

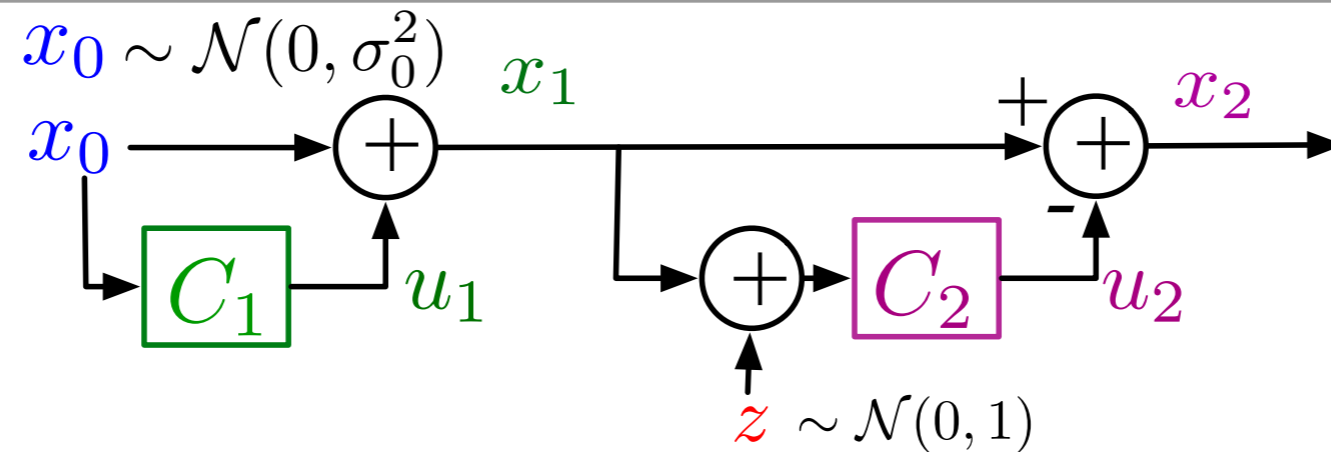
Is Witsenhausen's counterexample a relevant toy?

Pulkit Grover
UC Berkeley

with Anant Sahai

Acknowledgments: NSF grant CNS - 0932410 "Cyber-physical systems"

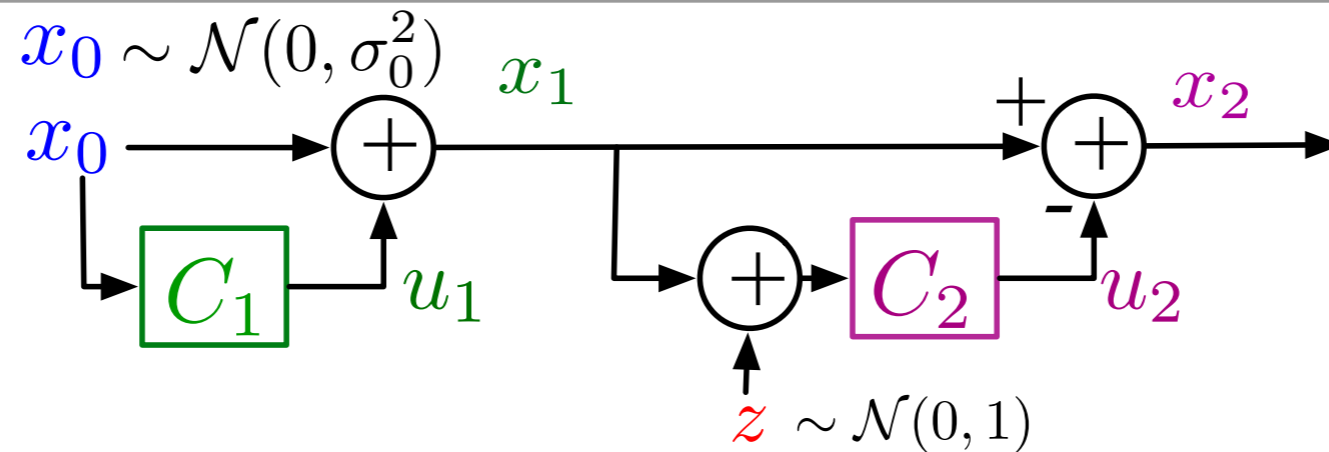
Witsenhausen's counterexample: a brief history



$$\min \{ k^2 \mathbb{E} [u_1^2] + \mathbb{E} [x_2^2] \}$$

Linear, Quadratic, Gaussian

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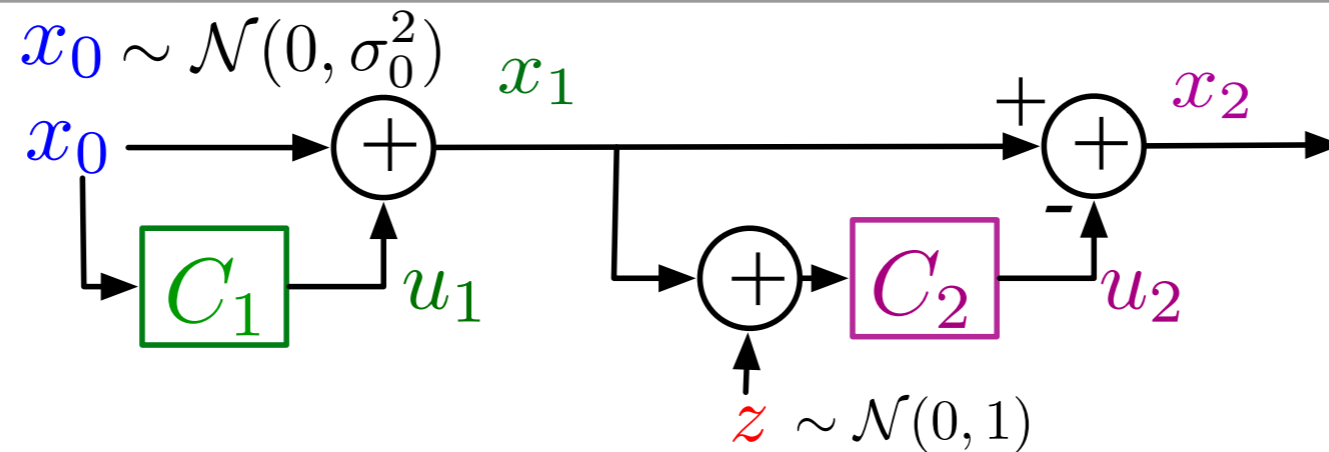


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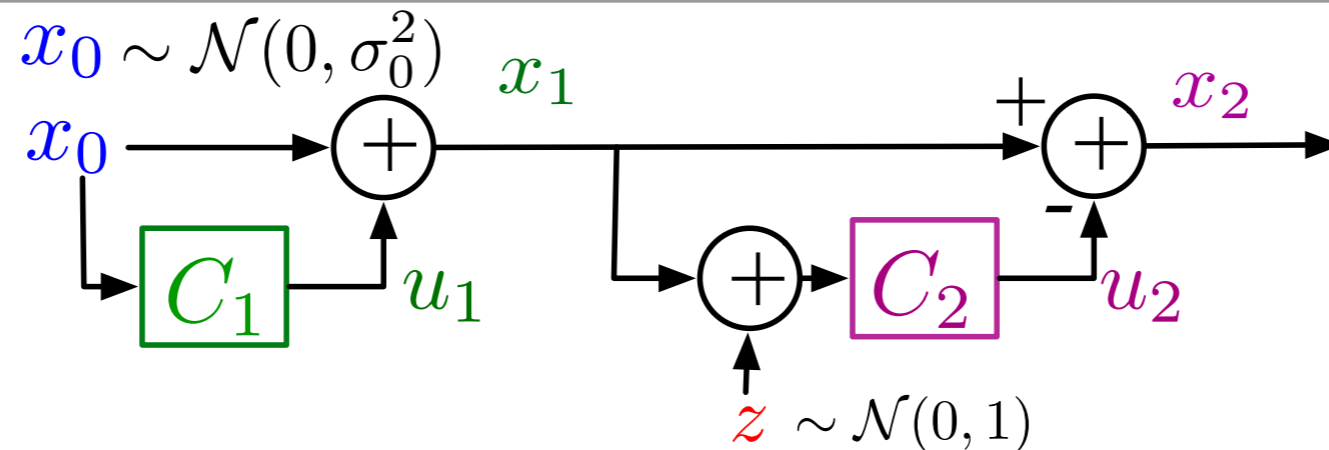
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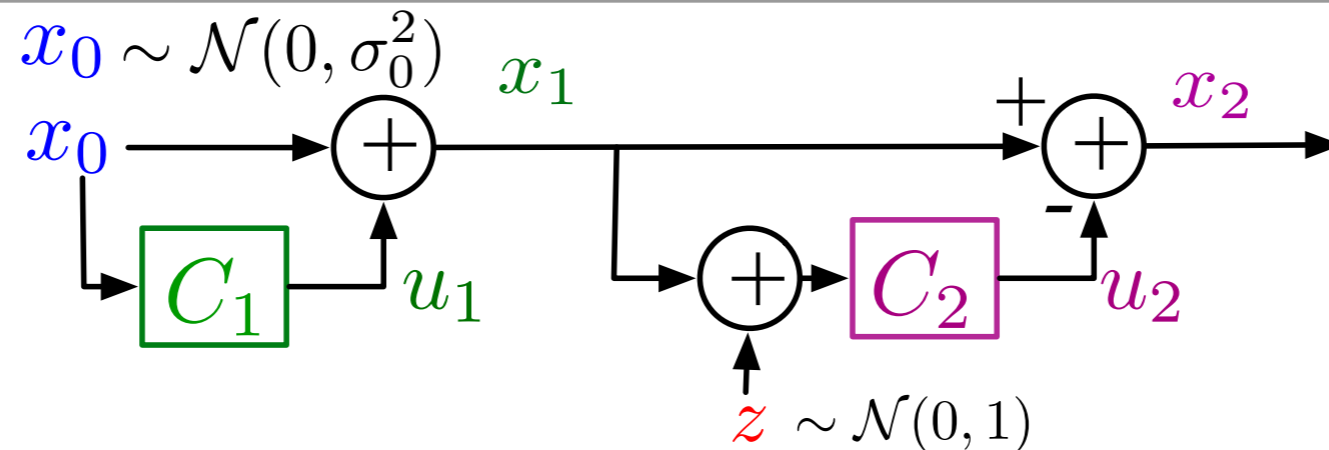
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Semi-exhaustive search techniques [Deng, Ho '99]

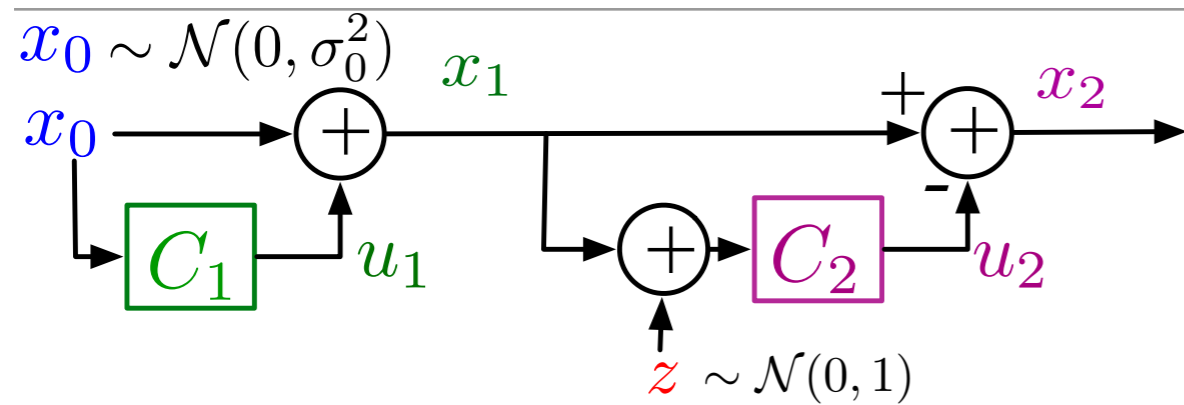
[Baglietto, Parisini, Zoppoli '01]

[Lee, Lau, Ho '01]

[Lee, Marden, Shamma '09]

[Karlsson, Gattami, Oechtering, Skoglund '10]

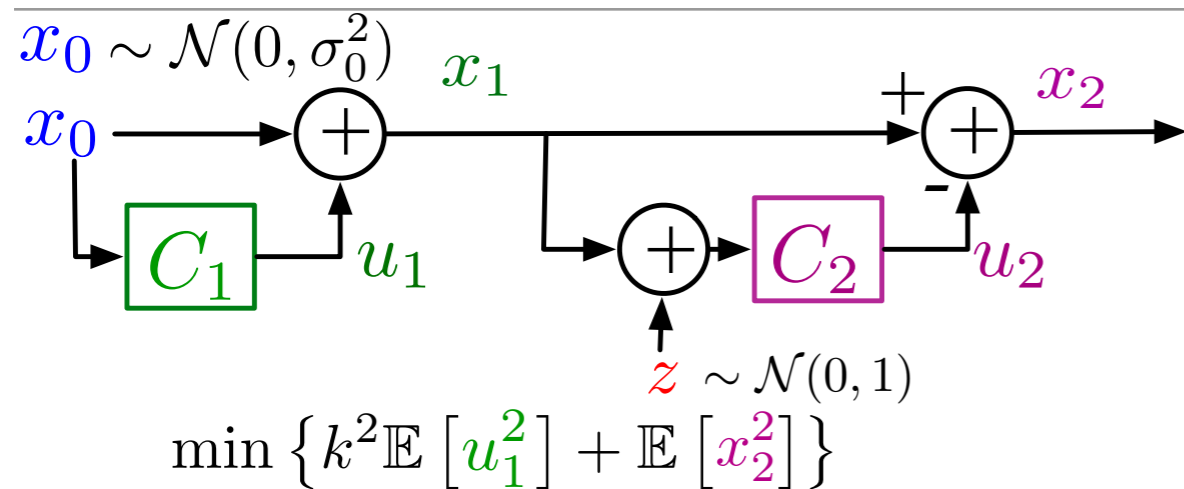
Witsenhausen's counterexample: a signaling perspective



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The counterexample

Witsenhausen's counterexample: a signaling perspective



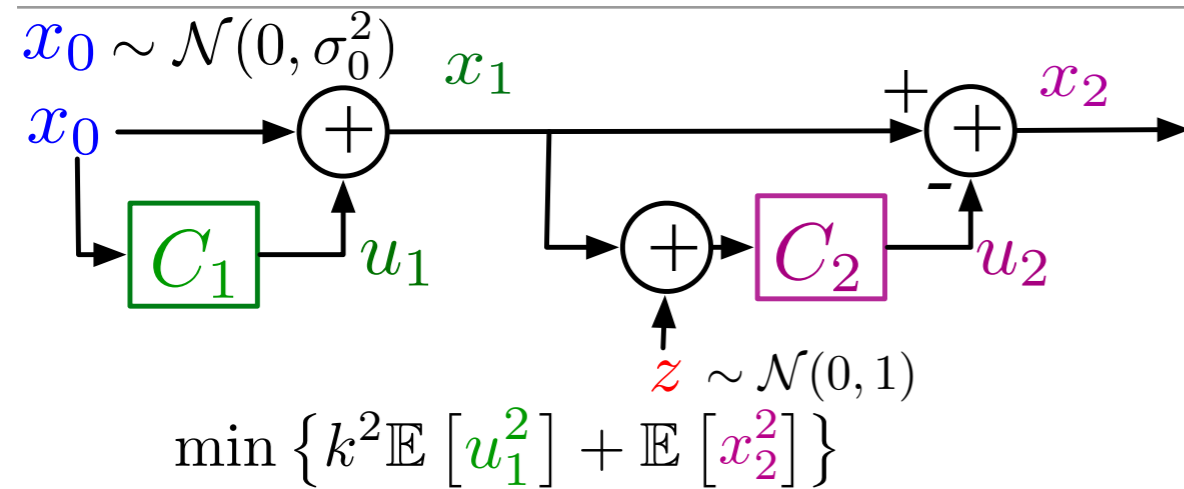
The counterexample

possibility of using **actions** to “signal”

[Ho, Kastner, Wong '78]

[Bansal, Başar '87]

