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Equilibrium $\implies R = C$!

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Doesn't matter who plays first!

Summary and..

Zero sum game:

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Zero sum game: $m \times n$ matrix A

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row minimizes.

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row minimizes. strategy: m-dimensional vector x

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Best row is worse under y_2.

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Best row is worse under y_2.
$\implies$ Column player has incentive to change.
x_1, y_1 is not equilibrium.

An "asymptotic" game.

"Catch me."

An “asymptotic” game.

“Catch me.”

Given: $G = (V, E)$.

An “asymptotic” game.

“Catch me.”

Given: $G = (V, E)$.
Given $a, b \in V$.

An “asymptotic” game.

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Given: $G = (V, E)$.
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Row (“Catch me”): choose path from a to b.

An “asymptotic” game.

“Catch me.”

Given: $G = (V, E)$.
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Row (“Catch me”): choose path from a to b.
Column(“Catcher”): choose edge.

An “asymptotic” game.

“Catch me.”

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Given $a, b \in V$.
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Row pays if column chooses edge on path.

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Matrix:
row for each path: p

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An "asymptotic" game.

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Row ("Catch me"): choose path from a to b.
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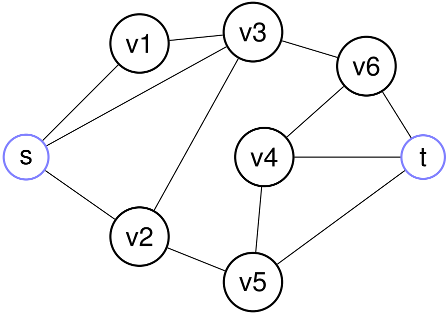
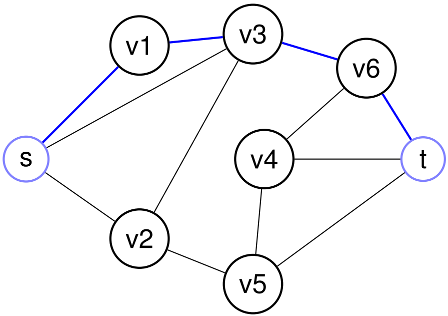
Row pays if column chooses edge on path.

Matrix:
row for each path: p
column for each edge: e
$A[p, e] = 1$ if $e \in p$.



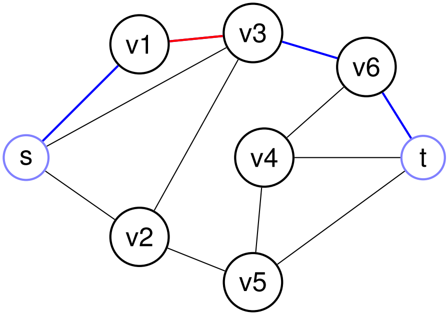
Catchme:

Catcher:



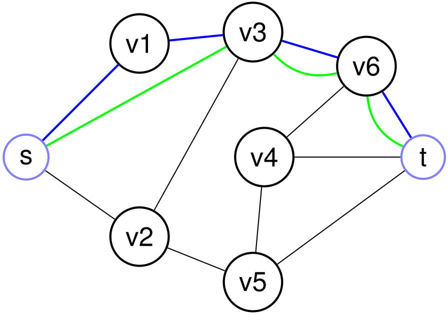
Catchme:
Use Blue Path.

Catcher:



Catchme:
Use Blue Path.

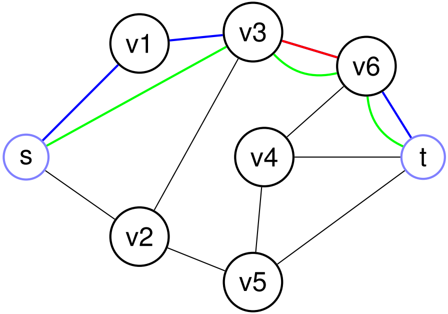
Catcher:
Caught!



Catchme:

Blue with prob. 1/2.
Green with prob. 1/2.

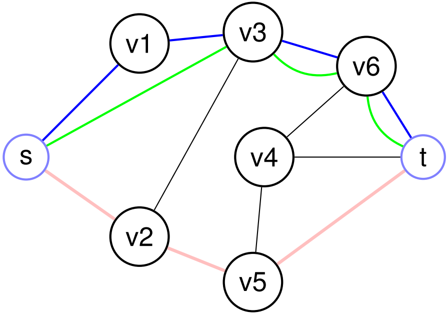
Catcher:



Catchme:

Blue with prob. 1/2.
Green with prob. 1/2.

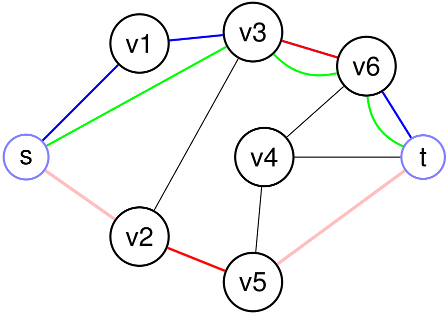
Catcher:
Caught!



Catchme:

Blue with prob. 1/3.
Green with prob. 1/6.
Pink with prob. 1/2.

Catcher:



Catchme:

Blue with prob. 1/3.
Green with prob. 1/6.
Pink with prob. 1/2.

Catcher:
Caught, sometimes.
With probability $1/2$.

Example.

Row solution: $Pr[p_1] = 1/2$, $Pr[p_2] = 1/3$, $Pr[p_3] = 1/6$.

