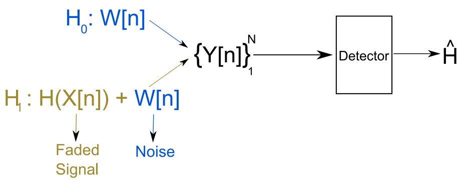
Outline:

Rahul Tandra, and Anant Sahai,

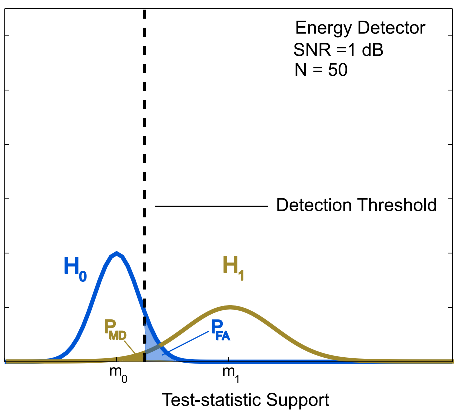
“Noise calibration, delay coherence and SNR walls for signal detection,”
IEEE DySpAN, Oct. 2008.

Paper available: http://www.eecs.berkeley.edu/∼sahai/Papers/DelayCoherenceDySpAN08.pdf
Slides available: . . . /∼sahai/Presentations/DelayCoherenceDySpAN08.pdf
This Handout: . . . /∼sahai/Presentations/DelayCoherenceDySpAN08.H.pdf
Further discussion, related work, and references can be found in the paper.

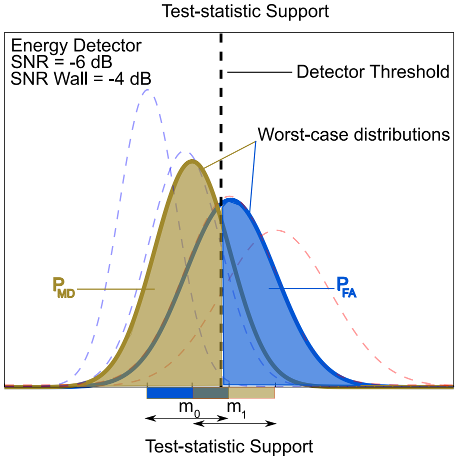
1 SNR Wall Review



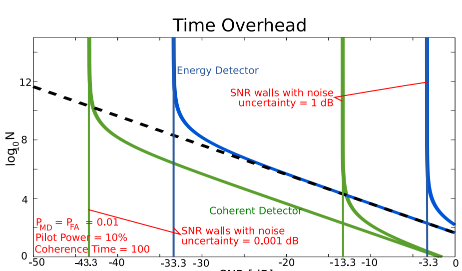
The basic problem of existence-based spectrum sensing is to discriminate between two hypotheses: that the possibly faded primary user signal is present at an adequately high power level or it is not there. The sensitivity of a detector is *the weakest signal it can reliably detect*.



The two hypotheses are usually distinguished by means of a test-statistic which is *an “averaged” quantity computed by combining many observations*. When a completely trustworthy ergodic probabilistic model exists for the fading and noise processes, then the probabilities of detection error will drop to zero with the number of observations as the test-statistic concentrates and a threshold can be set appropriately.