Witsenhausen's counterexample: a signaling perspective

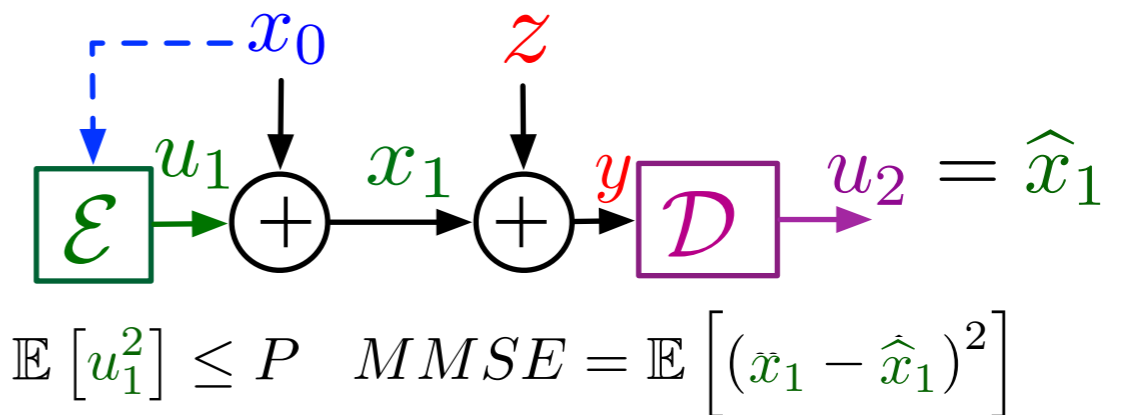


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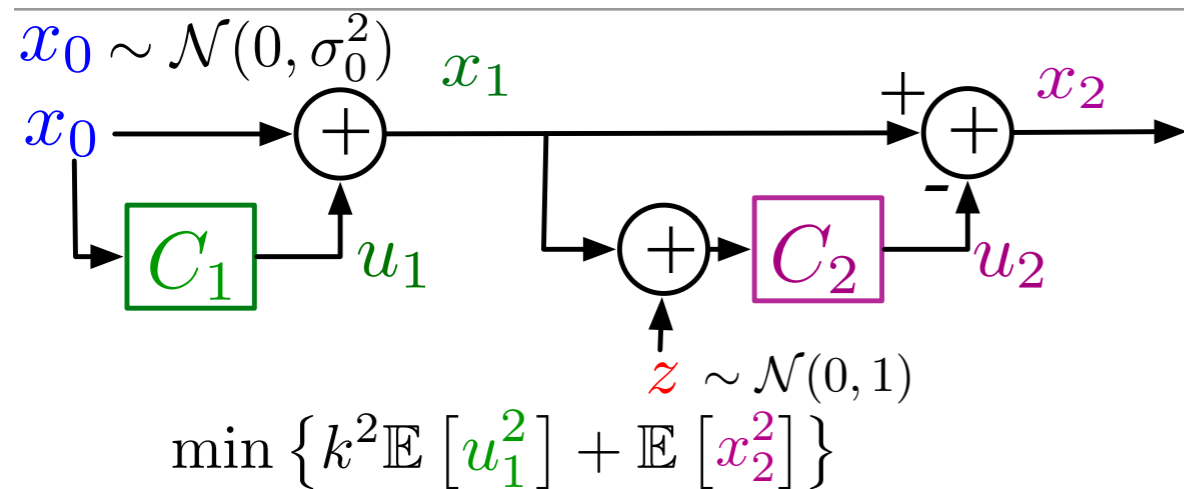
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An information-theoretic interpretation

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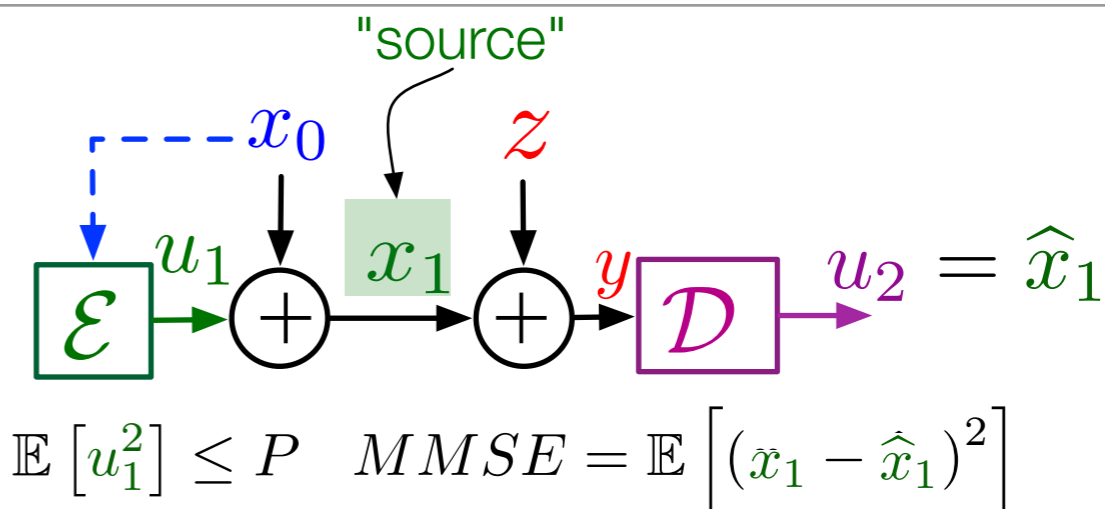


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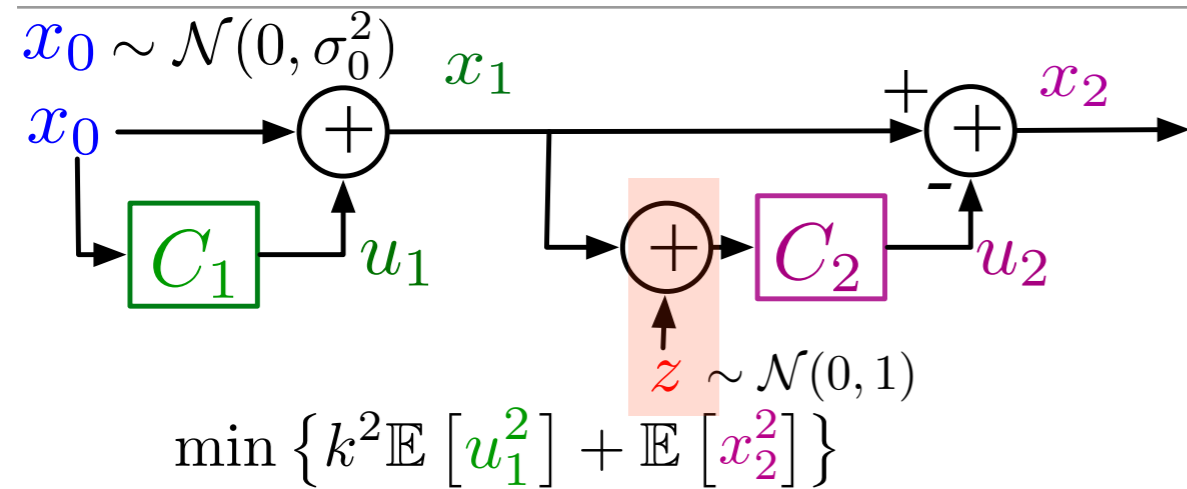
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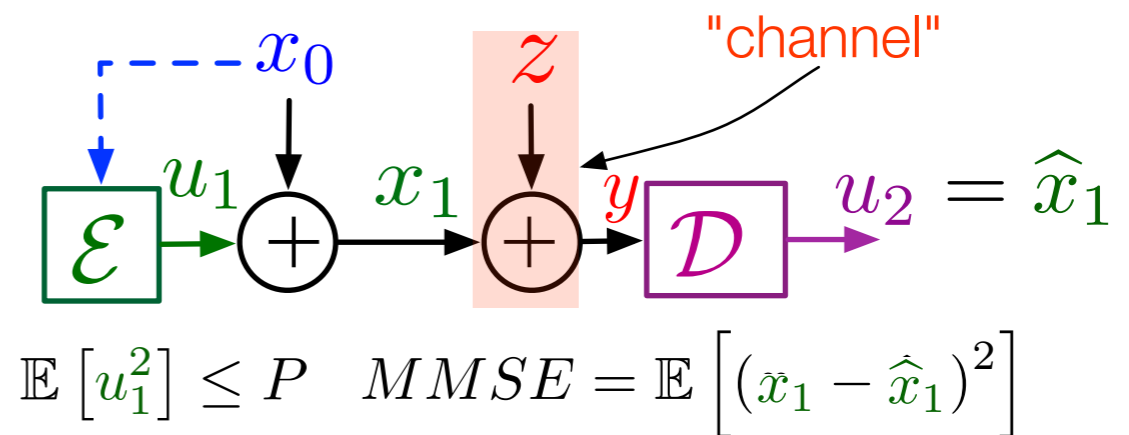


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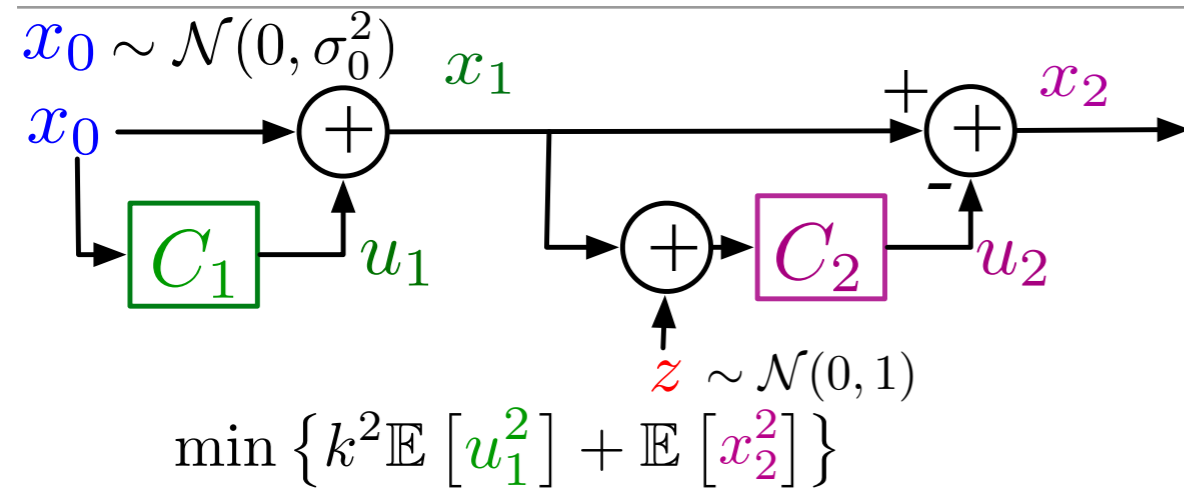
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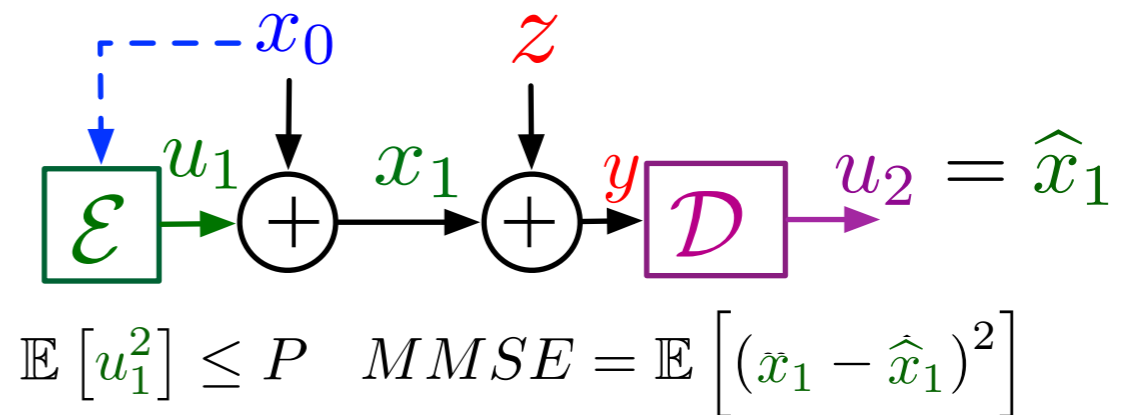


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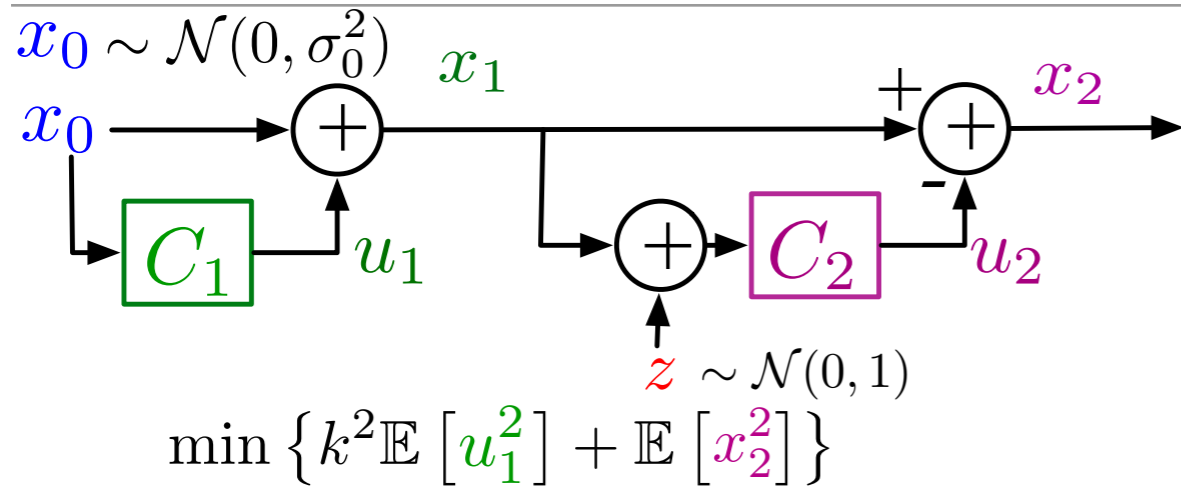
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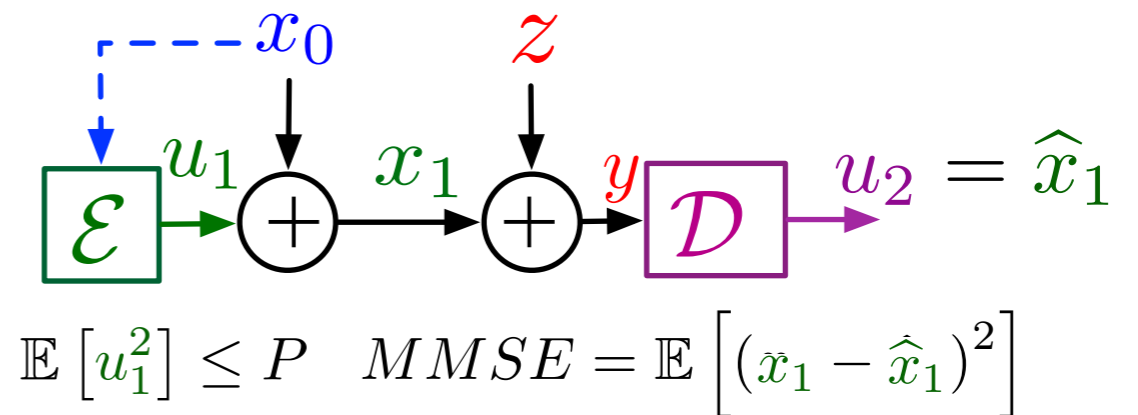


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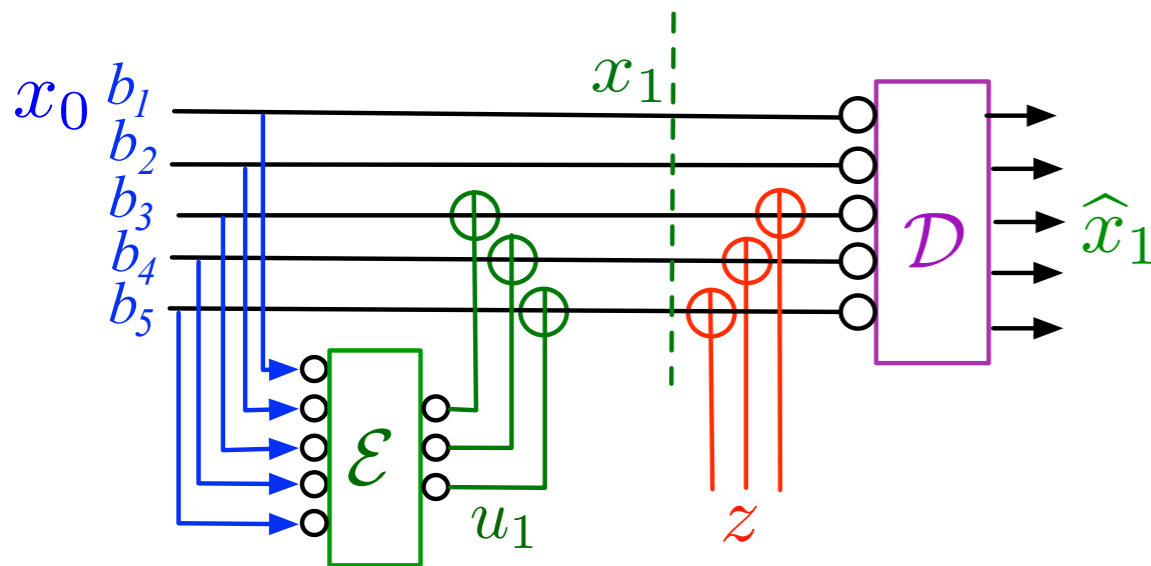
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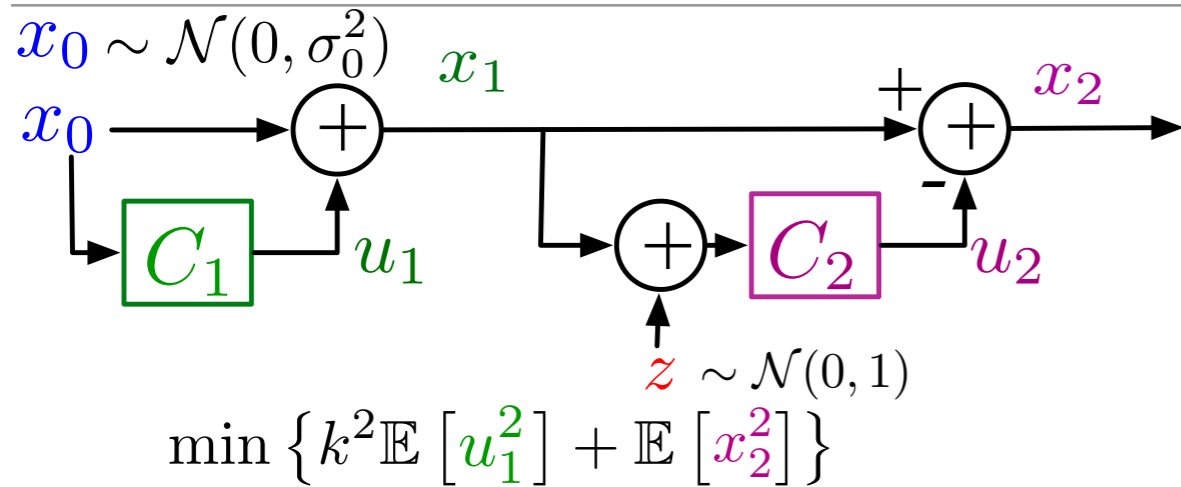