Example.

Row solution: $Pr[p_1] = 1/2$, $Pr[p_2] = 1/3$, $Pr[p_3] = 1/6$.
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Offense

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Offense (Best Response.):

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Row solution: $Pr[p_1] = 1/2$, $Pr[p_2] = 1/3$, $Pr[p_3] = 1/6$.
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Offense (Best Response.):

Catch me: route along shortest path.

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Offense (Best Response.):

Catch me: route along shortest path.
(Knows catcher's distribution.)

Example.

Row solution: $Pr[p_1] = 1/2$, $Pr[p_2] = 1/3$, $Pr[p_3] = 1/6$.
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Offense (Best Response.):

Catch me: route along shortest path.
(Knows catcher's distribution.)
Catcher: raise toll on most congested edge.

Example.

Row solution: $Pr[p_1] = 1/2$, $Pr[p_2] = 1/3$, $Pr[p_3] = 1/6$.
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Offense (Best Response.):

Catch me: route along shortest path.
(Knows catcher's distribution.)
Catcher: raise toll on most congested edge.
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Defense:

Where should "catcher" play to catch any path?

Example.

Row solution: $Pr[p_1] = 1/2$, $Pr[p_2] = 1/3$, $Pr[p_3] = 1/6$.
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Catch me: route along shortest path.
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Defense:

Where should "catcher" play to catch any path? a cut.

Example.

Row solution: $Pr[p_1] = 1/2$, $Pr[p_2] = 1/3$, $Pr[p_3] = 1/6$.
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Offense (Best Response.):

Catch me: route along shortest path.
(Knows catcher's distribution.)
Catcher: raise toll on most congested edge.
(Knows catch me's distribution.)

Defense:

Where should "catcher" play to catch any path? a cut.
Minimum cut allows the maximum toll on any edge!

Example.

Row solution: $Pr[p_1] = 1/2$, $Pr[p_2] = 1/3$, $Pr[p_3] = 1/6$.
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Catcher: raise toll on most congested edge.
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Defense:

Where should "catcher" play to catch any path? a cut.
Minimum cut allows the maximum toll on any edge!

What should "catch me" do to avoid catcher?

Example.

Row solution: $Pr[p_1] = 1/2$, $Pr[p_2] = 1/3$, $Pr[p_3] = 1/6$.
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Where should "catcher" play to catch any path? a cut.
Minimum cut allows the maximum toll on any edge!

What should "catch me" do to avoid catcher?
minimize maximum load on any edge!

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What should "catch me" do to avoid catcher?
minimize maximum load on any edge!
Max-Flow Problem.

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Minimum cut allows the maximum toll on any edge!

What should "catch me" do to avoid catcher?
minimize maximum load on any edge!
Max-Flow Problem.

Note: exponentially many strategies for "catch me"!

Toll/Congestion

Given: $G = (V, E)$.
Given $(s_1, t_1) \dots (s_k, t_k)$.
Row: choose routing of all paths.
Column: choose edge.
Row pays if column chooses edge on any path.

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row for each routing: r

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$A[r, e]$ is congestion on edge e by routing r

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Offense: (Best Response.)

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Offense: (Best Response.)
Router: route along shortest paths.

Toll/Congestion

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Row: choose routing of all paths.
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Matrix:
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$A[r, e]$ is congestion on edge e by routing r

Offense: (Best Response.)
Router: route along shortest paths.
Toll: charge most loaded edge.

Toll/Congestion

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Row: choose routing of all paths.
Column: choose edge.
Row pays if column chooses edge on any path.

Matrix:
row for each routing: r
column for each edge: e

$A[r, e]$ is congestion on edge e by routing r

Offense: (Best Response.)
Router: route along shortest paths.
Toll: charge most loaded edge.

Defense: Toll: maximize shortest path under tolls.

Toll/Congestion

Given: $G = (V, E)$.
Given $(s_1, t_1) \ldots (s_k, t_k)$.
Row: choose routing of all paths.
Column: choose edge.
Row pays if column chooses edge on any path.

Matrix:
row for each routing: r
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Route: minimize max loaded on any edge.

Again: exponential number of paths for route player.

Summary...

You should now know about

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Games

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Games
Nash Equilibrium

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Today

Today

Maximum Weight Matching

Today

Maximum Weight Matching
Undergraduate: saw maximum matching!

Today

Maximum Weight Matching
Undergraduate: saw maximum matching! (hopefully.)

Today

Maximum Weight Matching
Undergraduate: saw maximum matching! (hopefully.) Will review.

Matching.

Given a bipartite graph, $G = (U, V, E)$, with edge weights $w : E \to R$, find a maximum weight matching.

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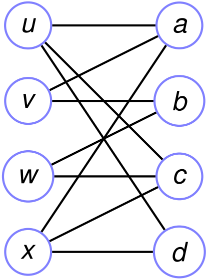

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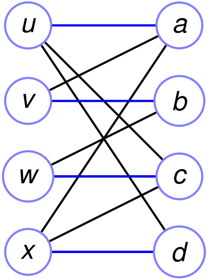

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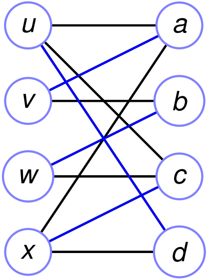

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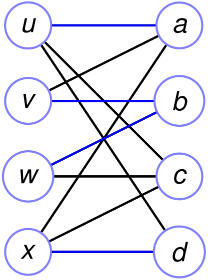

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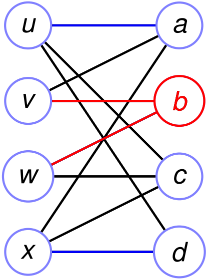

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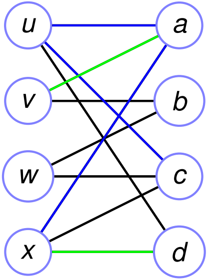

Blue – 3. Green - 2,
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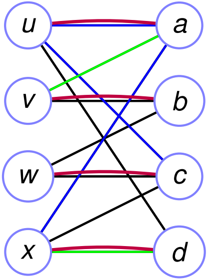

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Solution Value: 7.

