Anytime communication over the Gilbert-Eliot channel with noiseless feedback

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Outline

1. Review of feedback and reliability
2. Problem: Gilbert-Eliot channel
3. No fading — Schalkwijk-Kailath revisited
4. Fading case
5. Concluding remarks

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  - *Kramer (69) and SDG (05): Separate efficiency and reliability. Detect errors well and use increased power to do a retransmission.*

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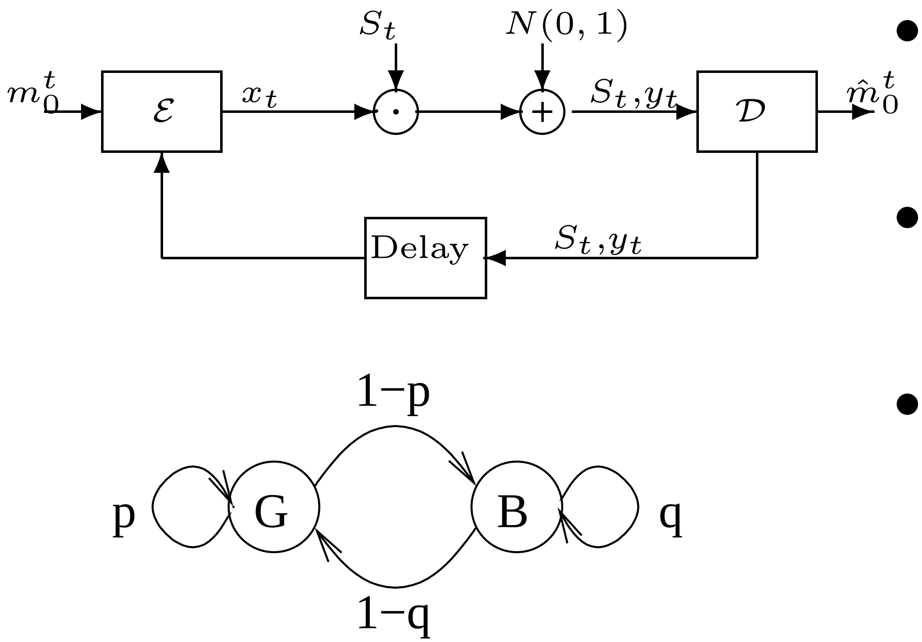
- DMC + fixed *delay* codes: large improvements possible.

  Key Lesson: use flow control to ameliorate channel atypicality.

Problem: Gilbert-Eliot channel with feedback



- First strategy: assume worst-case fades all the time
- "Naive" strategy: use more power in bad state
- Capacity-achieving strategy: use more power in good state

What causes an error?

- Two kinds of "atypicality"
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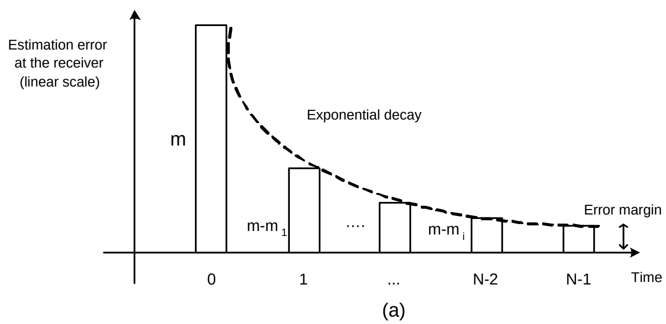
What causes an error and how to fix it

- Two kinds of "atypicality"
  - AWGN is atypically bad
  - Fading is atypically bad
- Can increase reliability using our "boosting" idea: *separate efficiency from reliability*
  - Operate near capacity in first part of block.
  - If net fading is atypically bad: increase power exponentially so as to guarantee reception in the rest of the block.
  - Meets average power constraint since probability of a bad net fade is exponentially small.
- But inelegant...