An information-theoretic interpretation



A semi-deterministic model

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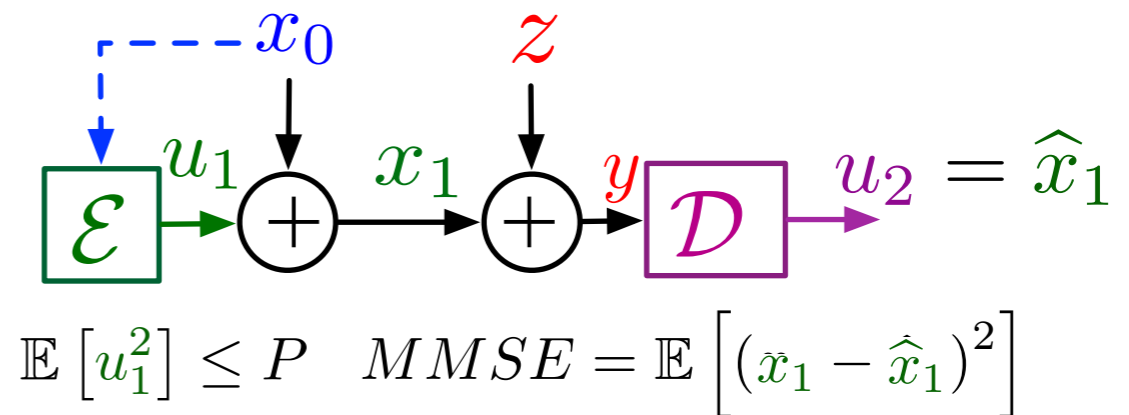


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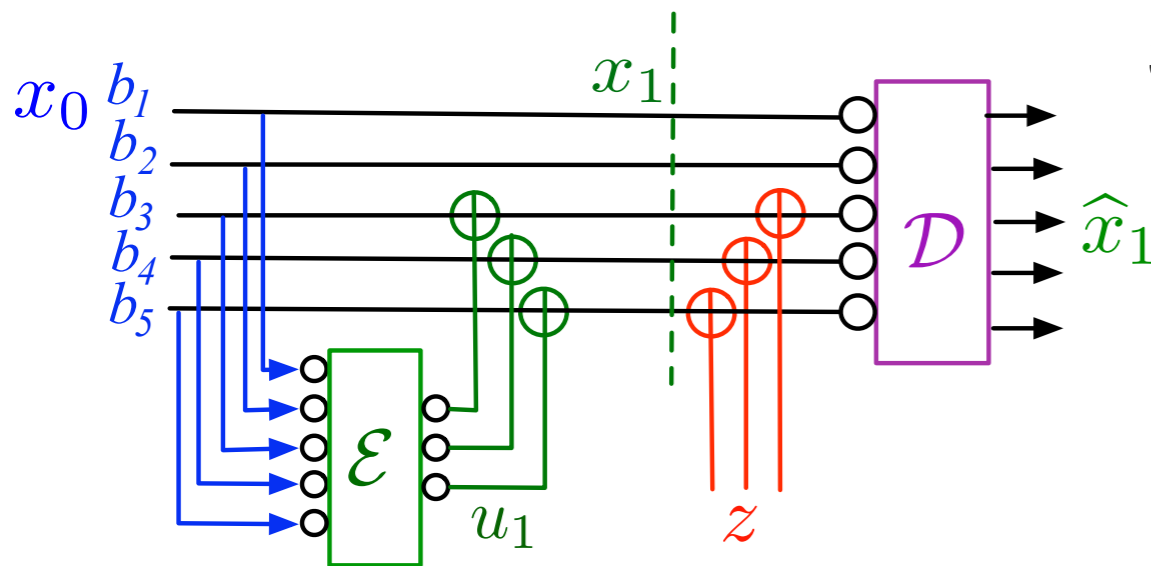
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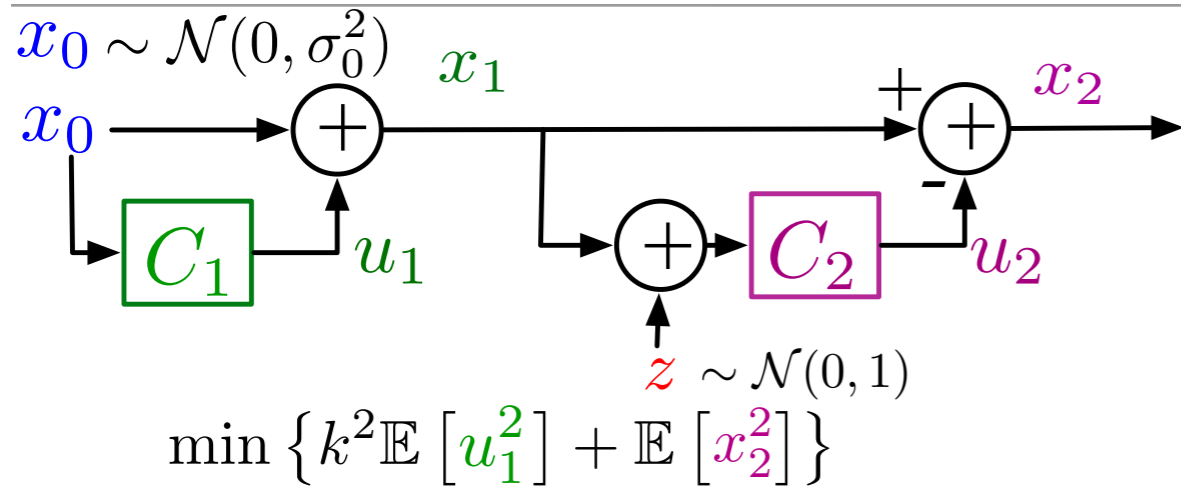
An information-theoretic interpretation



A semi-deterministic model

Strategies for “**signaling**”: quantization

Witsenhausen's counterexample: a signaling perspective

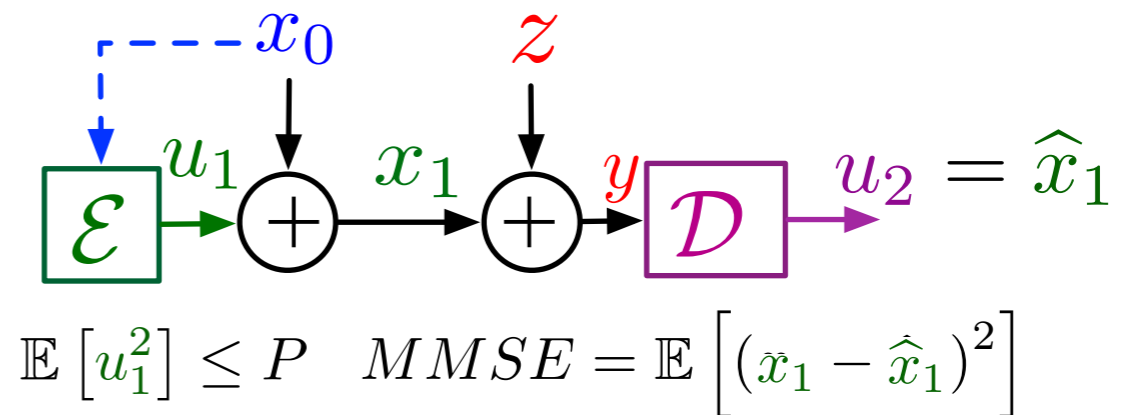


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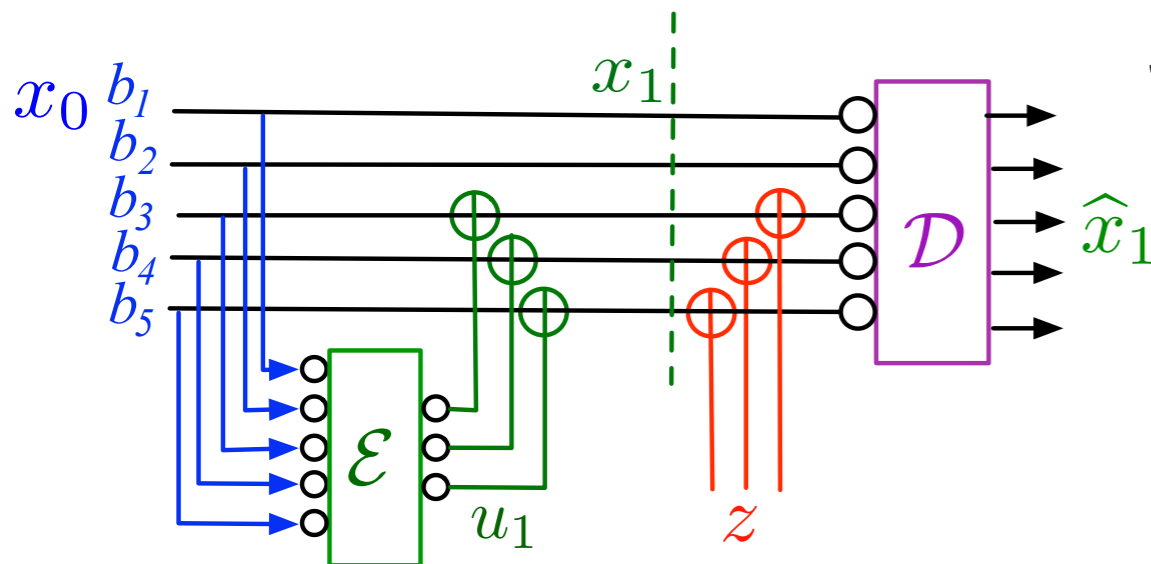
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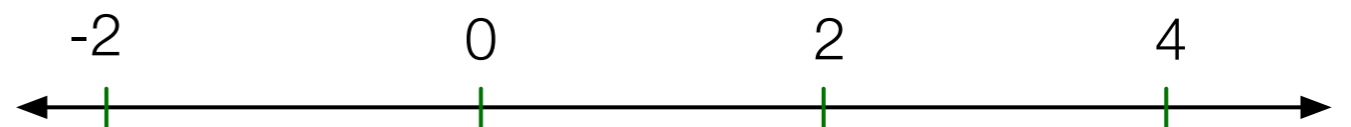


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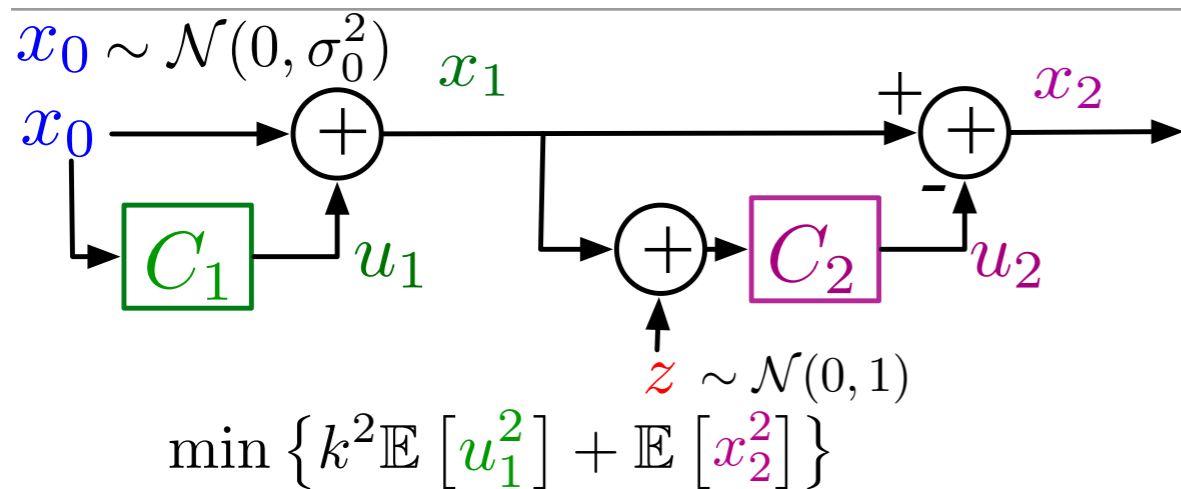
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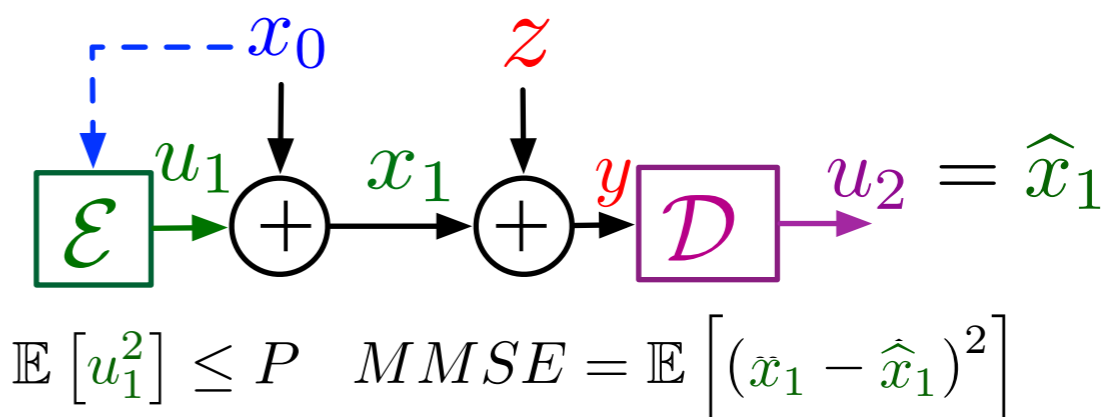


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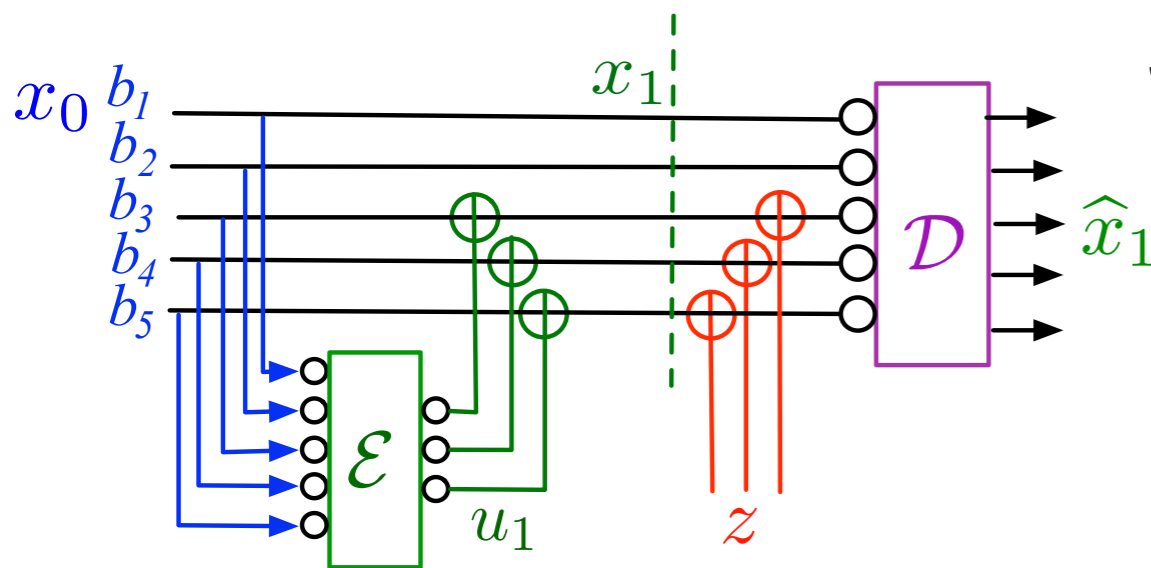
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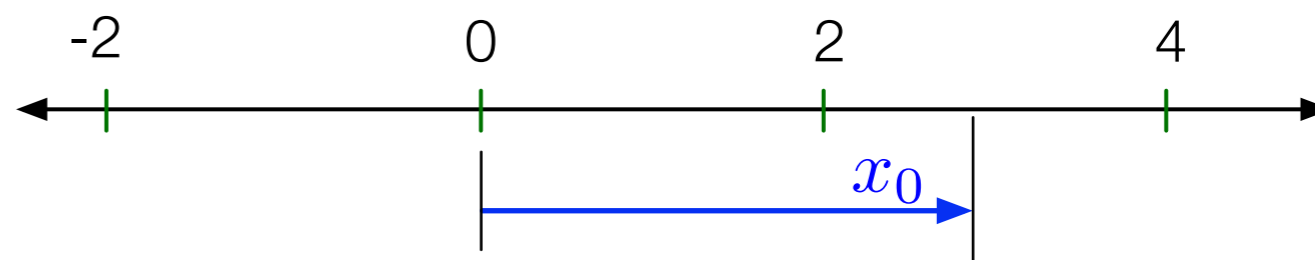