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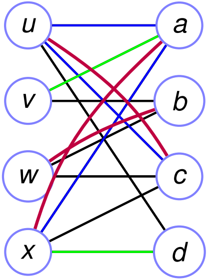

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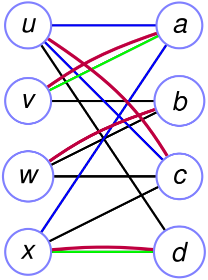

Blue – 3. Green - 2, Black - 1, Non-edges - 0.

Solution Value: 7.

Solution Value: 7.

Solution Value: 8.

Applications

Jobs to workers.

Applications

Jobs to workers.

Teachers to classes.

Applications

Jobs to workers.

Teachers to classes.

Classes to classrooms.

Applications

Jobs to workers.

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"The assignment problem"

Applications

Jobs to workers.

Teachers to classes.

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“The assignment problem”

Min Weight Matching.

Applications

Jobs to workers.

Teachers to classes.

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"The assignment problem"

Min Weight Matching.
  Negate values and find maximum weight matching.

Vertex Cover

Given a bipartite graph, $G = (U, V, E)$, with edge weights $w : E \rightarrow R$, find an vertex cover function of minimum total value.

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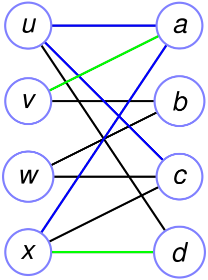
Minimize $\sum_{v \in U \cup V} p(u)$.



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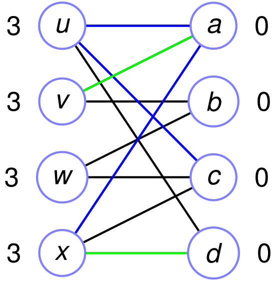

Solution Value: 12.

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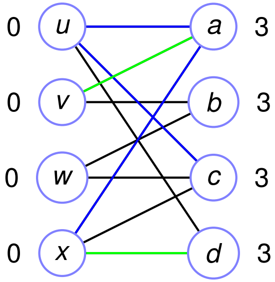

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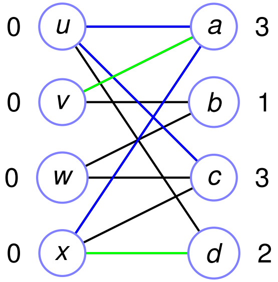

Solution Value: 12.

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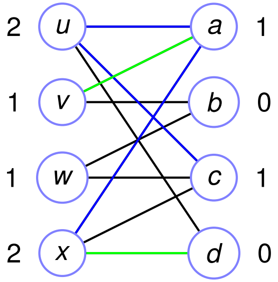

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Cover is upper bound.

Feasible $p(\cdot)$,

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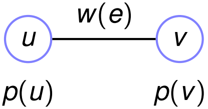

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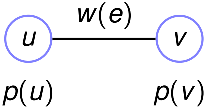
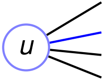


For a matching M, each u is the endpoint of at most one edge in M.



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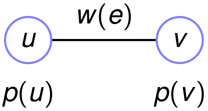

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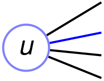

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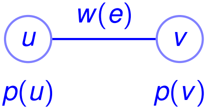

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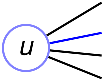

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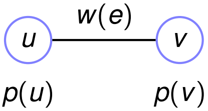

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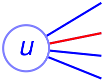

$$\sum_{e=(u,v)\in M} w(e) \leq \sum_{e=(u,v)\in M} (p(u) + p(v)) \leq \sum_{u \in U} p(u) + \sum_{v \in V} p(v)$$

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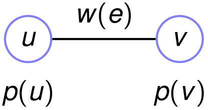

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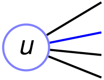

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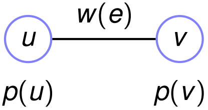

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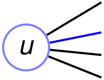

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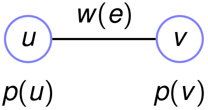

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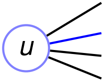

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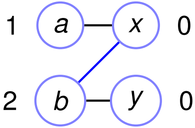
for $e \in M$, $w(e) = p(u) + p(v)$ (Defn: tight edge.) and

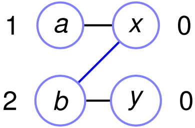
perfect matching.

Simple example.



Blue edge – 2, Others – 1.

Simple example.

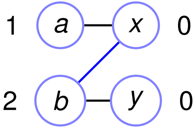


Blue edge – 2, Others – 1.

Using max incident edge.

Value: 3.

Simple example.



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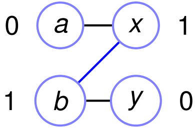
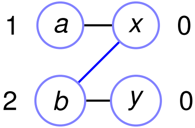


Value: 2.
Same as optimal matching!