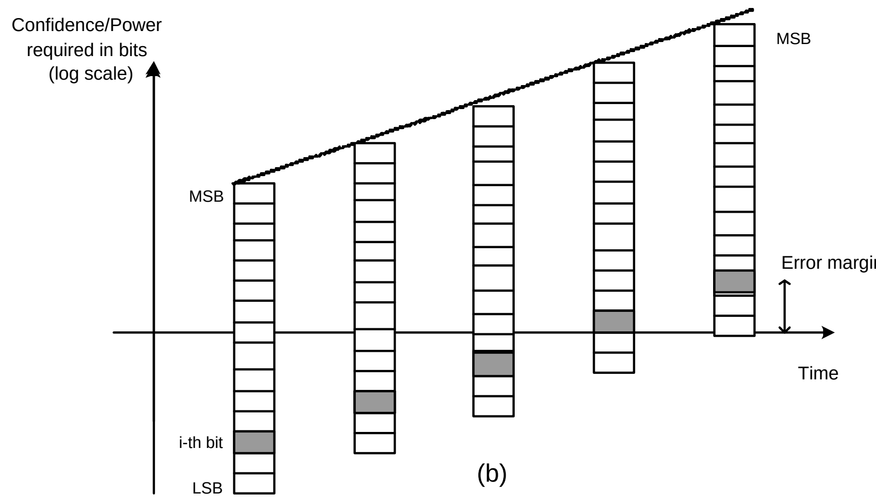
Revisit the no-fading case: Schalkwijk-Kailath

- Encode message bits into binary: $m = 0.B_1 B_2 B_3 \ldots B_{NR}$
- Transmit m, receiver gets $m + N_0$
- Gaussian uncertainty N_0. Transmit $\sqrt{P} N_0$. Receiver gets $\sqrt{P} N_0 + N_1$ and forms estimate $\hat{N}_0(1)$
- Calculate Gaussian MMSE error $e(1)$, scale up to power constraint, and transmit $\gamma e(1)$. Receiver updates to get MMSE estimate $\hat{N}_0(2)$
- Repeat till block is finished. Error variance drops exponentially.

Schalkwijk-Kailath visualized



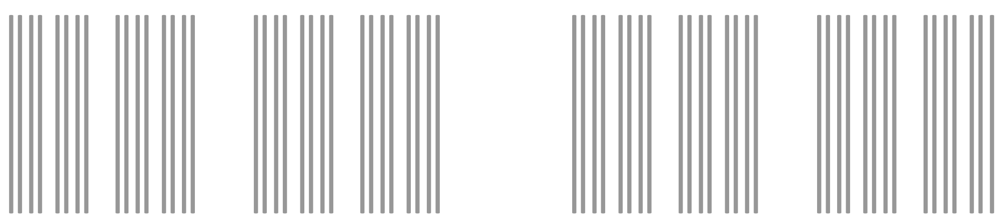
(a) Noise pumped out



(b) Log scale: linear confidence growth

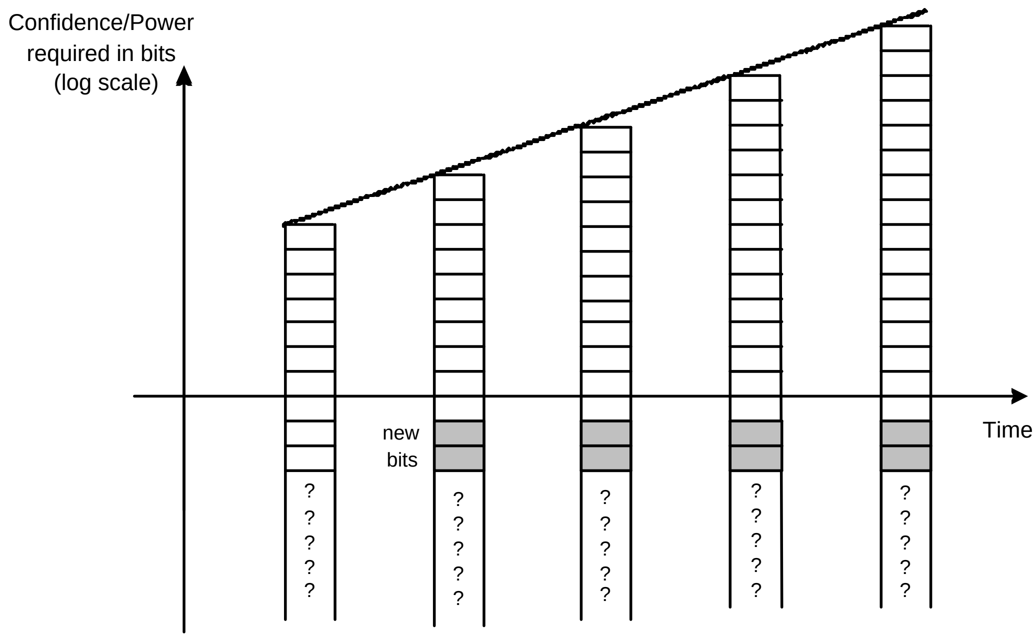
“Bitwise” Schalkwijk-Kailath

- Encode ∞ message bits: $m = 0.B_1 B_2 B_3 \ldots B_{i-1} B_i B_{i+1} \ldots$
- Suppose we have B_1^{i-1} correctly by time t.
  - Bit i is corrupted by $e(t) + \sum_{j=i+1}^{\infty} 2^{-j} B_j$
  - Gaussian error $e(t)$ has standard deviation like $O(2^{-Ct})$.
  - But “future” ISI can be arbitrarily close to 2^{-i} and dominates the probability of error.
- Bound “future” ISI away by giving up a small fraction of rate. Encode $m = \sum_{j=1}^{\infty} (2+\epsilon)^{-j} B_j$