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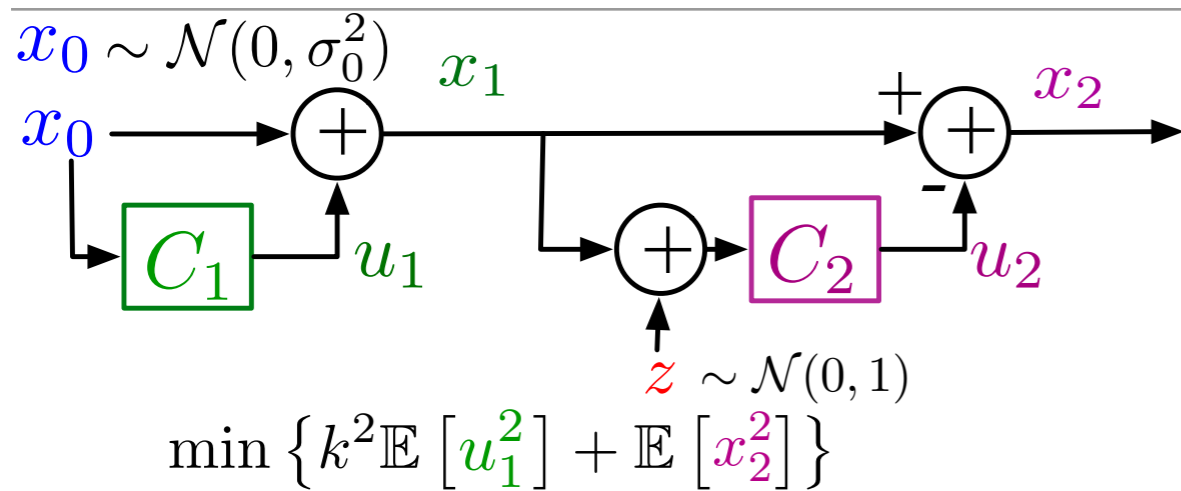
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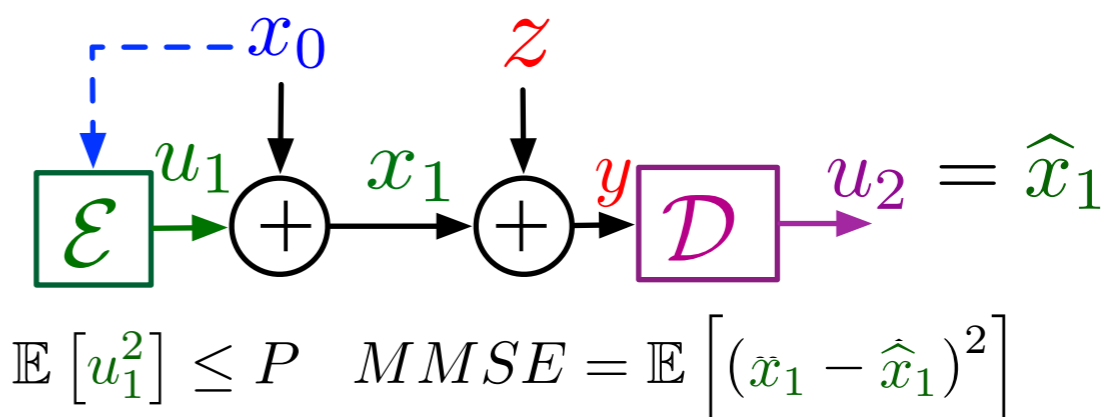


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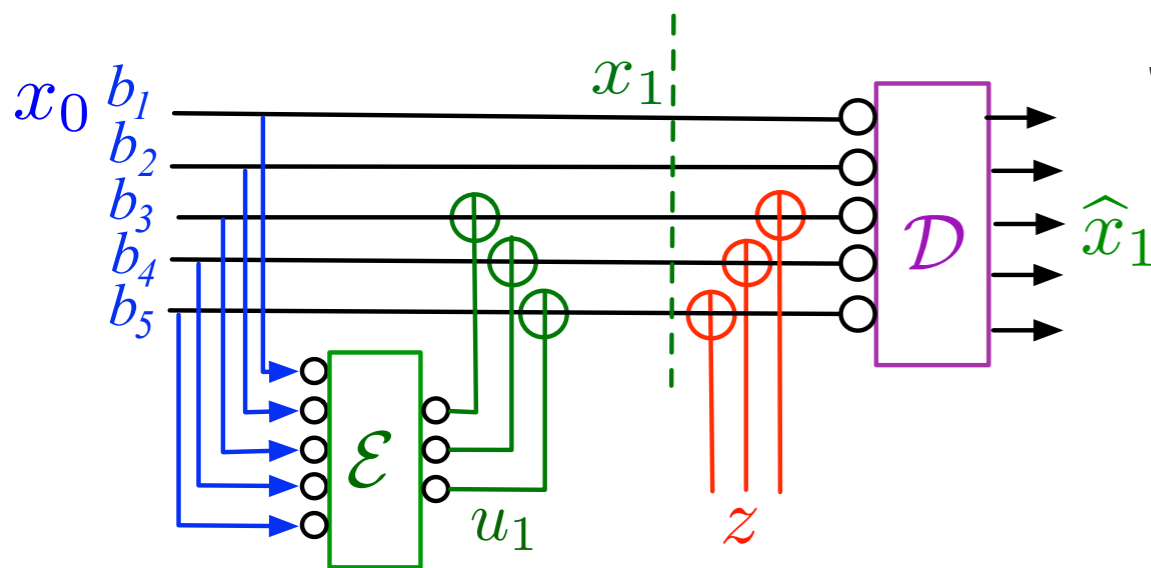
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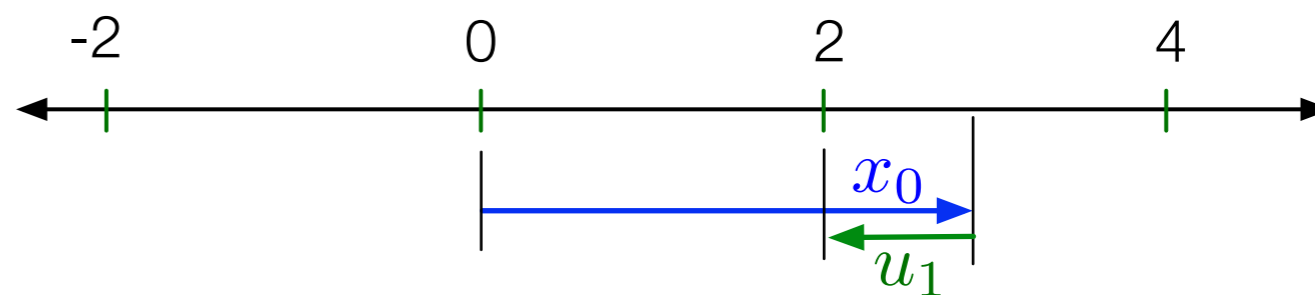


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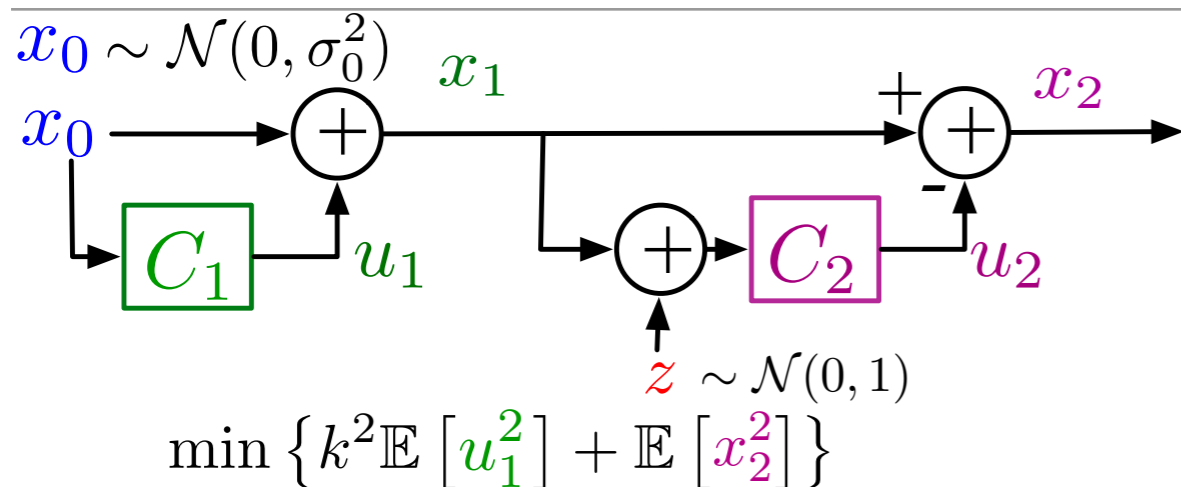
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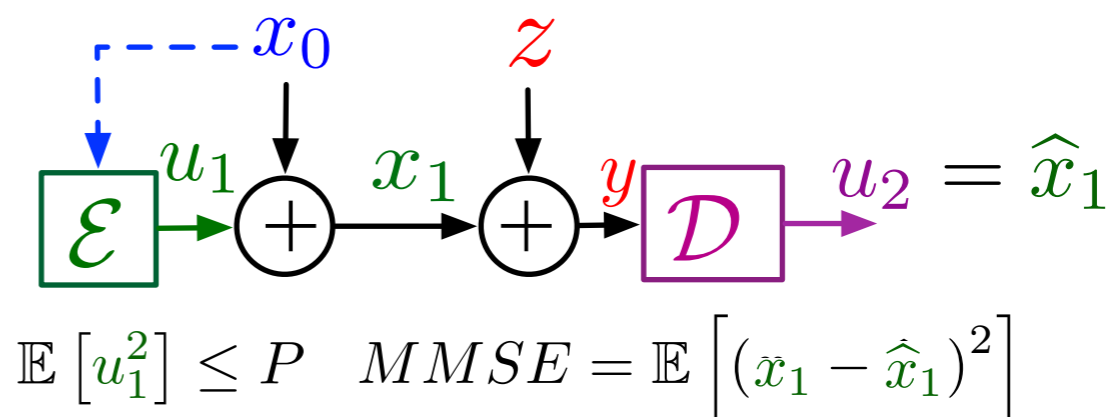


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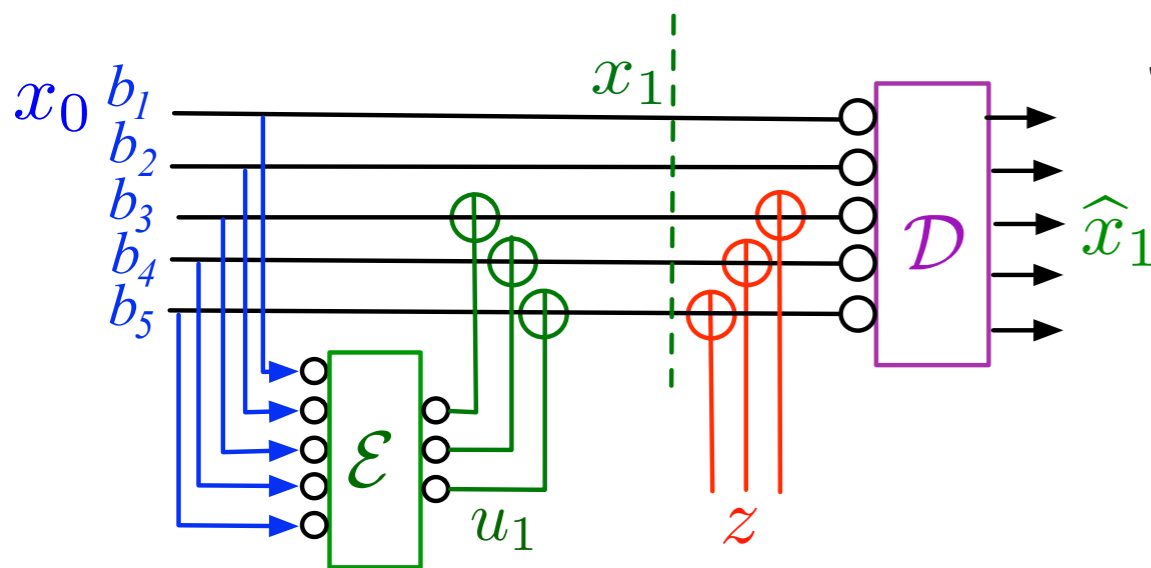
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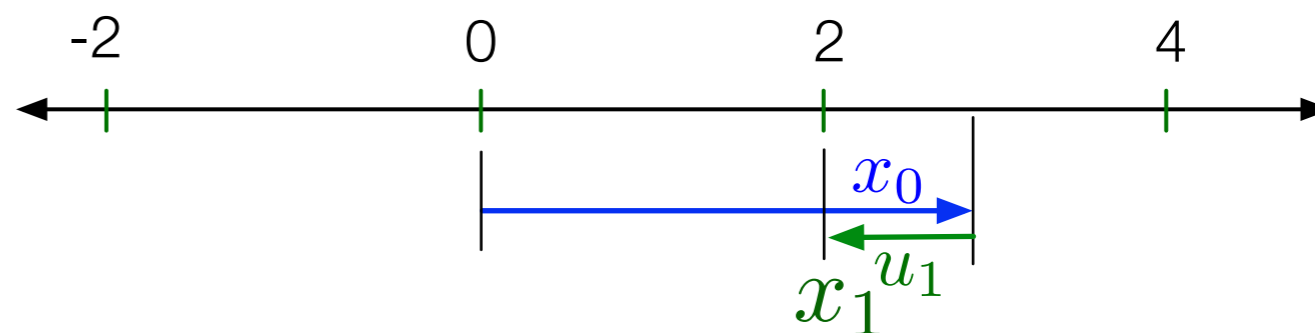


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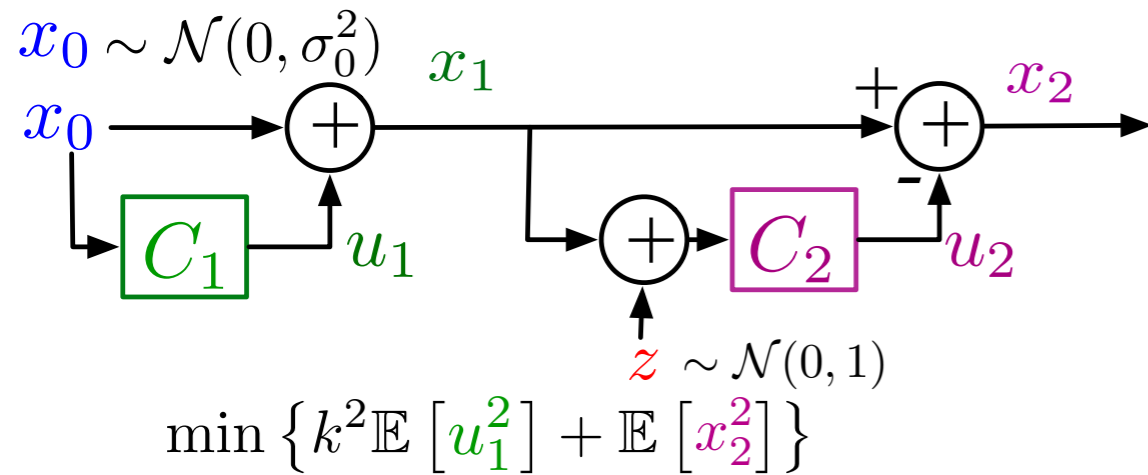
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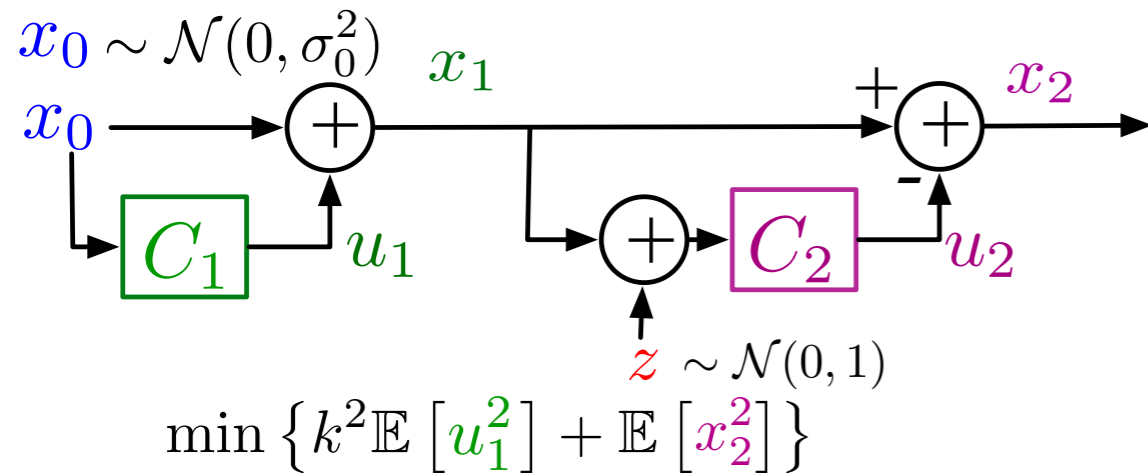
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Approximately-optimal solutions to Witsenhausen's counterexample