Proof of optimality.

Simple example.



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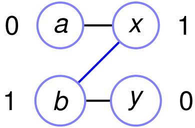

Matching and cover are optimal,

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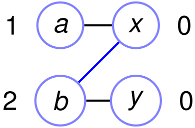

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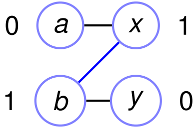

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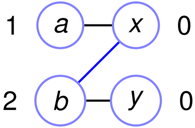

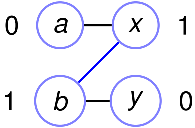

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  Same as optimal matching!

Proof of optimality.

Matching and cover are optimal,
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  all nodes are matched.

Maximum Matching

Given a bipartite graph, $G = (U, V, E)$, find a maximum sized matching.

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Key Idea: Augmenting Alternating Paths.

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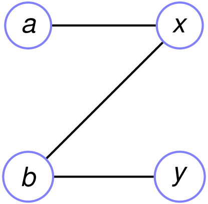

Maximum Matching

Given a bipartite graph, $G = (U, V, E)$, find a maximum sized matching.

Key Idea: Augmenting Alternating Paths.

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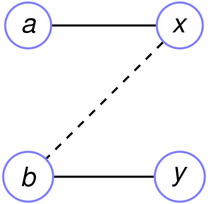

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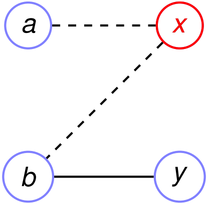

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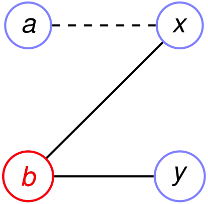

Start at unmatched node(s),
follow unmatched edge(s),
follow matched.
Repeat until an unmatched node.

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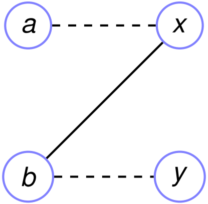

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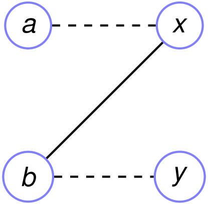

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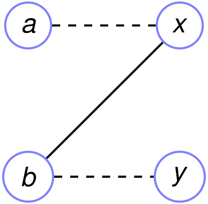

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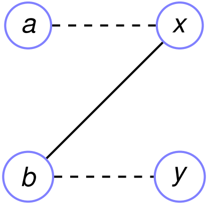

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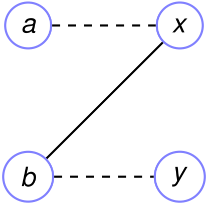

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Repeat until an unmatched node.

No perfect matching

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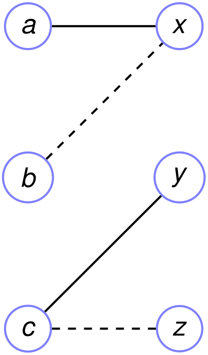
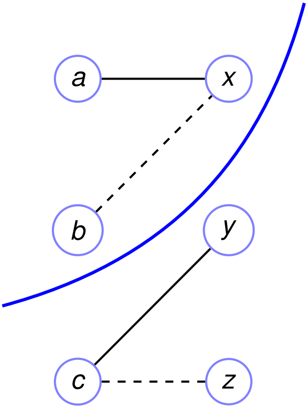


Can’t increase matching size.
No alternating path from (a) to (y).

No perfect matching

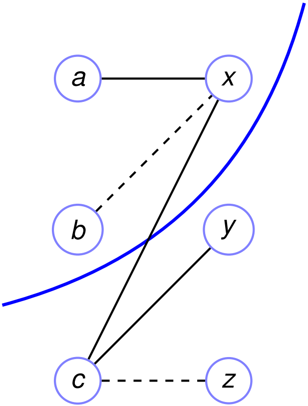


Can’t increase matching size.

No alternating path from (a) to (y).

Cut!

No perfect matching

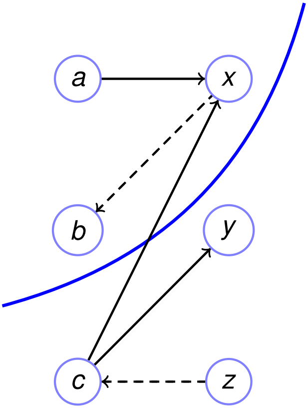


Can’t increase matching size.
  No alternating path from (a) to (y).

Cut!

Still no augmenting path.
  Still Cut?

No perfect matching



Algorithm:

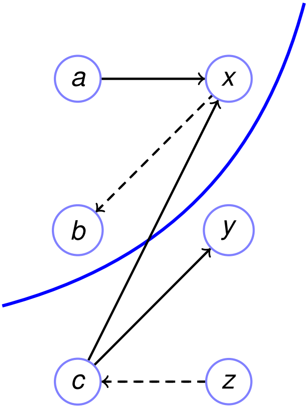
Can't increase matching size.
No alternating path from (a) to (y).

Cut!

Still no augmenting path.
Still Cut?

Use directed graph!
Cut in this graph.

No perfect matching



Can't increase matching size.
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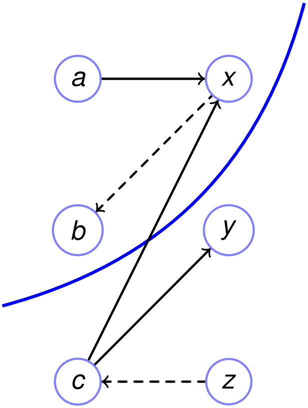
Algorithm:
Given matching.