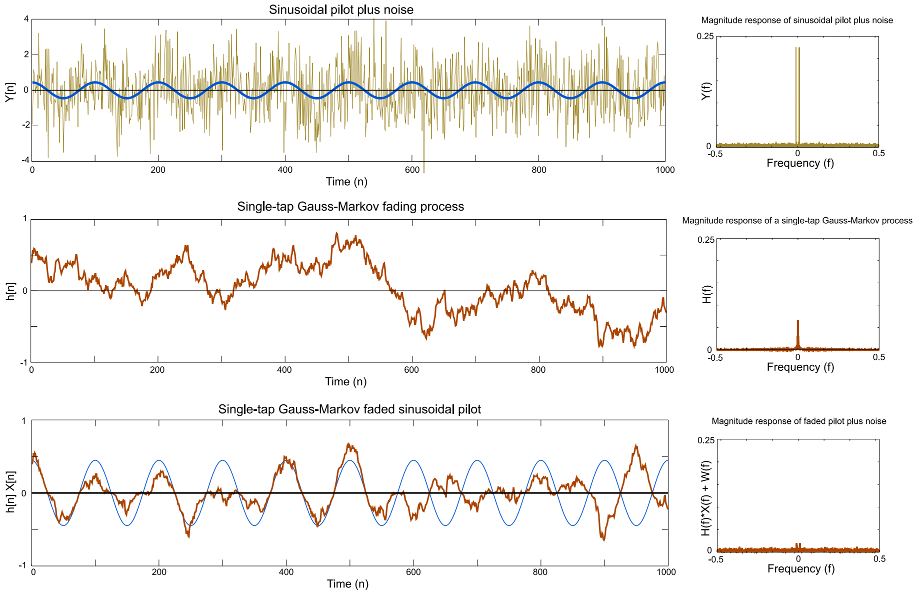


When the underlying distributions for noise and fading are uncertain, the concentration is not complete as the underlying uncertainty remains even with arbitrarily many observations. To robustly deal with this uncertainty, it is important to take the worst-case distributions when evaluating the different error probabilities. This leads to an SNR Wall which is *the SNR below which robust detection is impossible for the given detector.* The wall occurs where the means of the test-statistic begin to overlap under the two hypotheses.



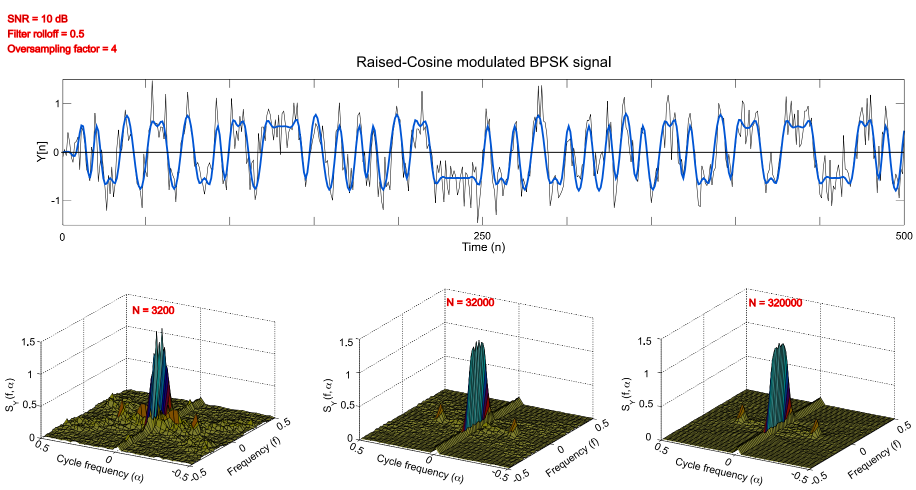
An alternative view of the SNR Wall can be seen at the system level by considering the sample complexity: *how many observations are required to meet target probabilities of error*. As the SNR approaches the SNR Wall, this sample complexity goes to infinity.

1.1 Pilot tones and coherent processing



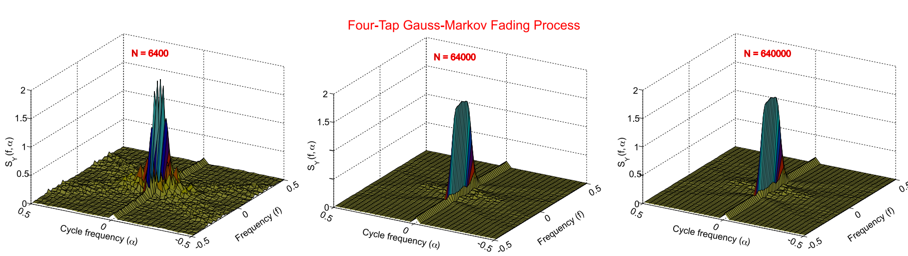
For the radiometer, the SNR Wall arises purely due to the uncertainty about the noise distribution. When considering a signal with a coherent feature like a pilot-tone, the potential time-varying fading process is also involved. The phase-variation in the fading process blurs the coherent feature. This limits the processing gain by blocking arbitrarily long coherent integration beyond the phase-coherence time which *is the length of time beyond which the fading can actually invert the phase from where it started*. The noise-level uncertainty then introduces an SNR Wall.