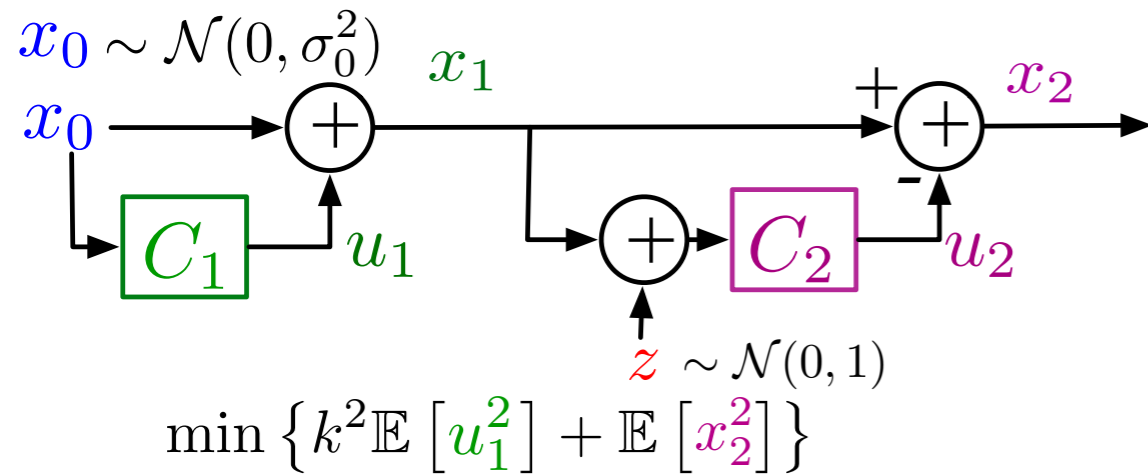


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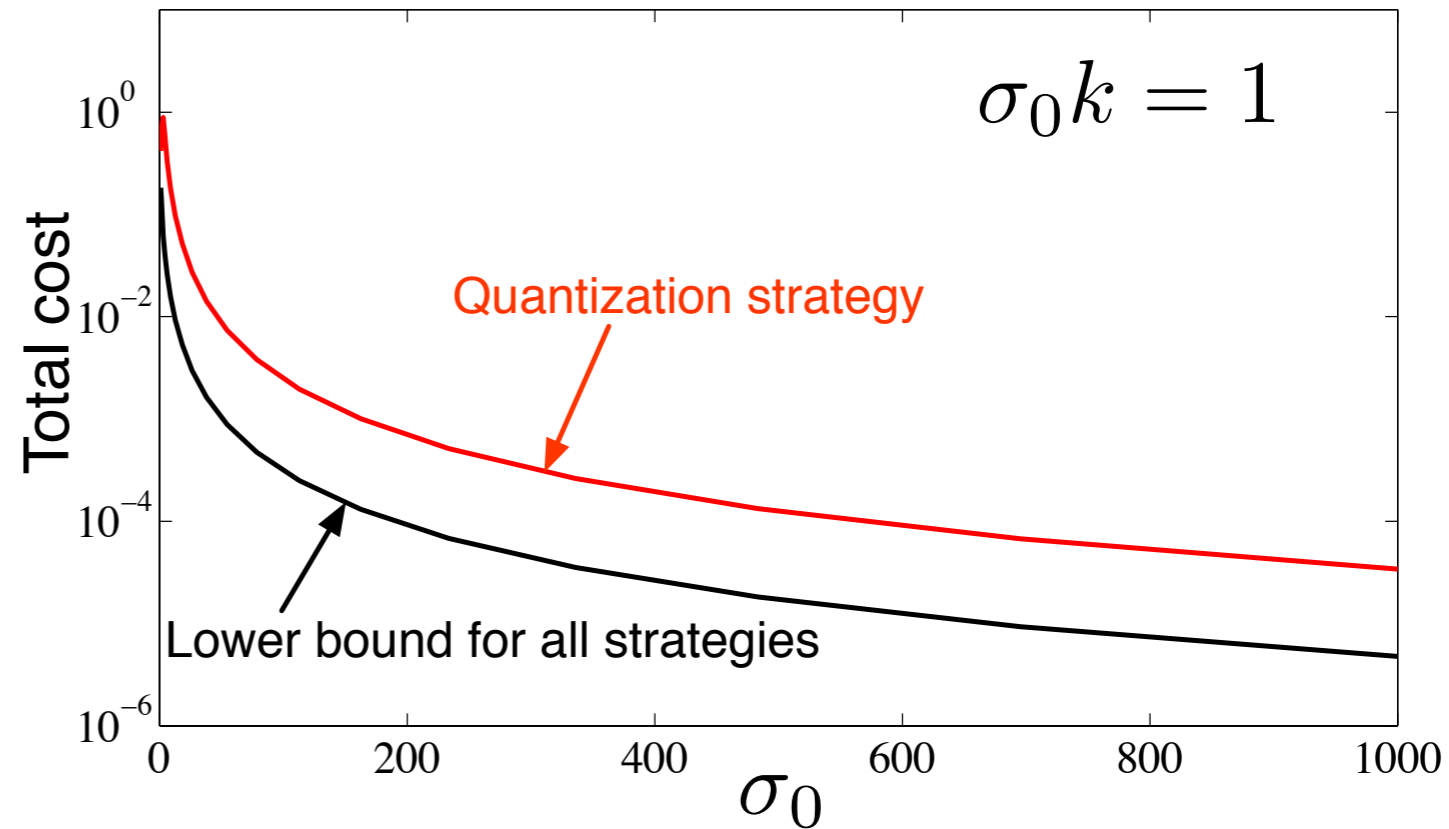


Information-theoretic [lower bound](#)
on costs for “signaling”
[\[Grover, Park, Sahai '09\]](#)

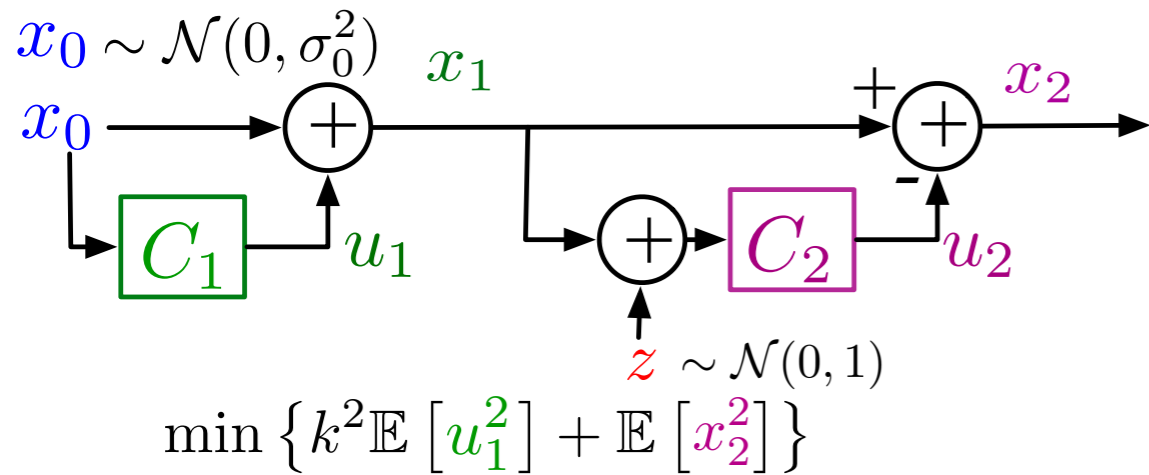
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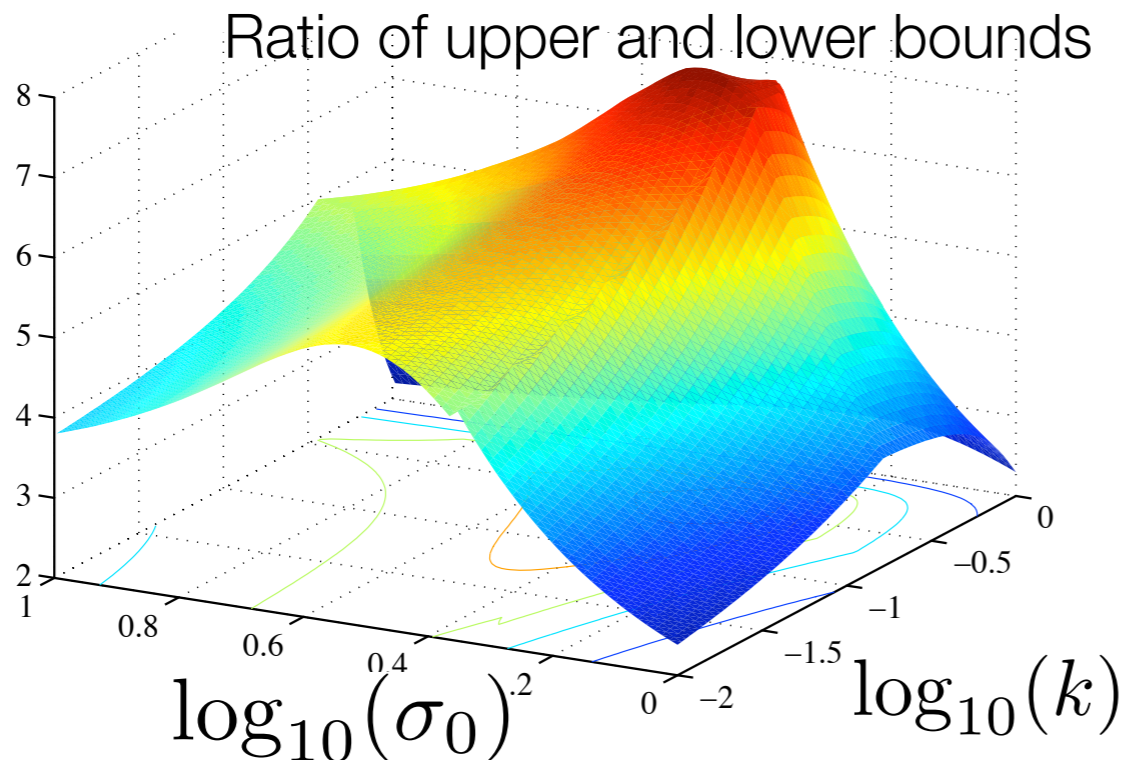
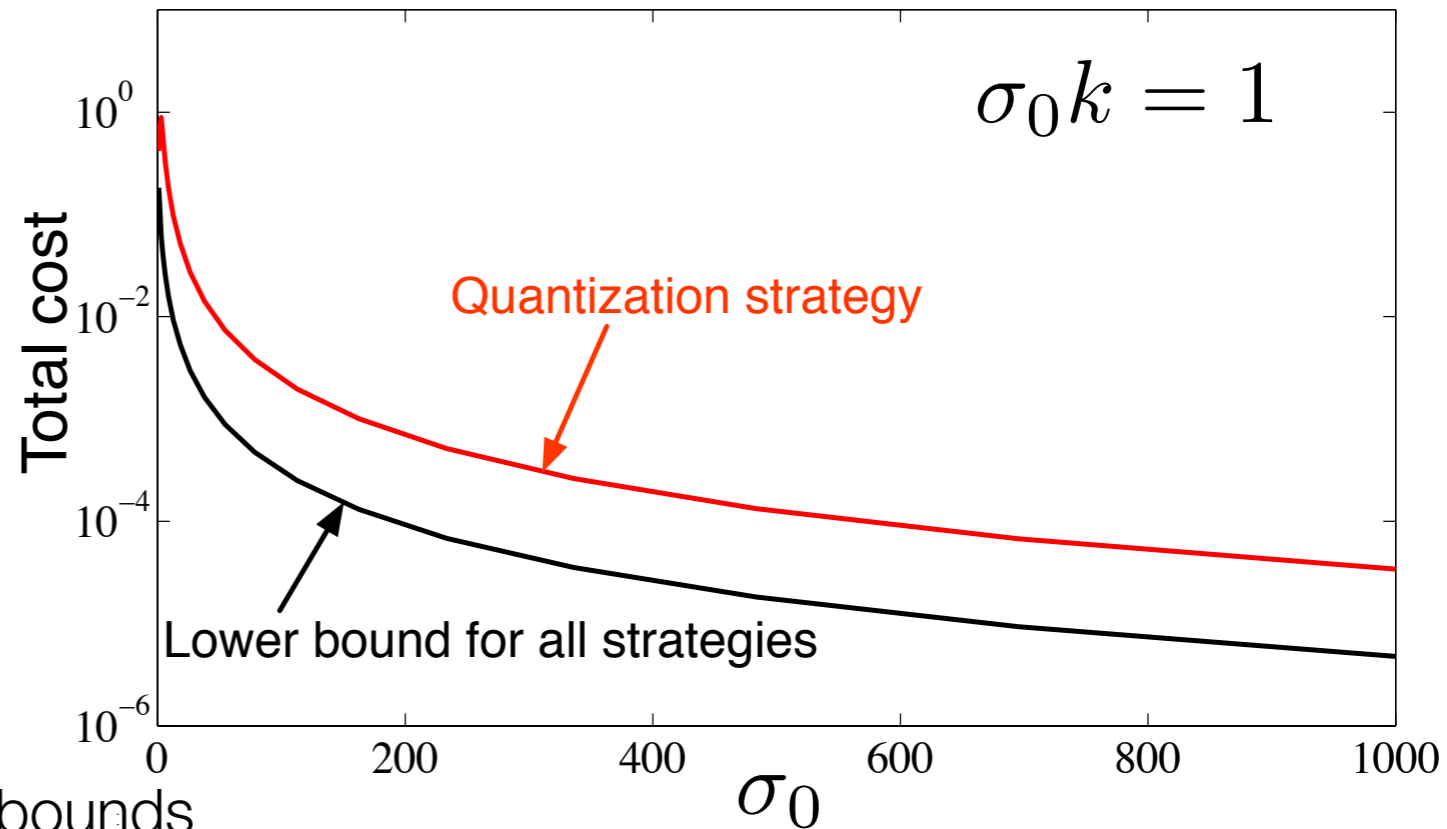
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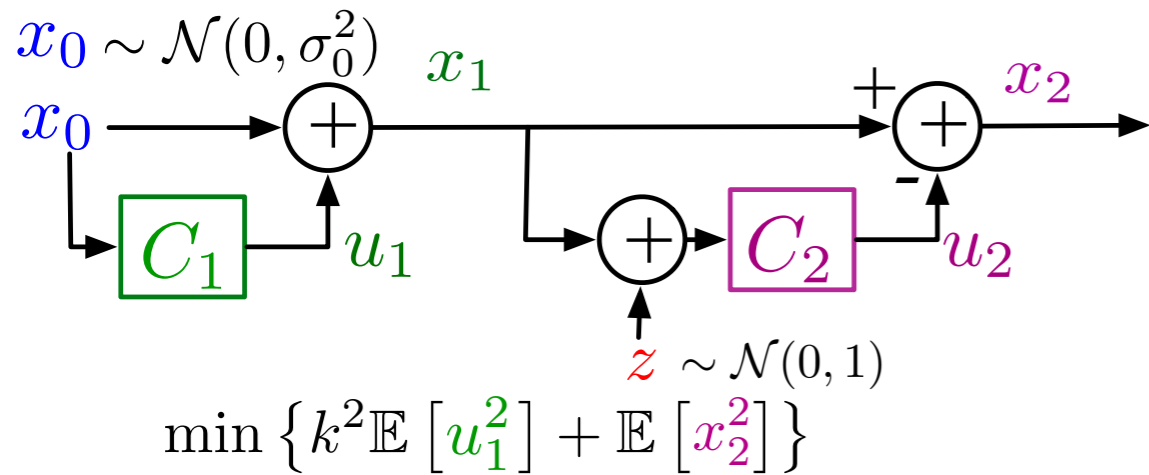
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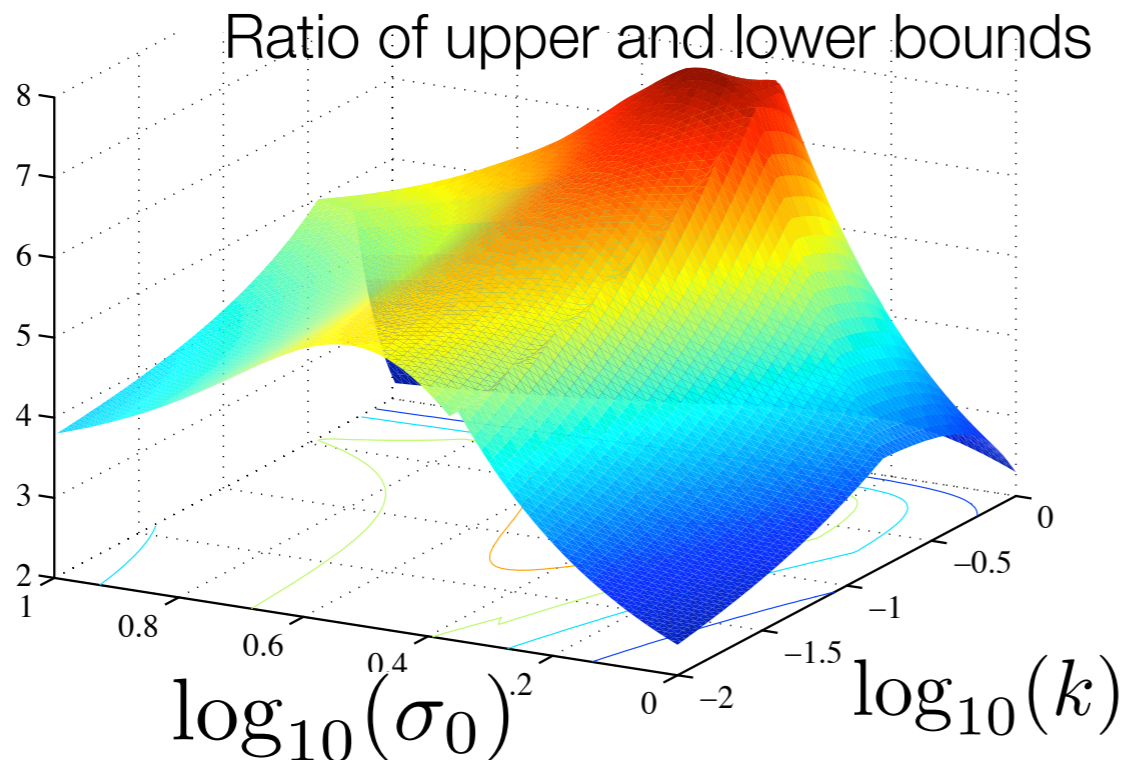
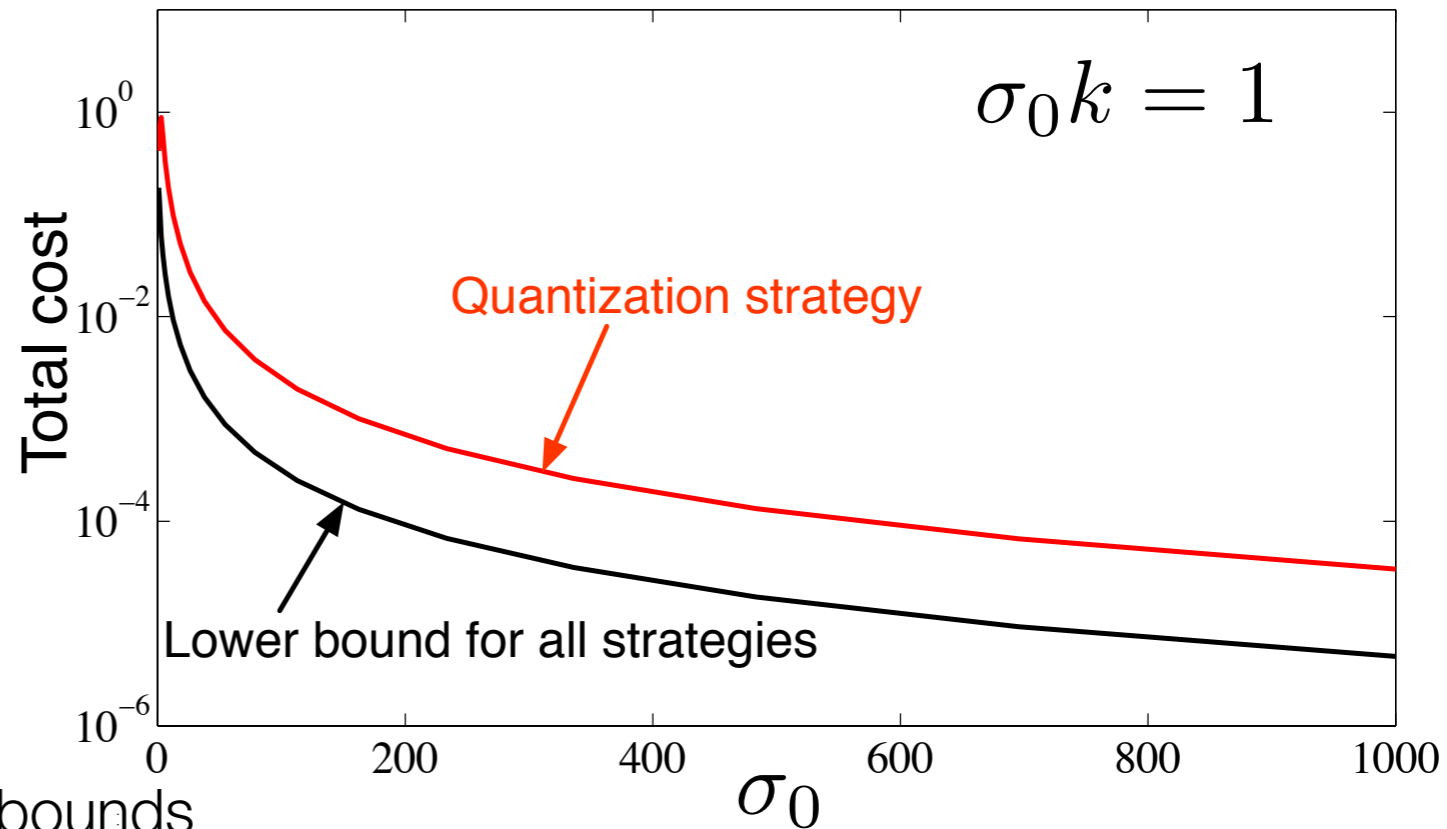
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Approximately-optimal solutions to Witsenhausen's counterexample