No perfect matching



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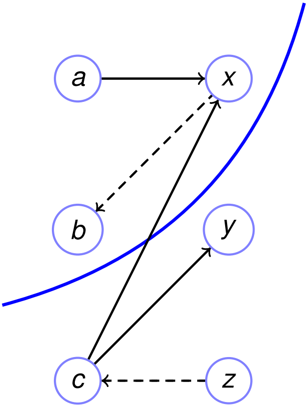
Algorithm:
Given matching.
Direct unmatched edges U to V, matched V to U.

No perfect matching



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Still Cut?

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Cut in this graph.

Algorithm:
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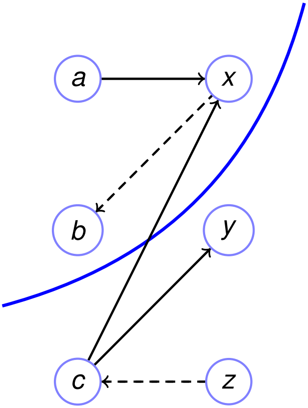
Direct unmatched edges U to V, matched V to U.
Find path between unmatched nodes on left to right. (BFS, DFS).

No perfect matching



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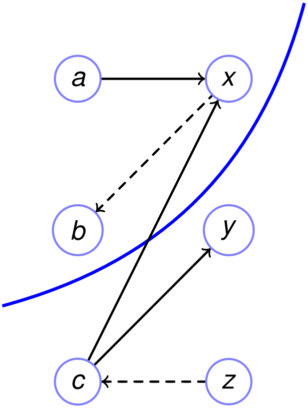
Until everything matched

No perfect matching



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Still Cut?

Use directed graph!
Cut in this graph.

Algorithm:
Given matching.
Direct unmatched edges U to V, matched V to U.
Find path between unmatched nodes on left to right. (BFS, DFS).
Until everything matched ... or output a cut.

Back to Maximum Weight Matching.

Want vertex cover (price function) $p(\cdot)$ and matching where.

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Optimal solutions to both if

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for $e \in M$, $w(e) = p(u) + p(v)$ (Defn: tight edge.) and

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Optimal solutions to both if

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Maximum Weight Matching

Goal: perfect matching on tight edges.

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Algorithm

Init: empty matching, feasible cover function ($p(\cdot)$)

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Add tight edges to matching.

Maximum Weight Matching

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Init: empty matching, feasible cover function ($p(\cdot)$)

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Use alt./aug. paths of tight edges.

Maximum Weight Matching

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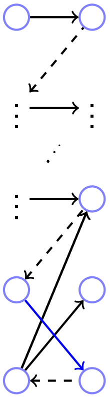

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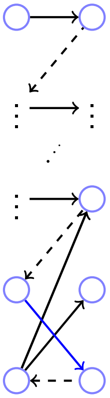

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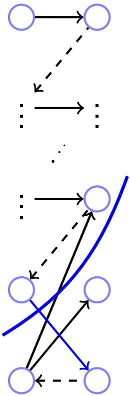

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No augmenting path.
  Cut, (S, T), in directed graph of tight edges!

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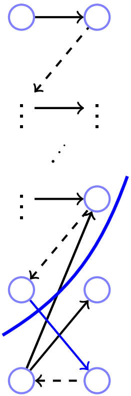

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All edges across cut are not tight. (loose?)

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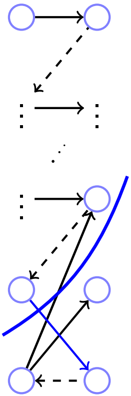

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Nontight edges leaving cut, go from S_U, T_V.

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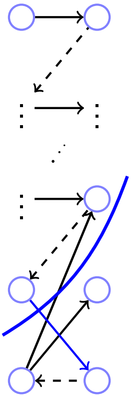
Goal: perfect matching on tight edges.



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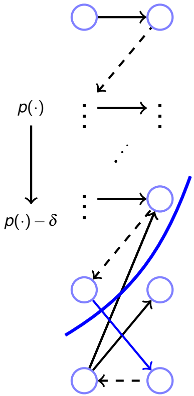

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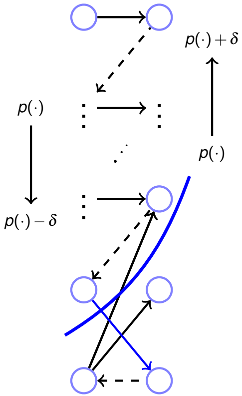
Nontight edges leaving cut, go from S_U, T_V.

Lower prices in S_U,

Maximum Weight Matching

Goal: perfect matching on tight edges.



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Add tight edges to matching.
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Lower prices in S_U, raise prices in S_V,

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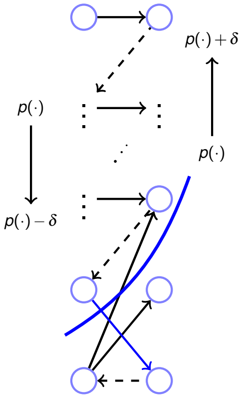
Goal: perfect matching on tight edges.



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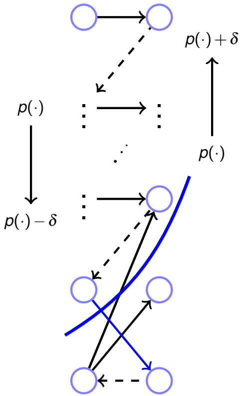
Nontight edges leaving cut, go from S_U, T_V.

