definition: For $x, y \in \Sigma^*$, $x \sim_L y$ iff $\forall z \in \Sigma^*$, $xz \in L \Leftrightarrow yz \in L$.

Let $\Sigma = \{0, 1\}$. Give a regular language L so that the minimum number of states for a DFA recognizing L is exactly 5.

Prove that your answer is correct, including all steps. Also draw the minimal DFA.

Problem 4: (25 points)

Define a *FIFO automaton* to be exactly like a pushdown automaton (PDA) except that it uses an *unbounded queue* instead of an unbounded stack. FIFO automata also use "push" and "pop" operations, only that "push" adds to the end of the queue, and "pop" removes from the front of the queue. The acceptance condition is the same as that of a PDA.

Define the *language class* C_{fifo} as the set $\{L \mid L \text{ is a language recognized by a FIFO automaton}\}$.

Answer the following questions, justifying your answer (a proof idea/sketch will suffice, but it must precisely state the key insights):

(a) If A is a regular language, is A in C_{fifo}?

(b) Recall the context-free language $B = \{0^n 1^n \mid n \geq 0\}$. Is B in C_{fifo}?

(c) Recall the language $D = \{ww \mid w \in \{0,1\}^*\}$. We showed in class that D is not context free. Is D in C_{fifo}?

(d) Using parts (a)-(c) and the course syllabus covered so far, what can you conclude about the subset relationships between C_{fifo}, the set of all regular languages C_{reg}, and the set of all CFLs C_{cf}?