The “Hallucination Bound” for the BSC

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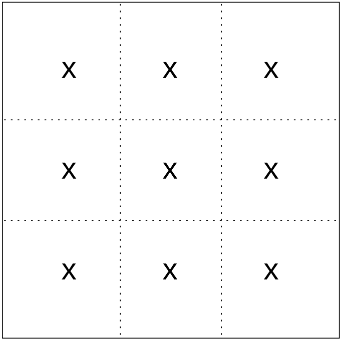
Outline

1. Motivation and review
2. Block-coding converse: minimum error probability
3. Streaming converse: bit error probability

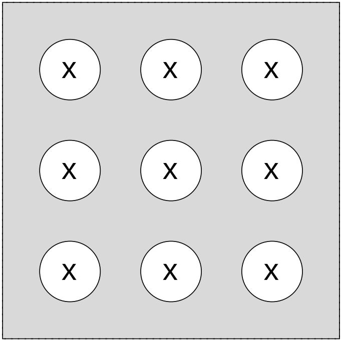
Review: Fixed blocks



- Hard decision regions cover space

- Feedback is pointless at high rates. (Dobrushin and Haroutunian)

Review: Fixed blocks: Forney-68



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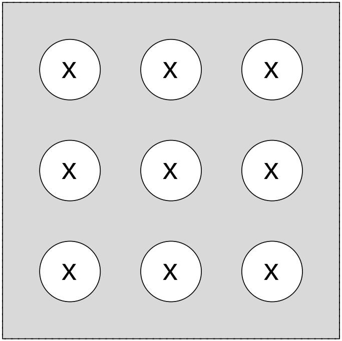

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- Can interpret as expected block-length

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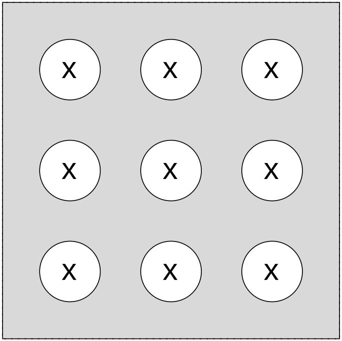
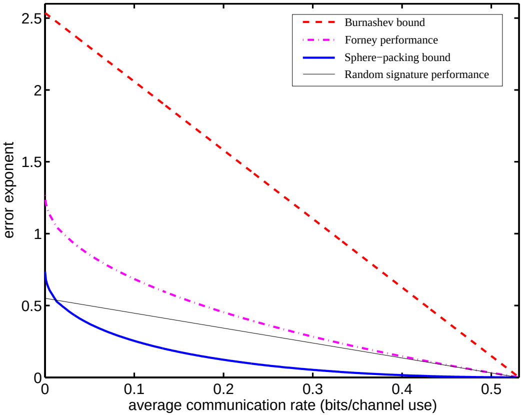


- Refuse to decide when ambiguous

- Decision regions catch the typical sets only
- 1 bit feedback can request retransmissions
- Can interpret as expected block-length
- No converse except at zero rate

Review: Fixed blocks, Soft deadlines: Burnashev-76



- Showed $C_1(1 - \frac{R}{C})$ was a bound where $C_1 = \max_{i,j} D(p_i||p_j)$
- Considered expected stopping time and used Martingale arguments.

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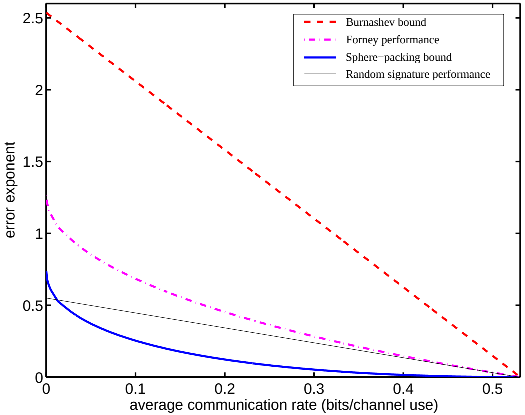
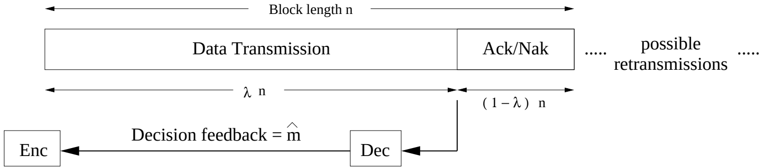
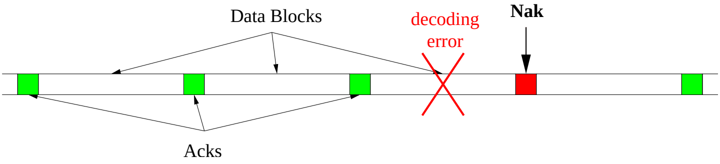