1.2 Cyclostationary features

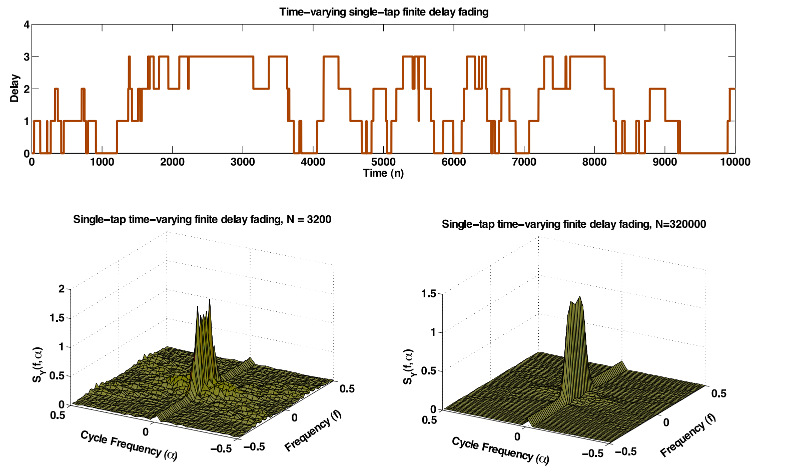


Many real-world signals exhibit other features beyond pilot tones. Cyclostationary features are those that *arise from the fact that the discrete-time information-carrying sequence is not modulated onto perfect sinc pulses and so the periodicity of the modulating pulse train is still present*. These structural redundancies can be revealed by oversampling the signal and then taking what is called the feature transform which is *a bank of correlations performed in the Fourier domain*. Here we see the features of the raised-cosine pulse. It turns out that a single fixed-tap channel filter does not destroy this feature no matter what the phase-coherence time turns out to be. Similarly, it is not corrupted by stationary noise.

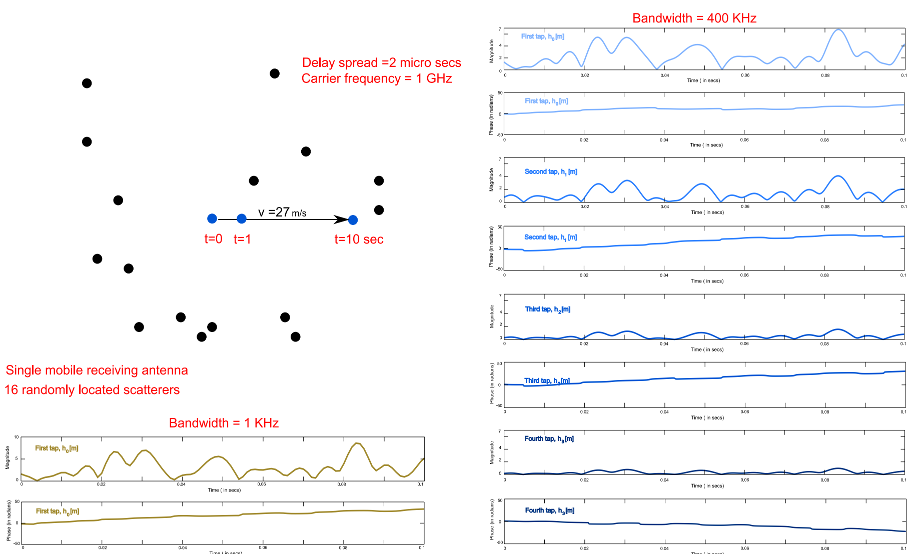
2 Delay-coherence time



To blur away the cyclostationary feature requires the potential delay-spread of the fading to be at least as large as the symbol periodicity. Here we see how multi-tap Gauss-Markov fading blurs away the feature. At one level this is not surprising because such time-varying fading can actually stationarize the received signal. At that point, the underlying noise uncertainty kicks in to impose an SNR Wall.