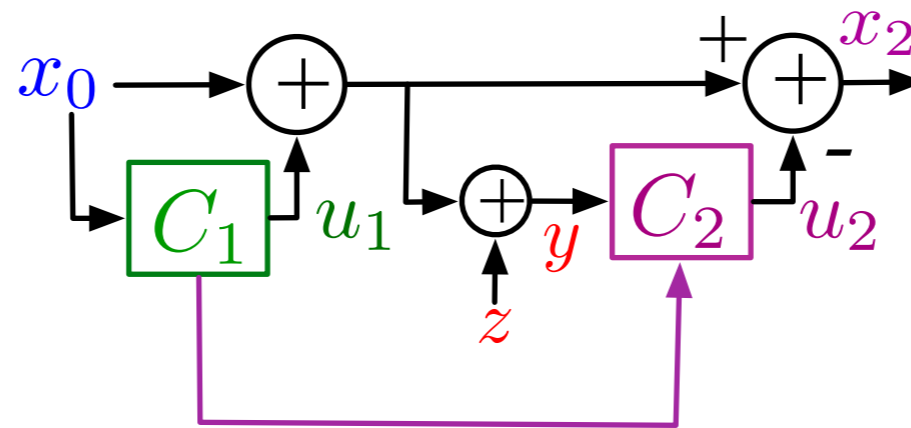
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Within a factor of 8 of optimal cost for all (k, σ_0)

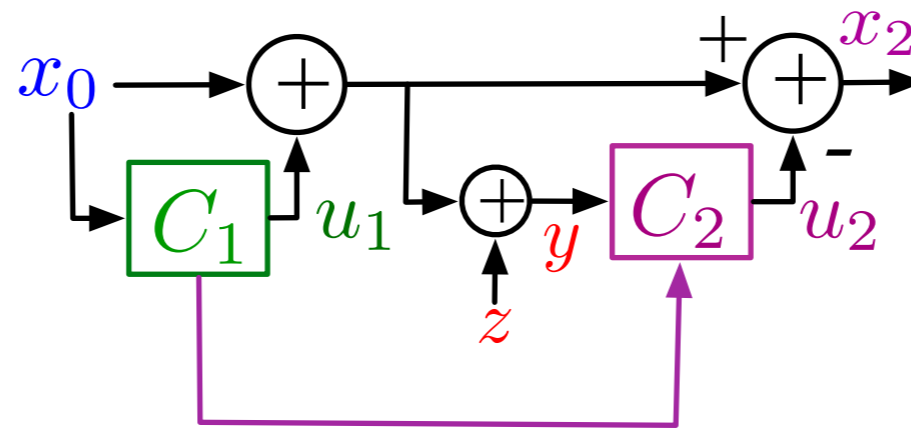
Is the counterexample **relevant**? Part I

Just connect them using an external channel!



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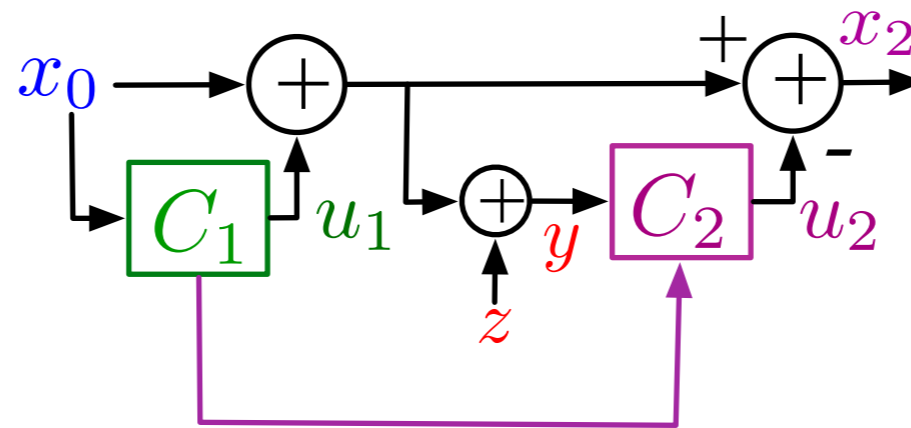
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partial-nestedness [Ho, Chu '72]

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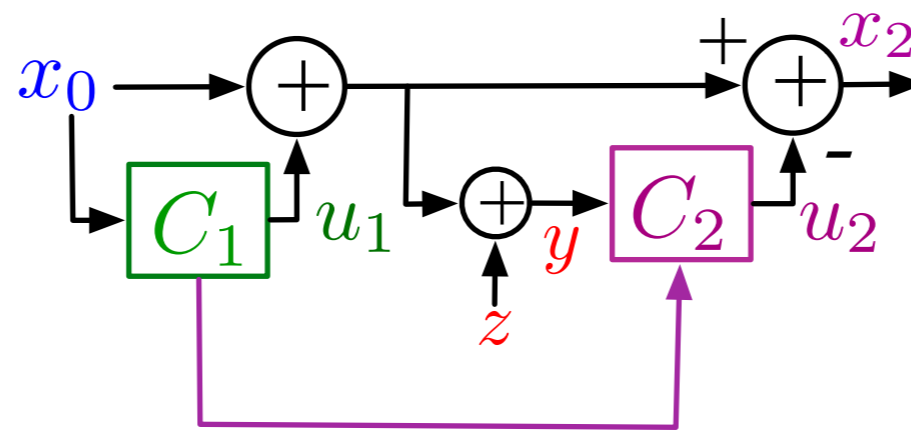


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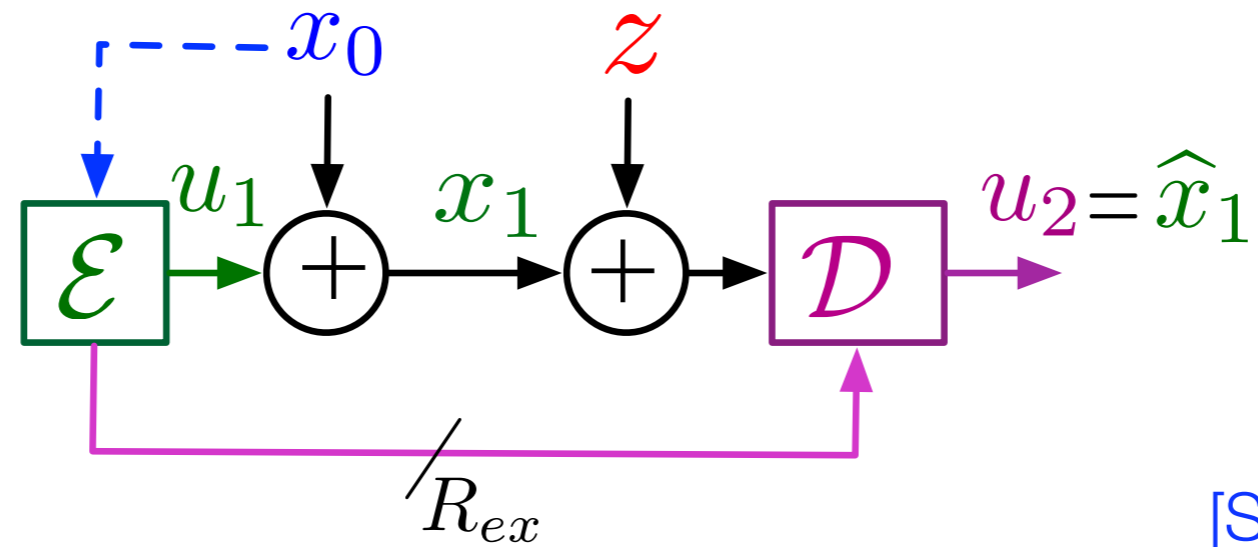
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Q1. Why should we use actions for signaling?

[John Doyle, "Paths ahead", MIT, 2009]

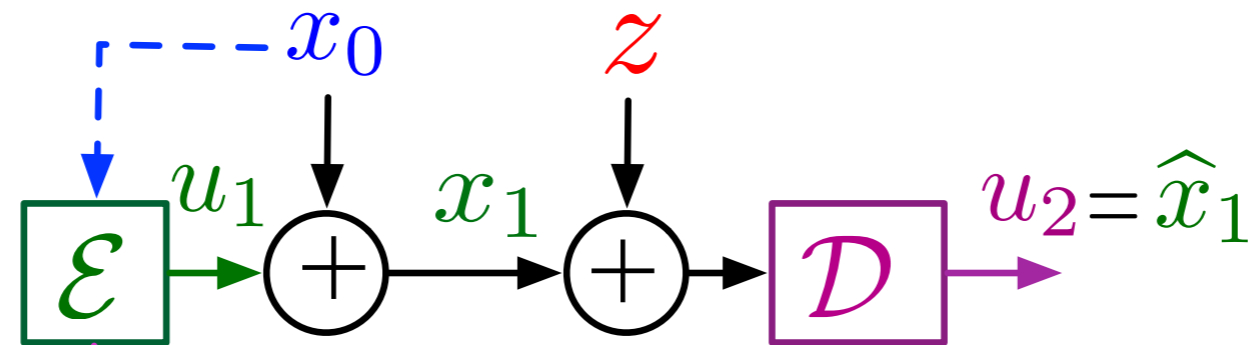
A **realistic** external channel has finite capacity!

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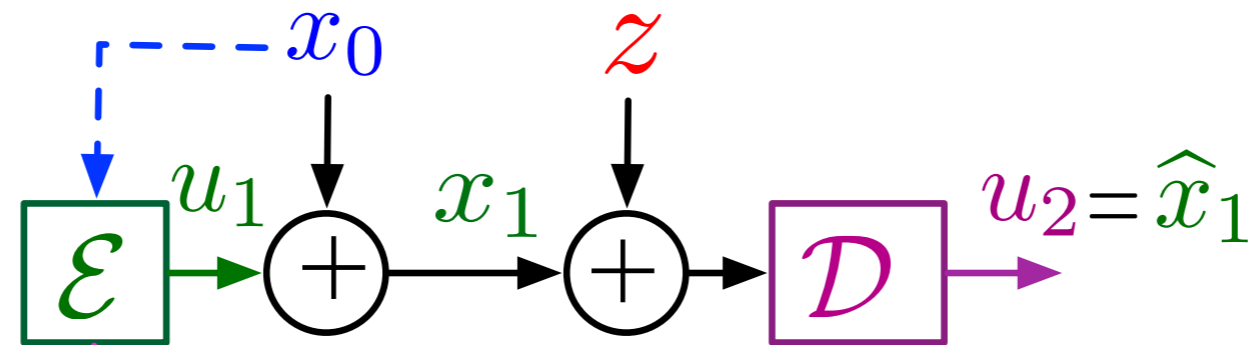
[Shoarinejad '02]
[Martins '06]

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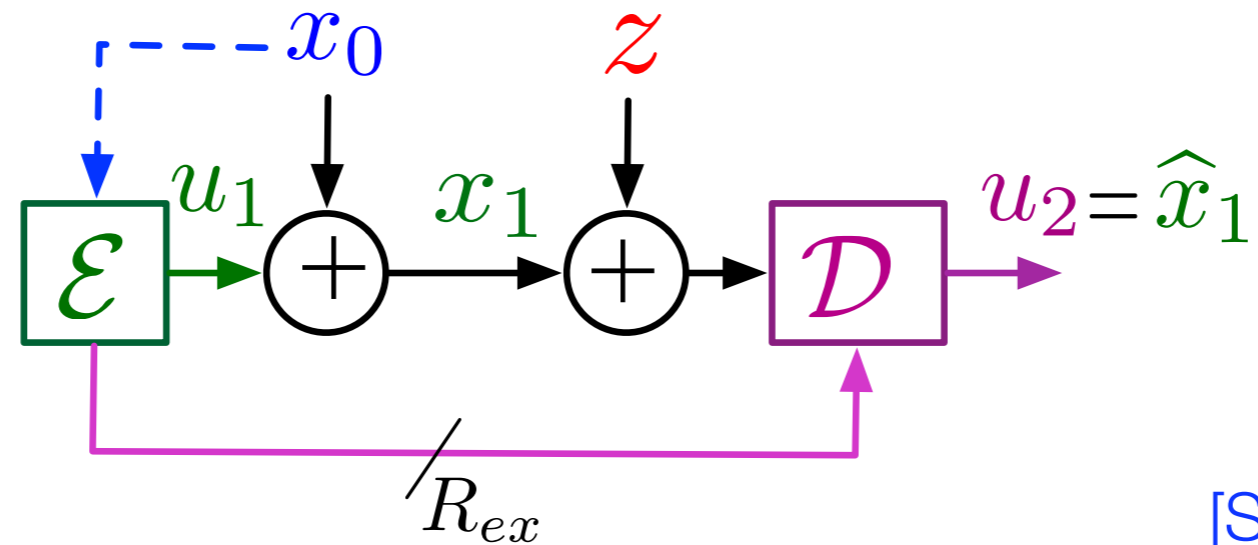
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Actions : quantize