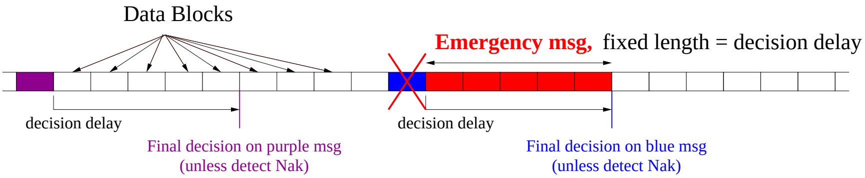


Streaming: an opportunity presents itself



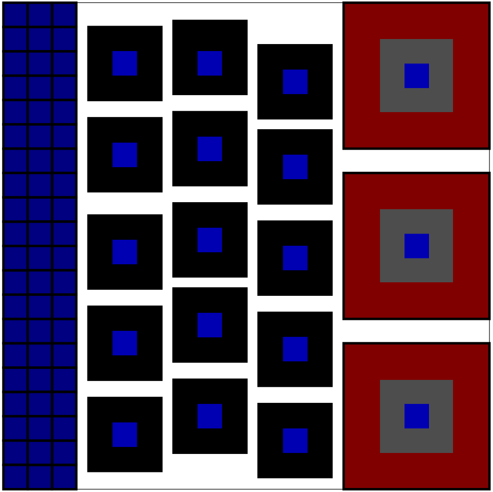
- What if we only sent NAKs when needed?

Sliding blocks with collective punishment only (Kudryashov-79)



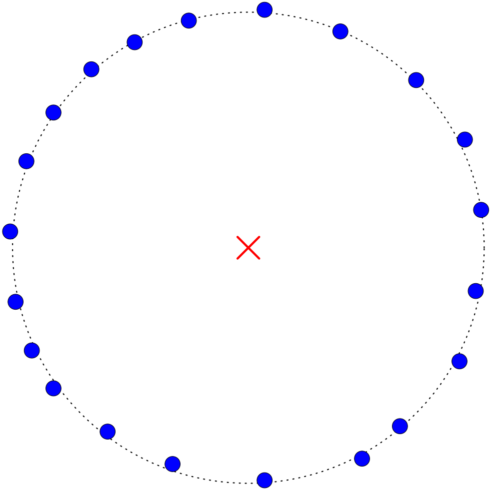
- Make packet length n much smaller than soft deadline Δ.
- A NAK collectively denies the past $\frac{\Delta}{n} - 1$ packets
- Error only if $\frac{\Delta}{n} - 1$ NAKs are all missed

Reason for hope: Csiszar's result '80



Can pack-in control messages at lower rates and give each subset their own random-coding bound!

Even simpler: need only one special message



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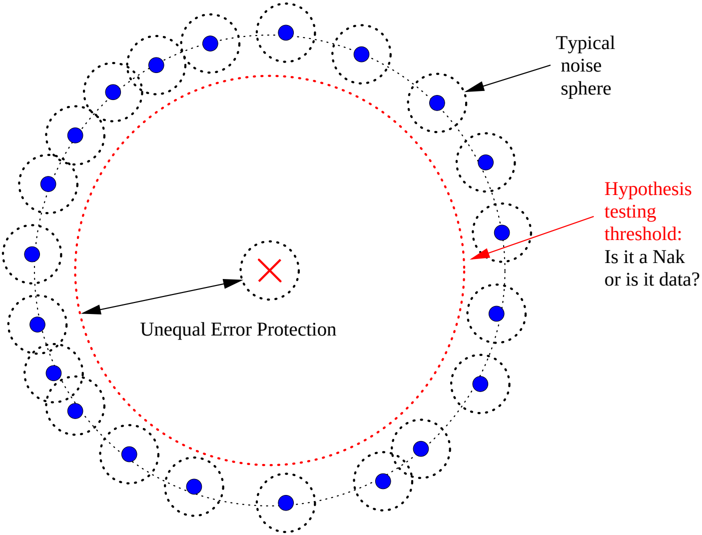
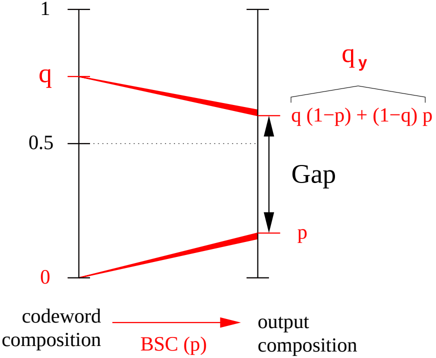