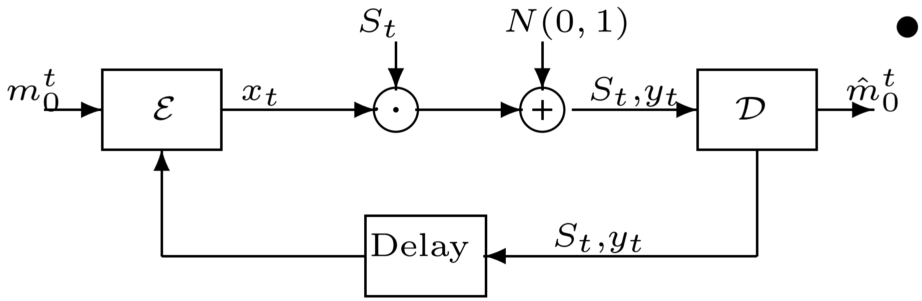
Sequentializing Schalkwijk-Kailath

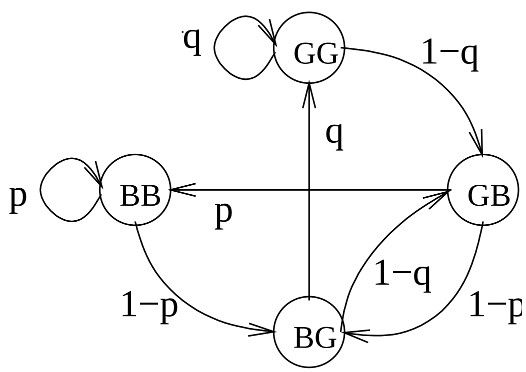
- Lazy initial message generation, but same receiver.
- Use a small amount of extra power to fill in bits by perturbing MMSE signal $e(t)$.



Can estimate bits at **any time**. *For any $R < C$, any bit i (arriving $\frac{i}{R}$), and all $t > \frac{i}{R}$, $P(\hat{B}_i(t) \neq B_i) \leq K e^{-c_1 e^{c_2 (t - \frac{i}{R})}}$.*