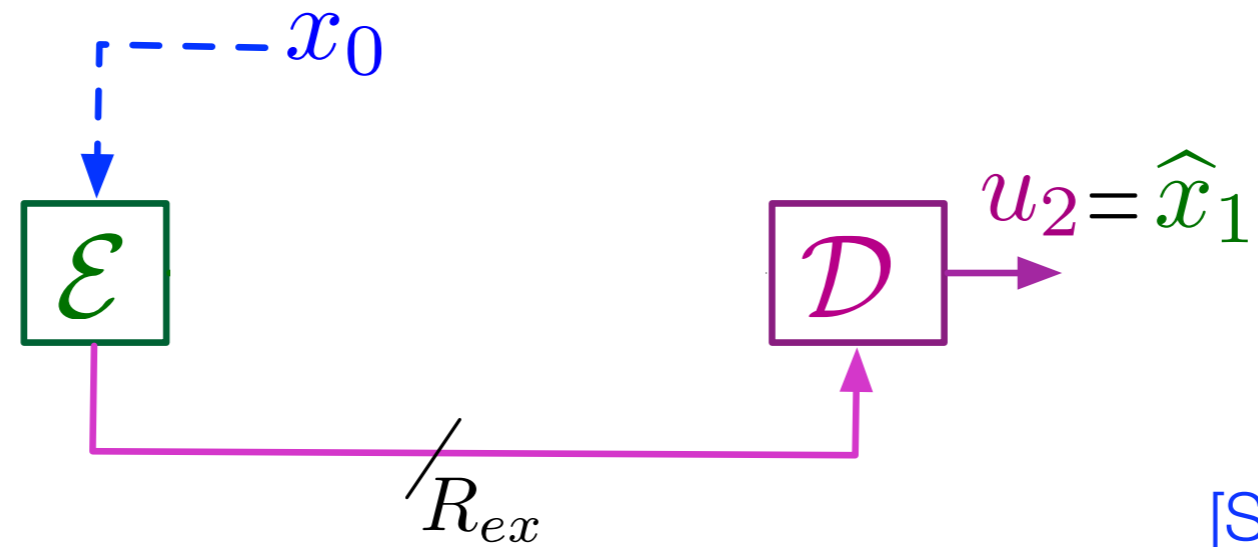
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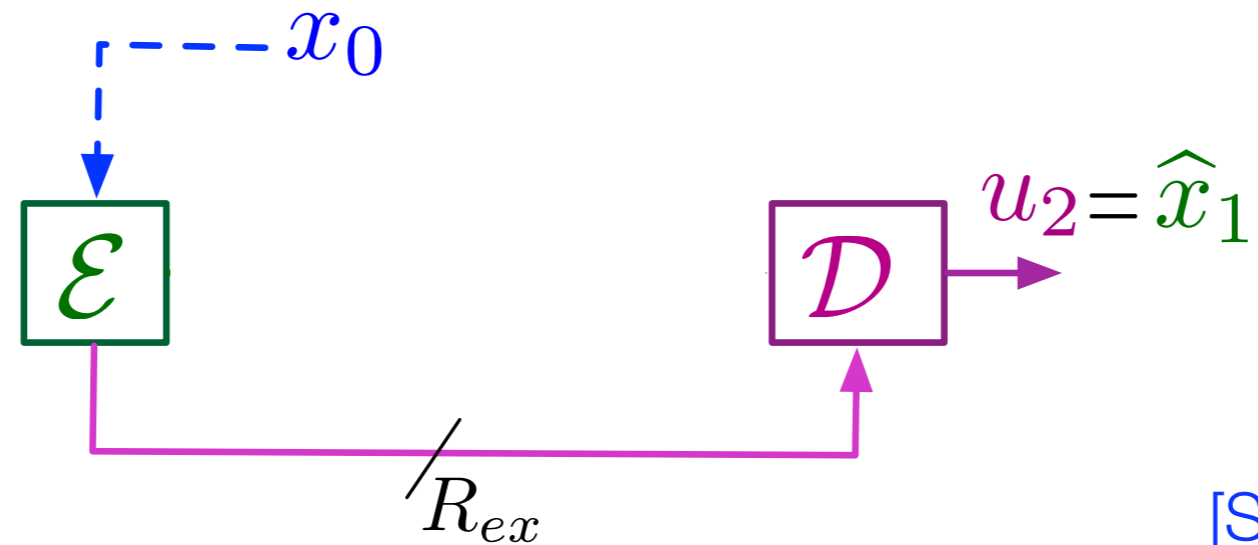
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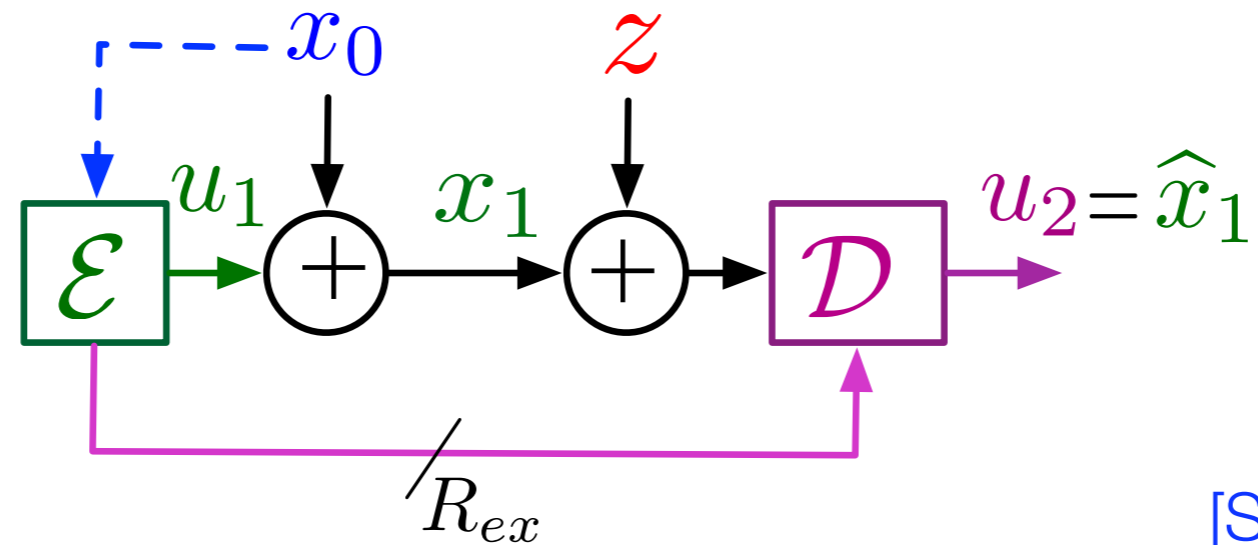


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Actions : quantize

External channel : linear

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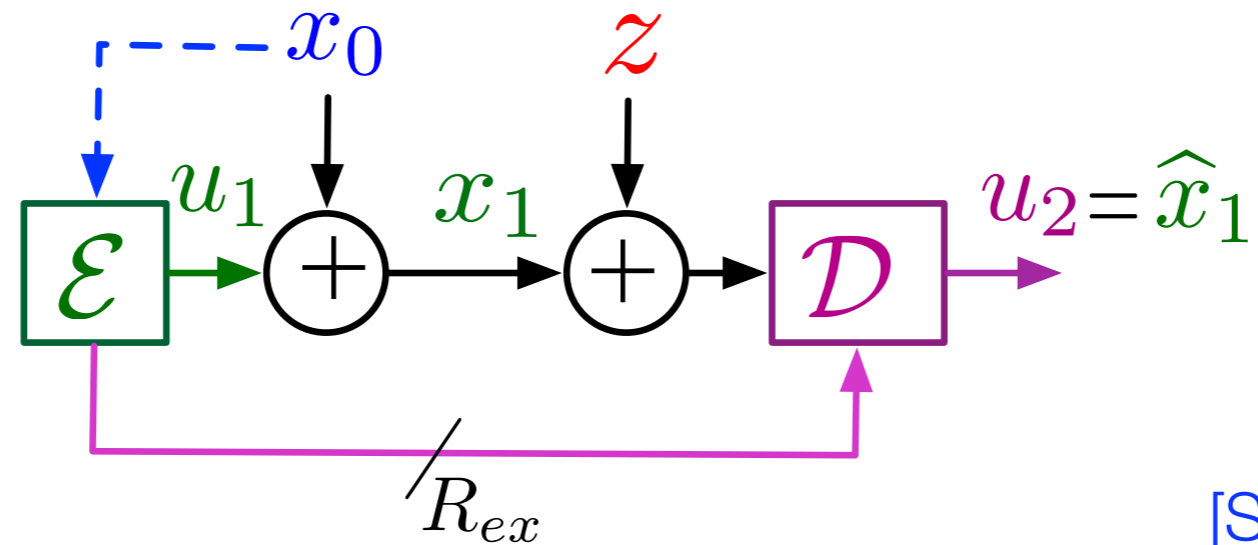


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Actions : quantize

External channel : linear

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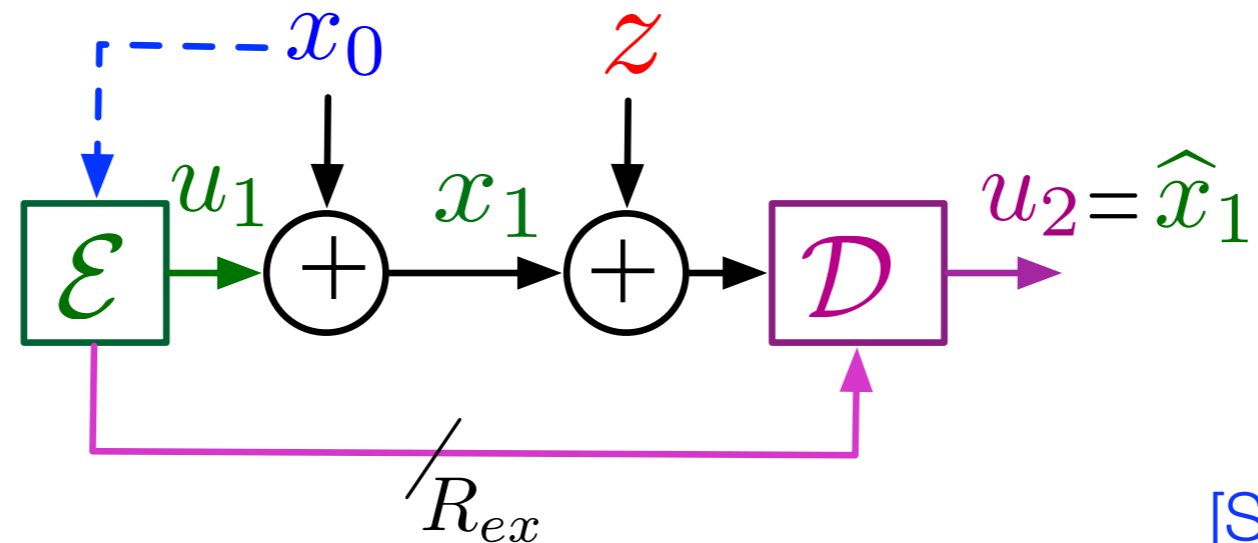
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Actions : quantize

External channel : linear

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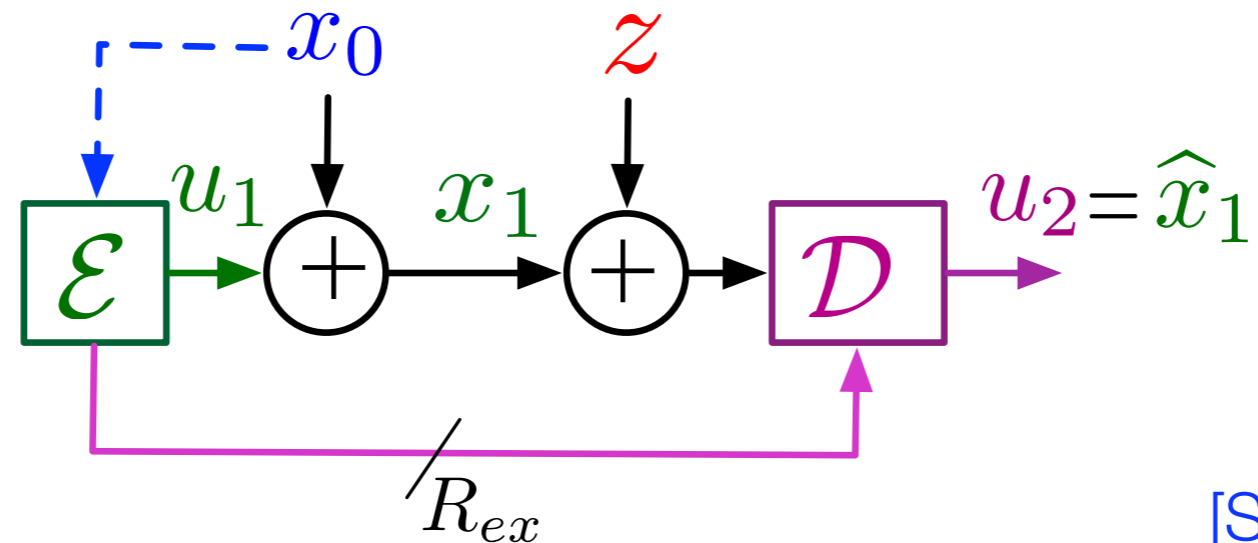
[Martins '06]

Actions : quantize

External channel : linear

Outperforms linear strategies even at high R_{ex}

A **realistic** external channel has finite capacity!



[Shoarinejad '02]
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[Martins '06]

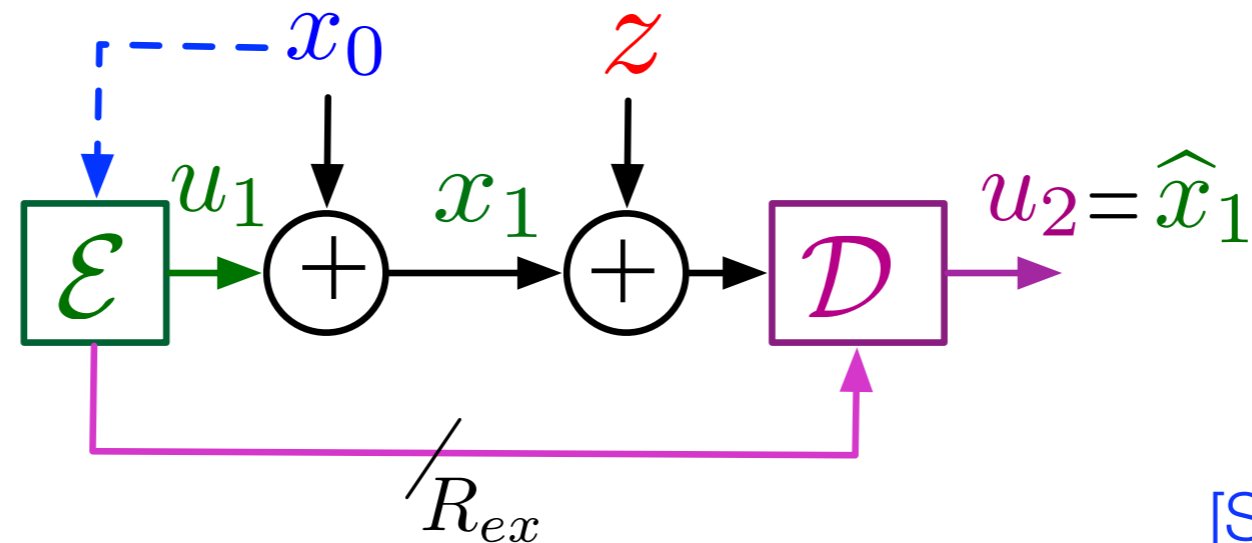
Actions : quantize

External channel : linear

Outperforms linear strategies even at high R_{ex}

“most-significant bits” through the plant

A **realistic** external channel has finite capacity!



