CS 172: Computability and Complexity, Spring 2008 **S. A. Seshia and A. Kulkarni**

Midterm 1

February 26, 2008

YOUR NAME:

Instructions:

This exam is closed-book, open-notes. Please turn off and put away electronic devices such as cell phones, laptops, etc.

You have a total of 60 minutes. There are 4 questions worth a total of 100 points. The questions vary in difficulty, so if you get stuck on any question, it might help to leave it and try another one.

Answer each question in the space provided below the question. If you need more space, you can use the reverse side of that page. *You can use without proof any result proved in class, in Sipser's book, or during discussion sections, but clearly state the result you are using.*

Do not turn this page until the instructor tells you to do so!

Problem 1	
Problem 2	
Problem 3	
Problem 4	
Total	

Problem 1: [True or False, with justification] (30 points)

For each of the following questions, state TRUE or FALSE. Justify your answer in brief, indicating only the "proof idea" or counterexample, drawing an annotated picture if needed.

(a) If the language L is context-free, then L^R is also context-free. (Recall that L^R is the language comprising reversals of all strings in L.)

(b) The class of regular languages is closed under the symmetric difference operator $\oplus$. (Definition: For any two sets A and B, $A \oplus B$ is the set of all items which are in either A or B but not both.)

(c) Let $\Sigma = \{0, 1\}$. Every language recognizable by a 1-state NFA is recognizable by a DFA with 1 state.

Problem 2: (20 points)

Let $\Sigma = \{0, 1, \#\}$. Draw a PDA for the following language L:
$L = \{w\#x \mid w, x \in \{0, 1\}^* \text{ where } w^R \text{ is a substring of } x\}$

Problem 3: (25 points)

Let $\Sigma = \{\texttt{0}, \texttt{1}, +, =\}$. Consider the language L where

$$L = \{\texttt{a=b+c} \mid \texttt{a}, \texttt{b}, \texttt{c} \in \{\texttt{0}, \texttt{1}\}^* \text{ where } \texttt{a} = \texttt{b} + \texttt{c} \text{ if } \texttt{a}, \texttt{b}, \texttt{c} \text{ are interpreted as unsigned binary integers and + and = are interpreted as addition } + \text{ and equality } =\}$$

For example, the string `10=01+01` is in L ($2 = 1 + 1$). But the string `10=11+01` is not in L ($2 \neq 3 + 1$).

Show L is not regular.

Since you have already solved the above problem on HW 3, do the one below instead:

If f is a function from nonnegative integers to nonegative integers, let L_f be all the strings of 1s whose length is $f(n)$ for some n.

Prove that if f grows faster than linearly, then L_f is not regular. Include all steps in your proof.

(Faster than linear means that for any c, there is some m so that for all $n > m$, you get $f(n) > cn$.)

Problem 4: (25 points)

An N-*constrained pushdown automaton* (N-CPA) is exactly like a pushdown automaton except that it can only perform at most N push, pop, or swap operations. (A swap is a simultaneous push and pop.) The acceptance condition and unbounded stack are just as in the PDA.

Consider a 1000-CPA. Define the *language class* $C_{1000-\text{CPA}}$ as the set
$\{L \mid L \text{ is a language recognized by a 1000-CPA}\}$.

Similarly, the set of all regular languages is C_{reg}, and the set of all CFLs is C_{cf}.

One of the following relationships is TRUE. State which one, and provide a complete proof for the truth of this statement. You don't need to justify falsity of the other statements.

1. $C_{reg} \subset C_{1000-\text{CPA}} \subset C_{cf}$. (Note: $\subset$ means "strict subset")
2. $C_{reg} = C_{1000-\text{CPA}}$.
3. $C_{1000-\text{CPA}} = C_{cf}$.

If you convert a DFA/PDA to a CPA or vice-versa, specify your construction precisely using the 5-tuple/6-tuple notation.