Lower prices in S_U, raise prices in S_V,
all explored edges still tight,
matched edges still tight

Maximum Weight Matching

Goal: perfect matching on tight edges.



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... and get new tight edge!

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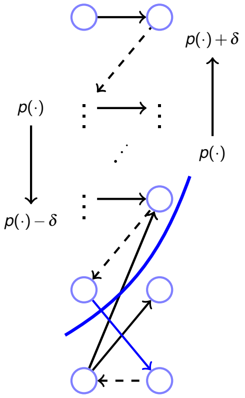

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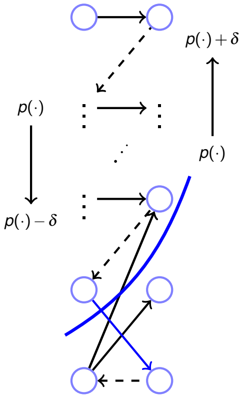
What's delta?

Maximum Weight Matching

Goal: perfect matching on tight edges.



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What's delta? $w(e) < p(u) + p(v) \rightarrow$

Maximum Weight Matching

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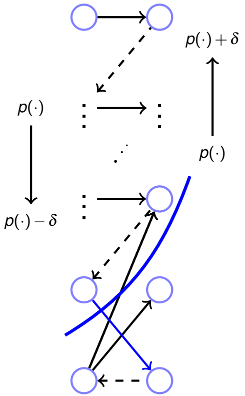

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What's delta? $w(e) < p(u) + p(v) \rightarrow$
$\delta = min_{e \in (S_U \times T_V)} p(u) + p(v) - w(e)$.

Some details

Add 0 value edges, so that optimal solution contains perfect matching.

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Beginning "Matcher" Solution: $M = \{\}$.

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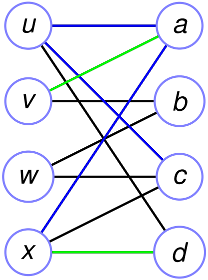
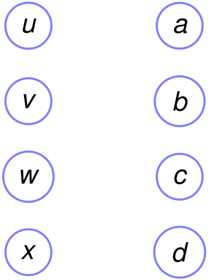
$O(n)$ iterations per augmentation.
$O(n)$ augmentations.

$O(n^2 m)$ time.

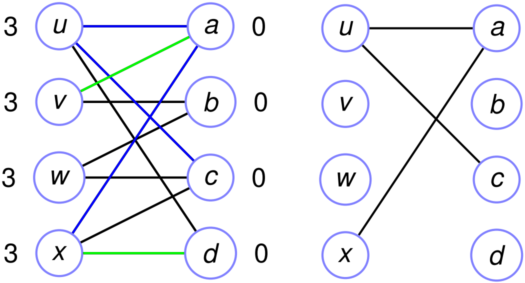
Example

Weight legend:
black 1, green 2, blue 3



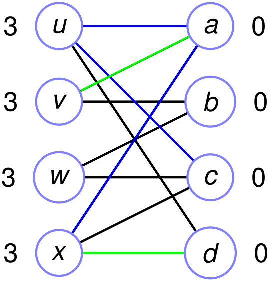


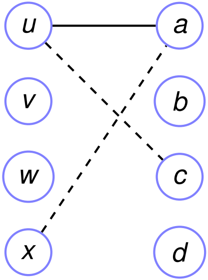
Example



Weight legend:
black 1, green 2, blue 3
Tight edges for inital prices.

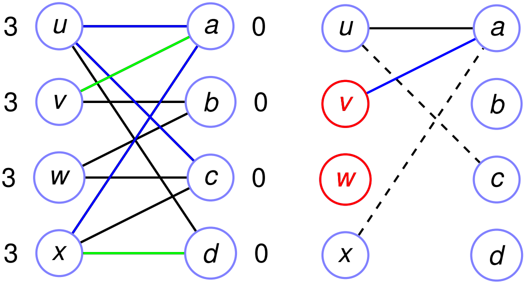
Example





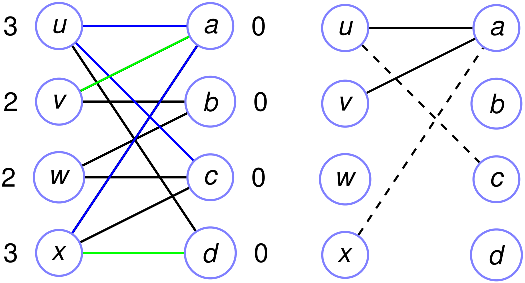
Weight legend:
black 1, green 2, blue 3
Tight edges for inital prices.
Max matching in tight edges.
dashed means matched.

Example



Weight legend:
black 1, green 2, blue 3
Tight edges for inital prices.
Max matching in tight edges.
dashed means matched.
No augmenting path $\rightarrow$
reachable: $S = \{u, v\}$
Blue edge on right soon
to be tight!

Example



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dashed means matched.
No augmenting path $\rightarrow$
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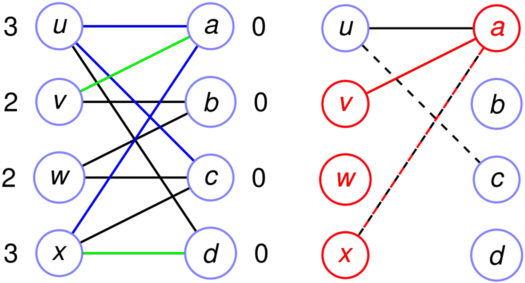
Blue edge on right soon
to be tight!
Adjust prices...$\delta = 1$
new tight edges.