Specialize to BSC case



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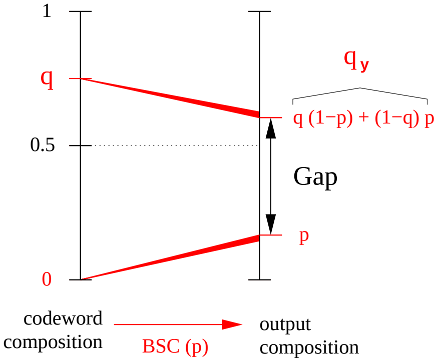

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  $R < H(q_y) - H(p)$

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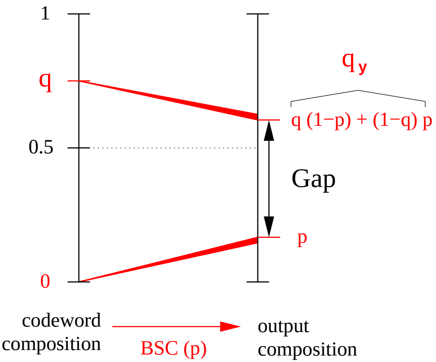
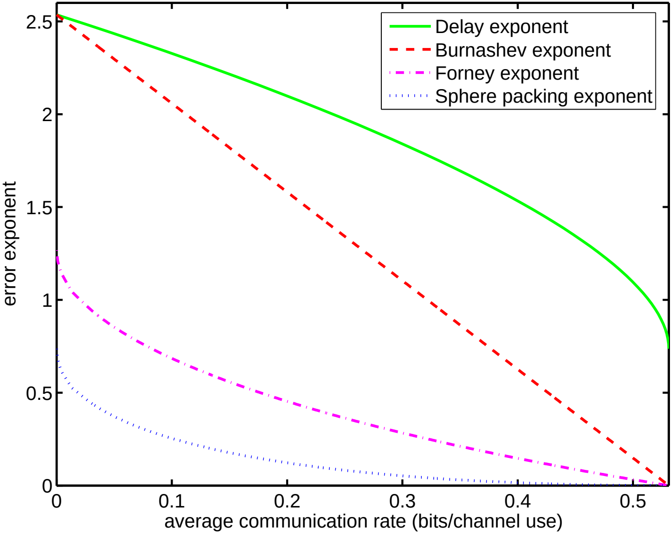


- Use all zero for NAK
- Use composition q code for data:
  $R < H(q_y) - H(p)$
- Probability of missed NAK is $2^{-nD(q_y||p)}$

Resulting exponents



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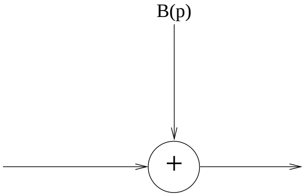

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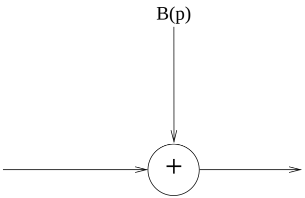

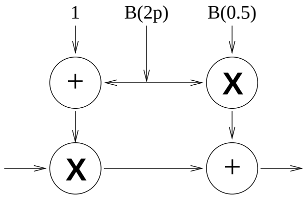

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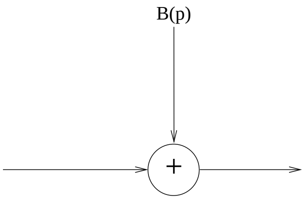

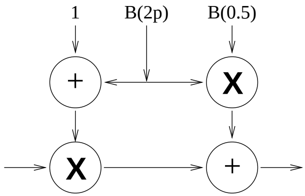

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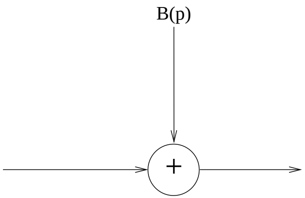

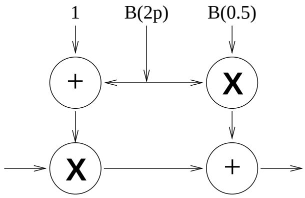

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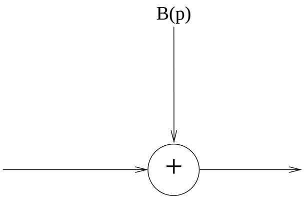

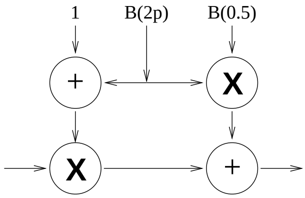

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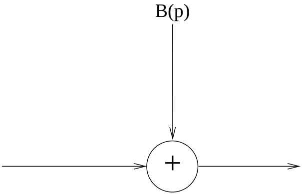