Apply these ideas to the Gilbert-Eliot channel

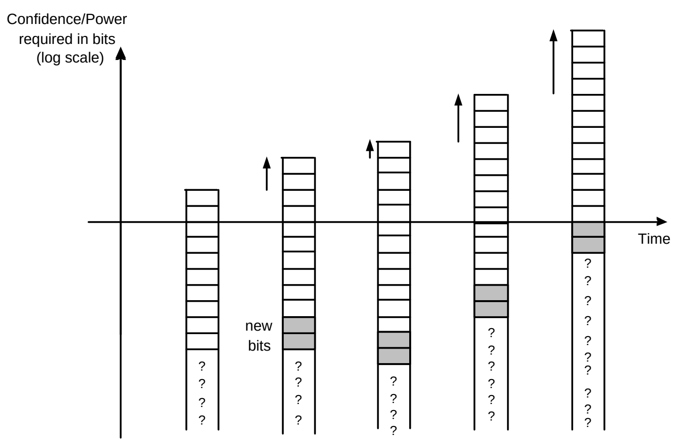


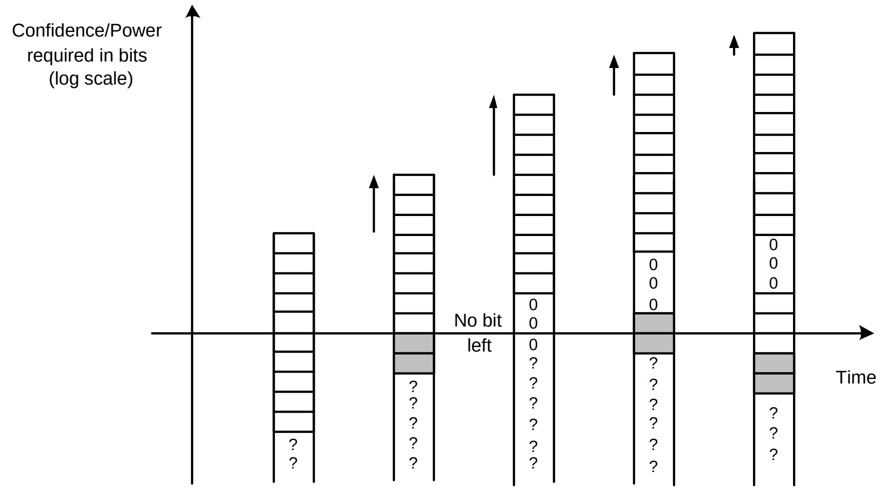


- Same initialization, but scale MMSE to meet the capacity-achieving power allocation for this state.
- MMSE factor reduction depends on current transition.
- Take logs — Markov-modulated random walk with net drift C.

No need to divide the message by states.

Sequentializing S-K for the Gilbert-Eliot channel





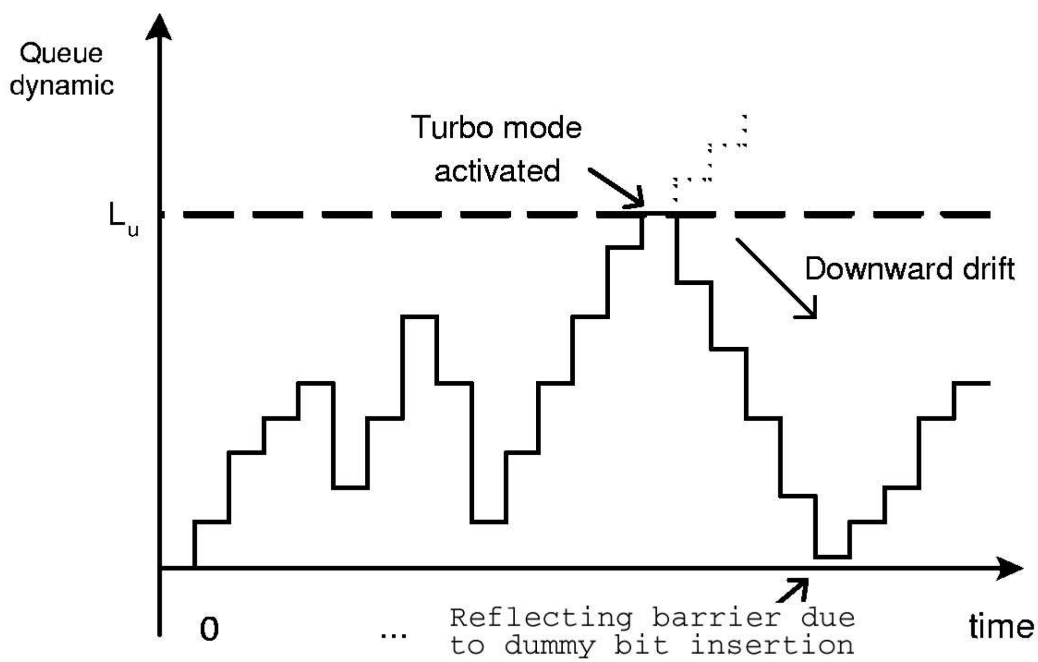
- Lazy initialization, but need to make sure perturbation terms are always small.
- Insert 0 dummy-bits in positions that are a function of the fading process.

What can go wrong?

- *Atypically Bad Gaussian Noise*: S-K style fights this implicitly — double-exponential.
- *Atypically Bad Fading*: desired bit is still underwater. Exponentially rare large-deviations type event.
  - Queue interpretation: too many bits still waiting in the send-queue.
  - Transmission power and fading state control service rate.
  - Under capacity-achieving power-control rule at "high" rates, queue has an exponential tail.

Clipping the tail of the queue

- If queue is too long, force sufficient service
  - Set power so that even the bad state can carry R bits.
  - Queue can not grow beyond this point.
- By adjusting how long is too long, keep expected power within average power constraint.



Conclusions

- For the Gilbert-Eliot channel with noiseless feedback and CSI, under an average power constraint, we can achieve a doubly-exponential drop in probability of error with fixed delay, *universally over all delays simultaneously.*
- Immediately extends to all Markov-fading channels without 0 states.
- For 0 states, only limited by the exponent corresponding to a long sequence of zero states.
- Can similarly sequentialize all S-K type schemes.
- Implications for automatic control over fading channels.