Example



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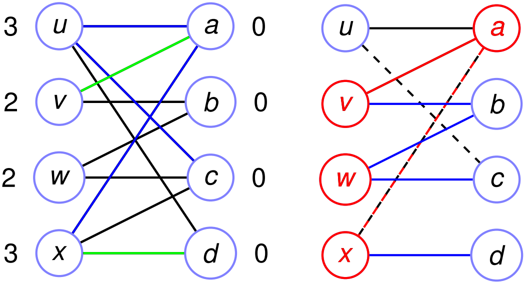
Still no augmenting path.
Reachable $S = \{v, w, x, a\}$

Example



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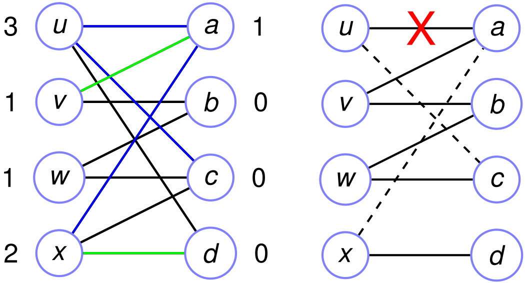
Adjust prices...$\delta = 1$
new tight edges.
Still no augmenting path.
Reachable $S = \{v, w, x, a\}$
Blue edges minimally non-tight.

Example



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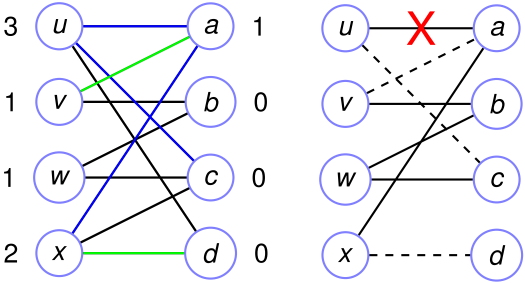
Blue edge on right soon
to be tight!
Adjust prices...$\delta = 1$
new tight edges.
Still no augmenting path.
Reachable $S = \{v, w, x, a\}$
Blue edges minimally non-tight.
Adjust prices.
Some more tight edges.
And X shows
a "new" nontight edge.

Example



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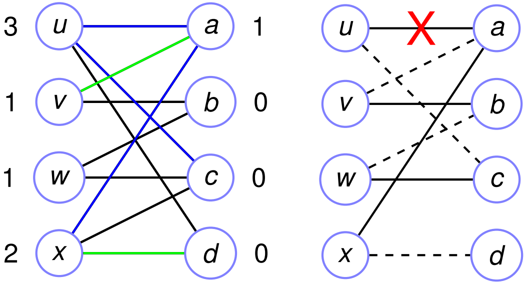
..and another augmentation...

Example



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Some more tight edges.
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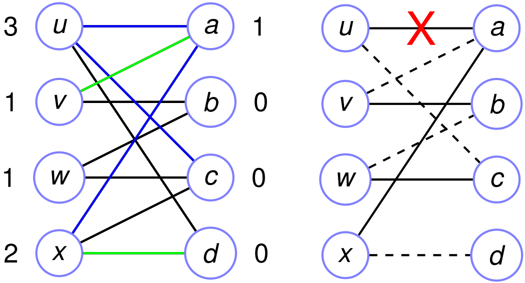
a “new” nontight edge.
..and another augmentation...

..and finally: a perfect matching.

Example



All matched edges tight.

Weight legend:
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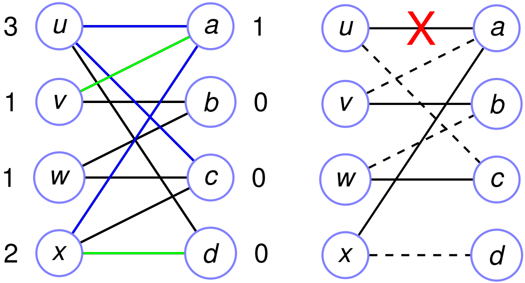

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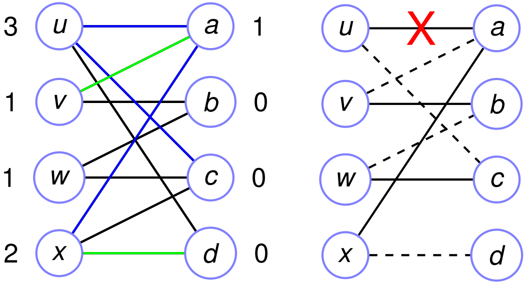
All matched edges tight.
Perfect matching.

Example



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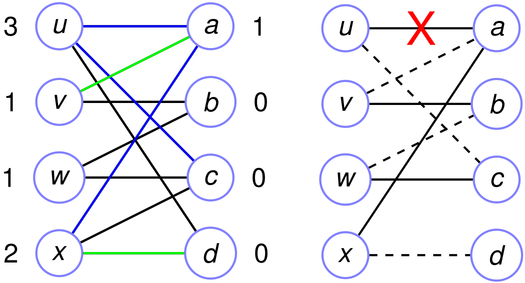
..and another augmentation...

..and finally: a perfect matching.

All matched edges tight.
Perfect matching. Feasible price function.