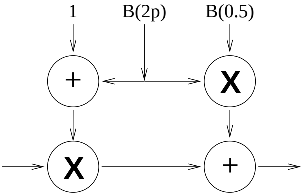

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- Exponent at most: $\log_2 \frac{1}{p} - R$

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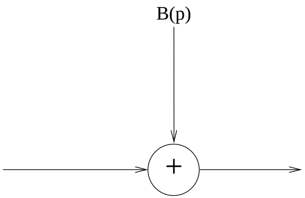

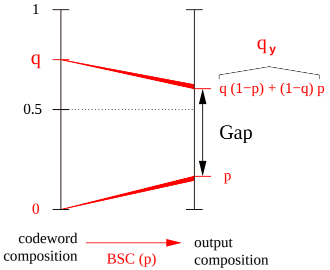

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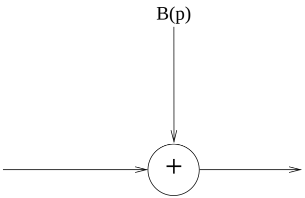

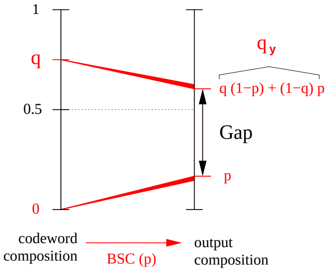

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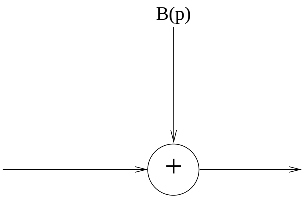

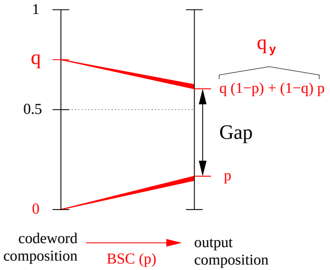

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- Matches achievability!

Generalizes to general symmetric channels

- Consider fixed block-length and moderate probability of correct decoding for “many” regular codewords.
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- On Friday Borade, *et al* will show a more general proof that holds with expected block-length.

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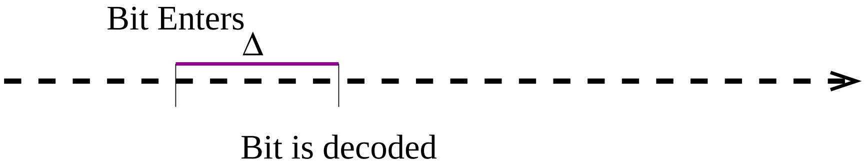
The basic intuition



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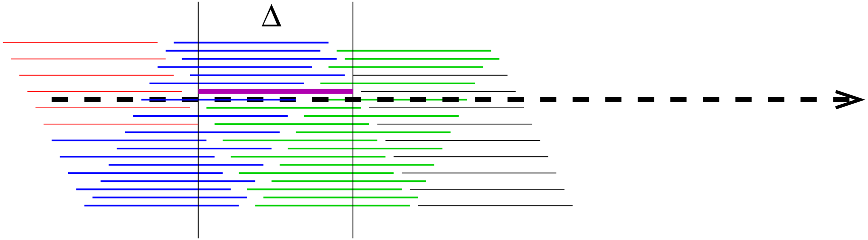

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- Challenge: overlaps with other bit-footprints.

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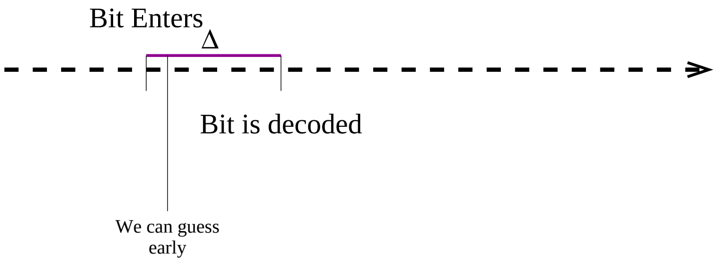

- View a block of many $n\Delta$.
- Count the volume of normal decoding regions in two different ways.
  - As a big block code based on correct decoding.
  - With a tree-structure based on the desired exponent.
- Technical condition: impose sequentiality on the code: we can usually guess the answer even before the deadline runs out.

Conclusions

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  - Eliminate the technical condition for the streaming case.
  - Get a two-way "Hallucination bound" for the case of noisy feedback.