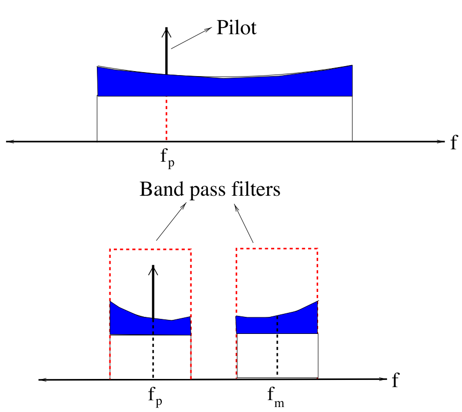


It turns out that even the simplest fading process with a finite delay spread is enough: a time-varying single tap delay. At one level this is not surprising because such time-varying fading can actually stationarize the received signal. At that point, the underlying noise uncertainty kicks in to impose an SNR Wall even if there is no ISI from the communication perspective.

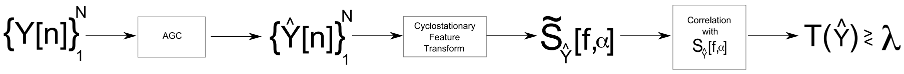


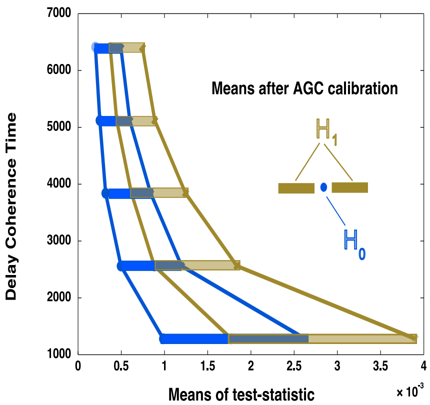
So what is the relevant coherence-time that limits cyclostationary processing gains? Physical intuition can be obtained by considering the scenario of a user moving through a cloud of scatterers. If the desired signal is narrowband, then all of the reflected paths will end up in the same tap. Although the phase-coherence time might be short because of the carrier frequency, the delay-coherence time could be very long. If the desired signal is wideband, then the reflected paths will end up in many different taps. At this point, both the phase-coherence time and the delay-coherence time will be the same.

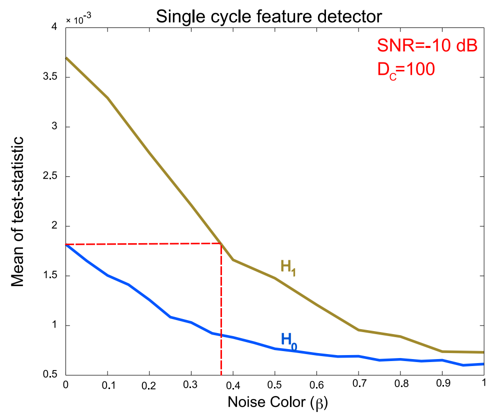
3 Noise Calibration



So far, the uncertainty in the noise-level has been the relevant uncertainty when it comes to the noise. However, one of the conceptual advantages of signal features is that they are in some places and not others. If the noise is considered to be basically white, then its level can be calibrated at runtime by looking at a place where the signal feature is not present. For pilot detectors, this is clear: look in a nearby band that does not contain the pilot. The post-calibration residual uncertainty comes from the color in the noise.







For cyclostationary detection, there is no particular time-segment or frequency-band that can be used to calibrate the noise-level. However, the average value of the received signal itself can serve this purpose. This calibration can be implemented simply as an automatic gain control. If the noise were guaranteed to be white, this would result in a clean separation of the two hypotheses. However, if the noise can be colored, there is still an SNR wall that comes from the uncertain color of the noise.

This talk is intended to bring out the following ideas:

- SNR Walls are the sensitivity limits to a particular detector for a particular class of signals. The uncertainties relevant to determining the position of the wall vary with the detector and signal family being detected.
- For cyclostationary signals, it is the delay-coherence time that is important in determining the limits to cyclostationary processing gain.
- For cyclostationary signals, noise-level calibration is quite simply achieved through the use of an AGC. This eliminates the fragility to pure noise-level uncertainty even if the cyclostationary processing gains are limited. However, an SNR Wall remains due to the uncertain noise color.