Example



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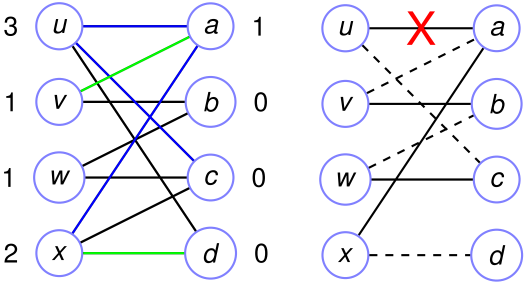
..and another augmentation...

..and finally: a perfect matching.

All matched edges tight.
Perfect matching. Feasible price function. Values the same.

Example



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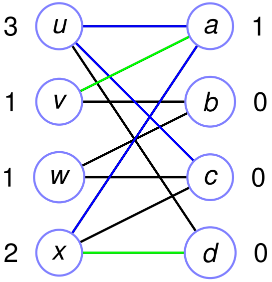
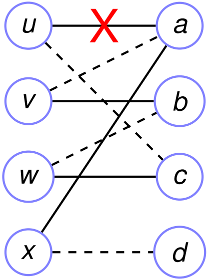
All matched edges tight.
Perfect matching. Feasible price function. Values the same. Optimal!

Example





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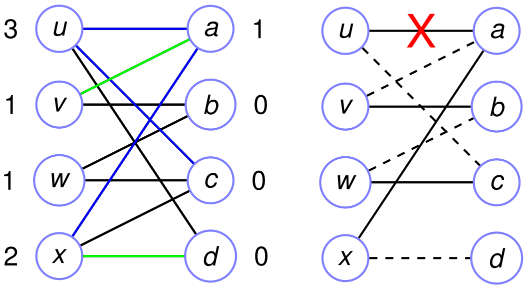
Notice:

Example



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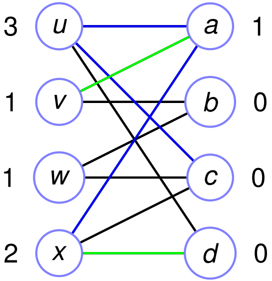
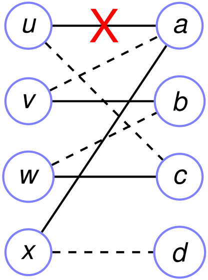
All matched edges tight.
Perfect matching. Feasible price function. Values the same. Optimal!

Notice:
no weights on the right problem.

Example





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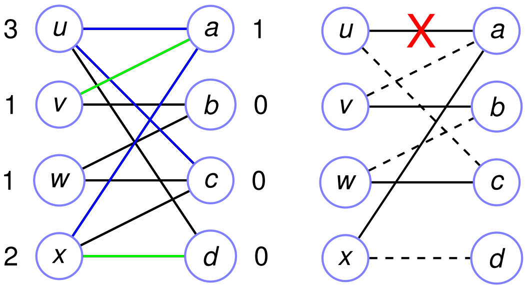
..and finally: a perfect matching.

All matched edges tight.
Perfect matching. Feasible price function. Values the same. Optimal!

Notice:
no weights on the right problem.
retain previous matching through price changes.

Example



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..and finally: a perfect matching.

All matched edges tight.
Perfect matching. Feasible price function. Values the same. Optimal!

Notice:
no weights on the right problem.
retain previous matching through price changes.
retains edges in failed search through price changes.

Some thoughts..

Unweighted matching algorithm to weighted.

Some thoughts..

Unweighted matching algorithm to weighted.

How?

Some thoughts..

Unweighted matching algorithm to weighted.

How?

Use duality.

Some thoughts..

Unweighted matching algorithm to weighted.

How?

Use duality.

In this case:

Some thoughts..

Unweighted matching algorithm to weighted.

How?

Use duality.

In this case:
  Dual feasible.

Some thoughts..

Unweighted matching algorithm to weighted.

How?

Use duality.

In this case:
  Dual feasible.
  Primal infeasible.

Some thoughts..

Unweighted matching algorithm to weighted.

How?

Use duality.

In this case:
  Dual feasible.
  Primal infeasible.
Primal only “plays” tight constraints.

Some thoughts..

Unweighted matching algorithm to weighted.

How?

Use duality.

In this case:
  Dual feasible.
  Primal infeasible.
Primal only “plays” tight constraints. Best offense.
Terminate when perfect matching.

Some thoughts..

Unweighted matching algorithm to weighted.

How?

Use duality.

In this case:
Dual feasible.
Primal infeasible.
Primal only “plays” tight constraints. Best offense.
Terminate when perfect matching.
$\rightarrow$ Dual only plays tight constraints.

Some thoughts..

Unweighted matching algorithm to weighted.

How?

Use duality.

In this case:
 Dual feasible.
 Primal infeasible.
Primal only "plays" tight constraints. Best offense.
Terminate when perfect matching.
  $\rightarrow$ Dual only plays tight constraints.
   Dual's best offense.

Some thoughts..

Unweighted matching algorithm to weighted.

How?

Use duality.

In this case:
  Dual feasible.
  Primal infeasible.
Primal only “plays” tight constraints. Best offense.
Terminate when perfect matching.
   $\rightarrow$ Dual only plays tight constraints.
      Dual’s best offense.

Some thoughts..

Unweighted matching algorithm to weighted.

How?

Use duality.

In this case:
  Dual feasible.
  Primal infeasible.
Primal only "plays" tight constraints. Best offense.
Terminate when perfect matching.
  $\rightarrow$ Dual only plays tight constraints.
    Dual's best offense.

Equilibrium.

...see you on Tuesday