

$In(1, X_1)$ to denote the first input terminal for circuit X_1 . A similar function $Out(n, c)$ is used for output terminals. The predicate $Arity(c, i, j)$ asserts that circuit c has i input and j output terminals. The connectivity between gates can be represented by a predicate, $Connected$, which takes two terminals as arguments, as in $Connected(Out(1, X_1), In(1, X_2))$.

Finally, we need to know whether a signal is on or off. One possibility is to use a unary predicate, $On(t)$, which is true when the signal at a terminal is on. This makes it a little difficult, however, to pose questions such as “What are all the possible values of the signals at the output terminals of circuit C_1 ?” We therefore introduce as objects two signal values, 1 and 0, representing “on” and “off” respectively, and a function $Signal(t)$ that denotes the signal value for the terminal t .

Encode general knowledge of the domain

One sign that we have a good ontology is that we require only a few general rules, which can be stated clearly and concisely. These are all the axioms we will need:

1. If two terminals are connected, then they have the same signal:

$$\forall t_1, t_2 \text{ Terminal}(t_1) \wedge \text{Terminal}(t_2) \wedge \text{Connected}(t_1, t_2) \Rightarrow \text{Signal}(t_1) = \text{Signal}(t_2) .$$

2. The signal at every terminal is either 1 or 0:

$$\forall t \text{ Terminal}(t) \Rightarrow \text{Signal}(t) = 1 \vee \text{Signal}(t) = 0 .$$

3. $Connected$ is commutative:

$$\forall t_1, t_2 \text{ Connected}(t_1, t_2) \Leftrightarrow \text{Connected}(t_2, t_1) .$$

4. There are four types of gates:

$$\forall g \text{ Gate}(g) \wedge k = \text{Type}(g) \Rightarrow k = \text{AND} \vee k = \text{OR} \vee k = \text{XOR} \vee k = \text{NOT} .$$

5. An AND gate’s output is 0 if and only if any of its inputs is 0:

$$\forall g \text{ Gate}(g) \wedge \text{Type}(g) = \text{AND} \Rightarrow \text{Signal}(Out(1, g)) = 0 \Leftrightarrow \exists n \text{ Signal}(In(n, g)) = 0 .$$

6. An OR gate’s output is 1 if and only if any of its inputs is 1:

$$\forall g \text{ Gate}(g) \wedge \text{Type}(g) = \text{OR} \Rightarrow \text{Signal}(Out(1, g)) = 1 \Leftrightarrow \exists n \text{ Signal}(In(n, g)) = 1 .$$

7. An XOR gate’s output is 1 if and only if its inputs are different:

$$\forall g \text{ Gate}(g) \wedge \text{Type}(g) = \text{XOR} \Rightarrow \text{Signal}(Out(1, g)) = 1 \Leftrightarrow \text{Signal}(In(1, g)) \neq \text{Signal}(In(2, g)) .$$

8. A NOT gate’s output is different from its input:

$$\forall g \text{ Gate}(g) \wedge \text{Type}(g) = \text{NOT} \Rightarrow \text{Signal}(Out(1, g)) \neq \text{Signal}(In(1, g)) .$$

9. The gates (except for NOT) have two inputs and one output.

$$\begin{aligned} \forall g \text{ Gate}(g) \wedge \text{Type}(g) = \text{NOT} &\Rightarrow \text{Arity}(g, 1, 1) . \\ \forall g \text{ Gate}(g) \wedge k = \text{Type}(g) \wedge (k = \text{AND} \vee k = \text{OR} \vee k = \text{XOR}) &\Rightarrow \text{Arity}(g, 2, 1) \end{aligned}$$

10. A circuit has terminals, up to its input and output arity, and nothing beyond its arity:

$$\begin{aligned} \forall c, i, j \text{ Circuit}(c) \wedge \text{Arity}(c, i, j) &\Rightarrow \\ \forall n (n \leq i \Rightarrow \text{Terminal}(In(n, c))) \wedge (n > i \Rightarrow \text{In}(n, c) = \text{Nothing}) \wedge \\ \forall n (n \leq j \Rightarrow \text{Terminal}(Out(n, c))) \wedge (n > j \Rightarrow \text{Out}(n, c) = \text{Nothing}) \end{aligned}$$