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Actions : quantize

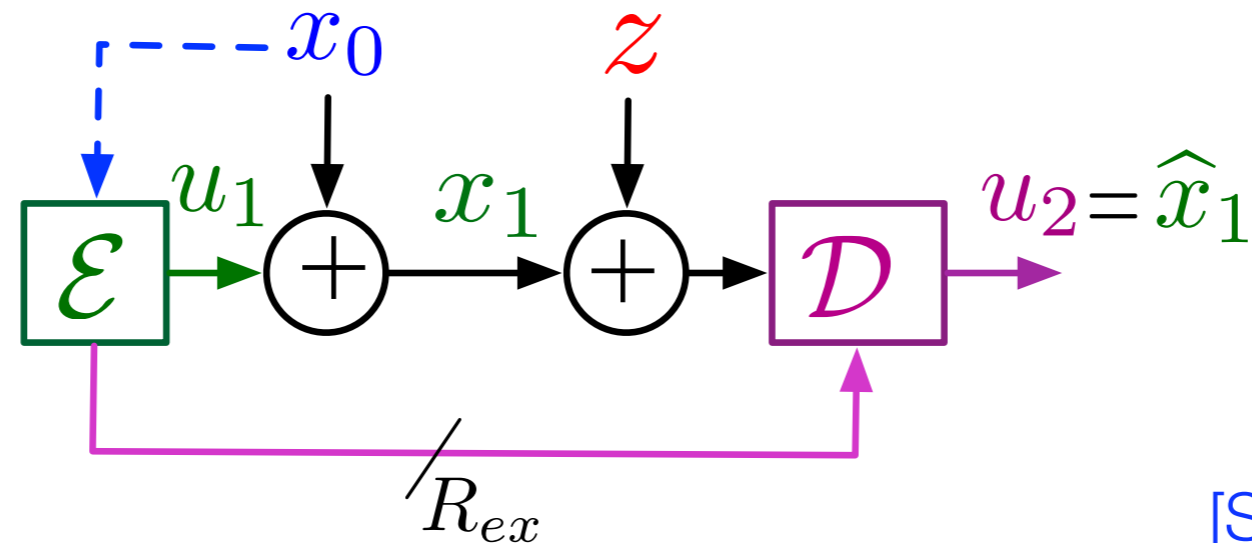
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“most-significant bits” through the plant

“most significant bits” also through the external channel

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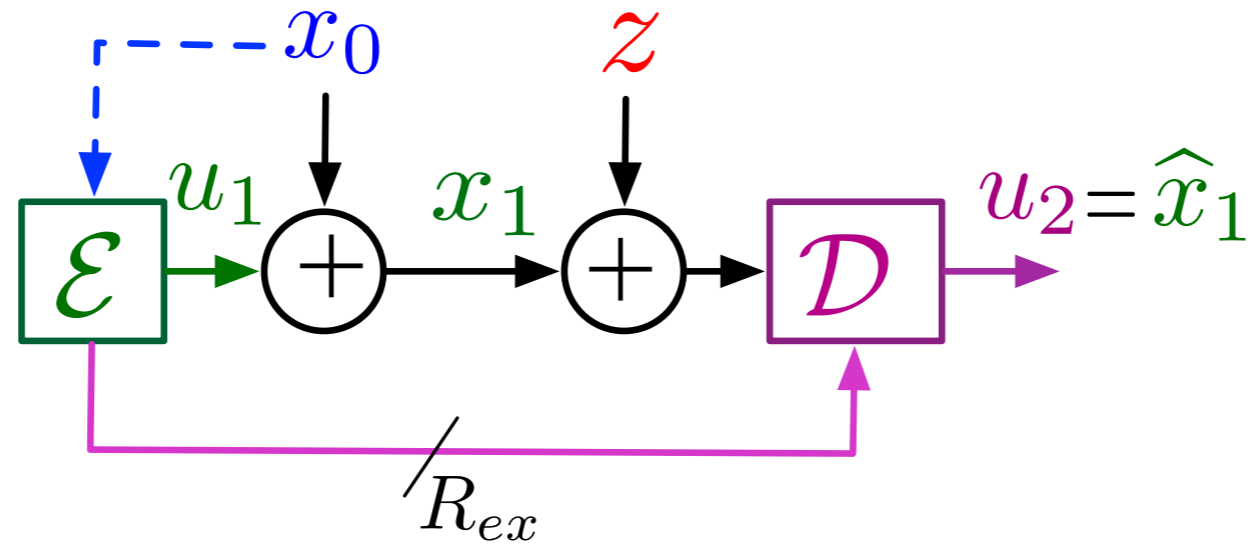
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Outperforms linear strategies even at high R_{ex}

“most-significant bits” through the plant

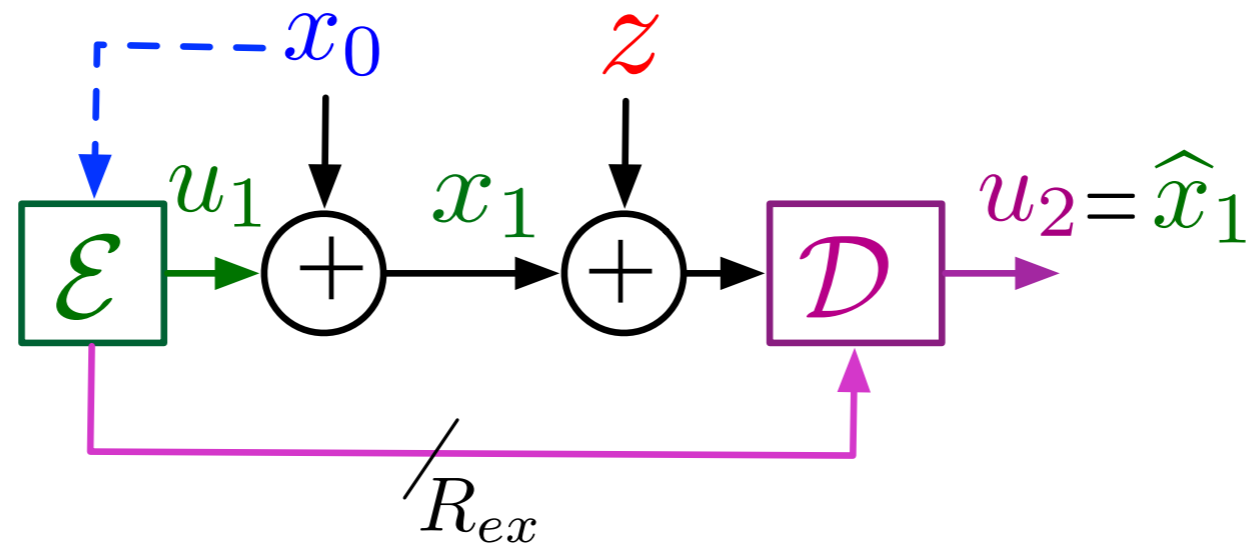
“most significant bits” also through the external channel : **redundant!**

A semi-deterministic model



[Grover, Sahai '10]

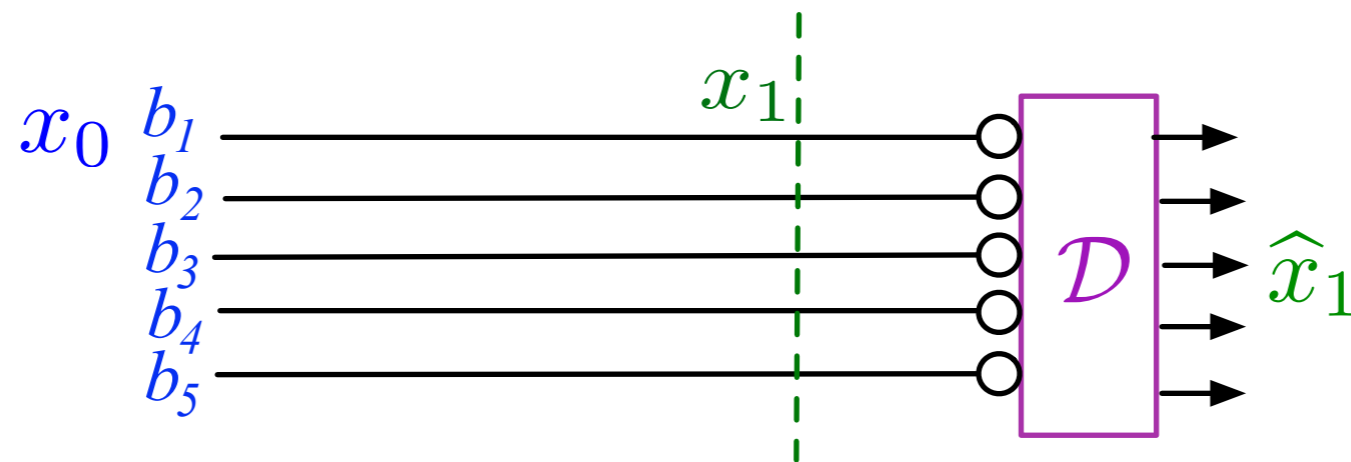
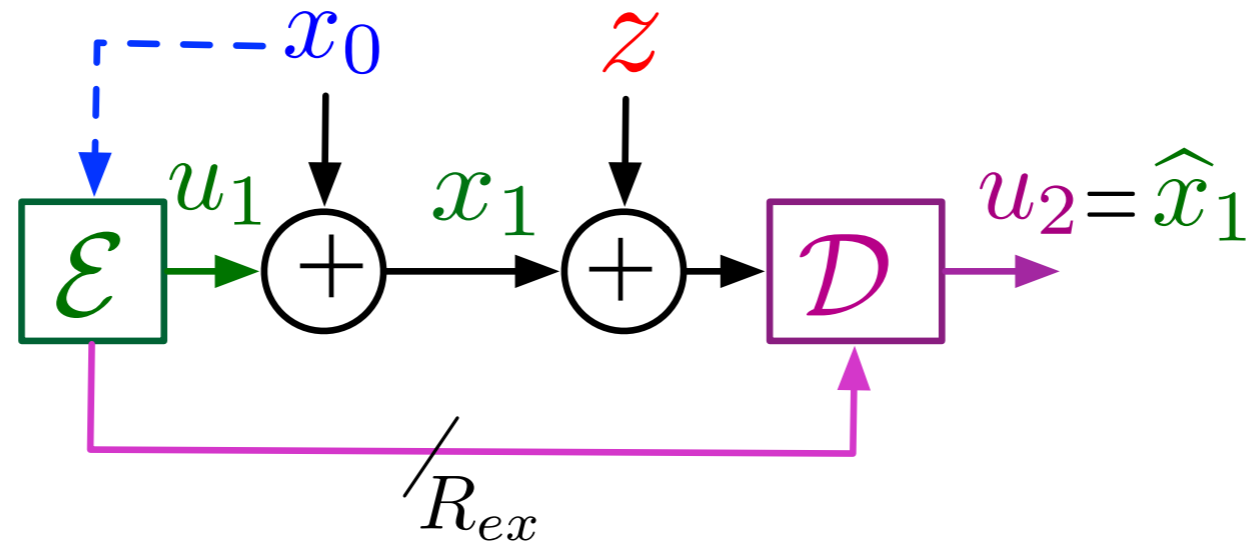
A semi-deterministic model



x_0 b_1
 b_2
 b_3
 b_4
 b_5

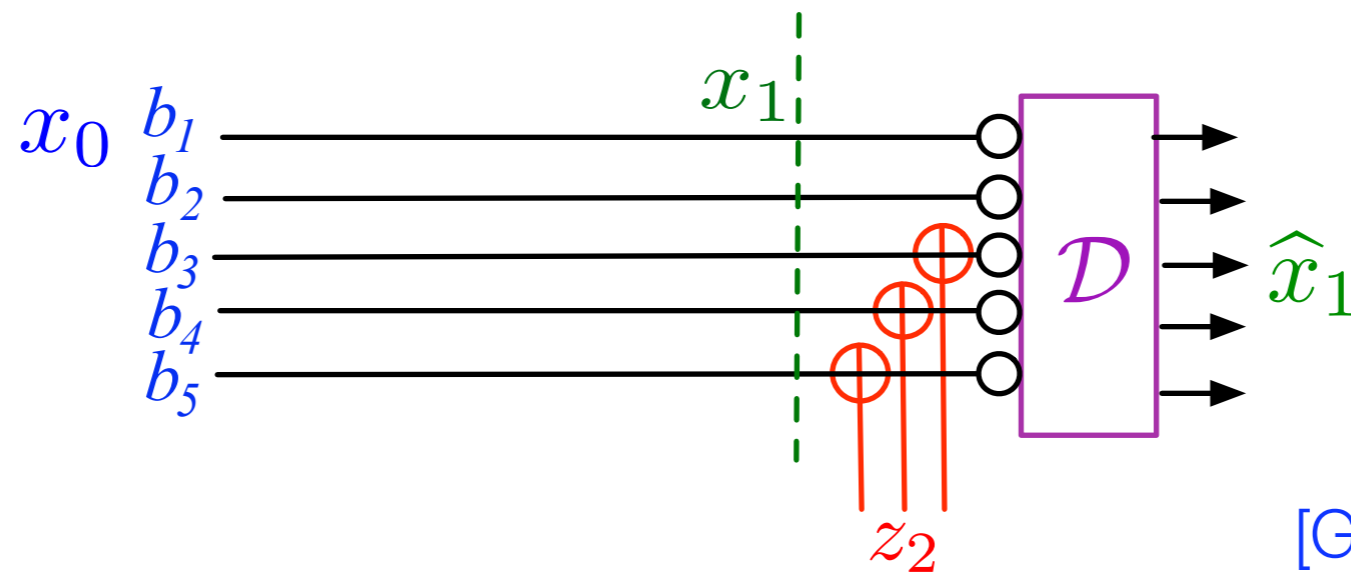
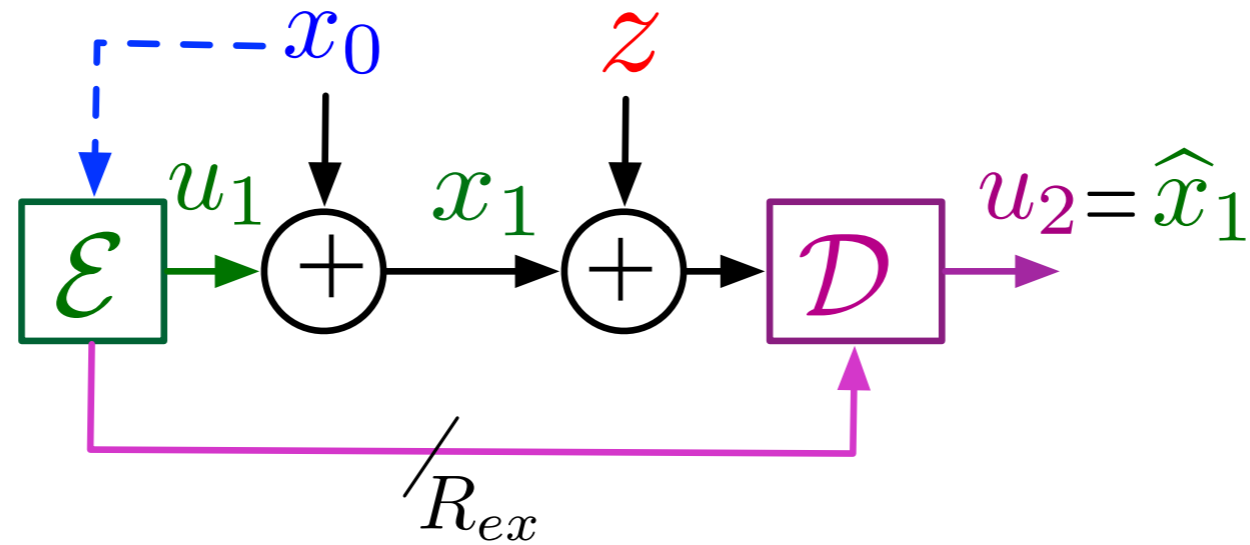
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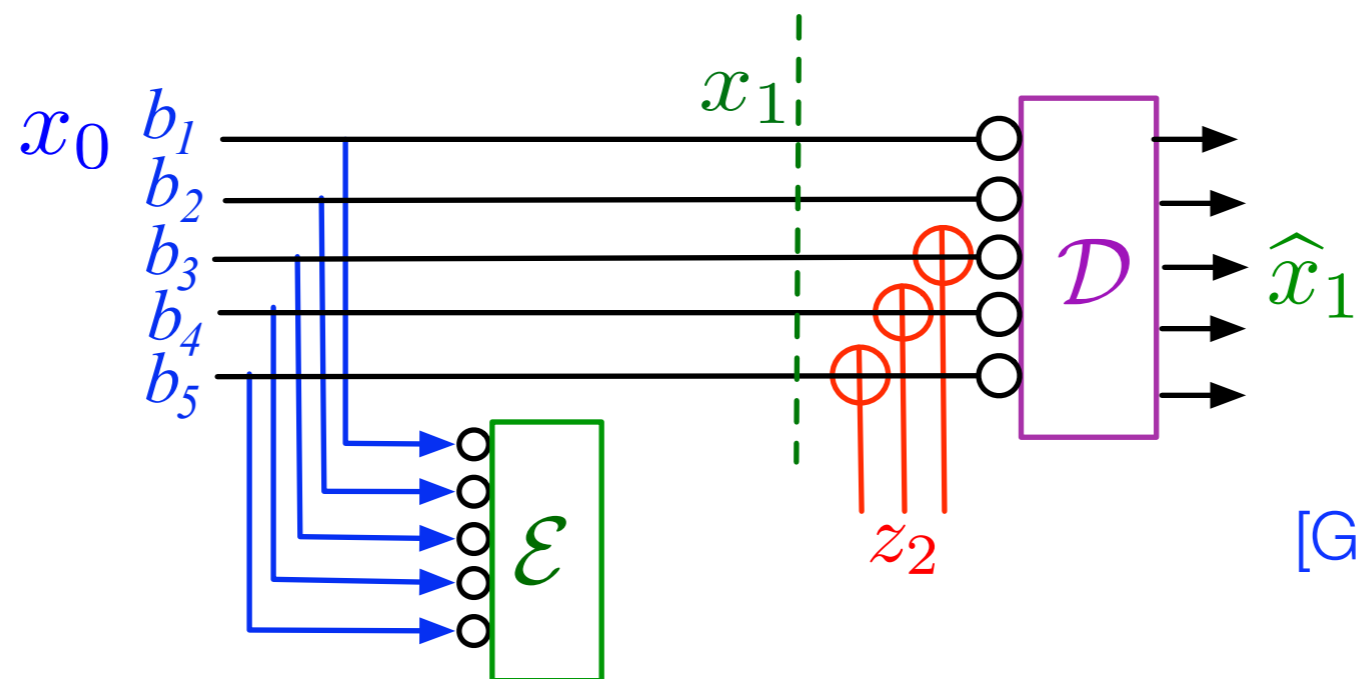
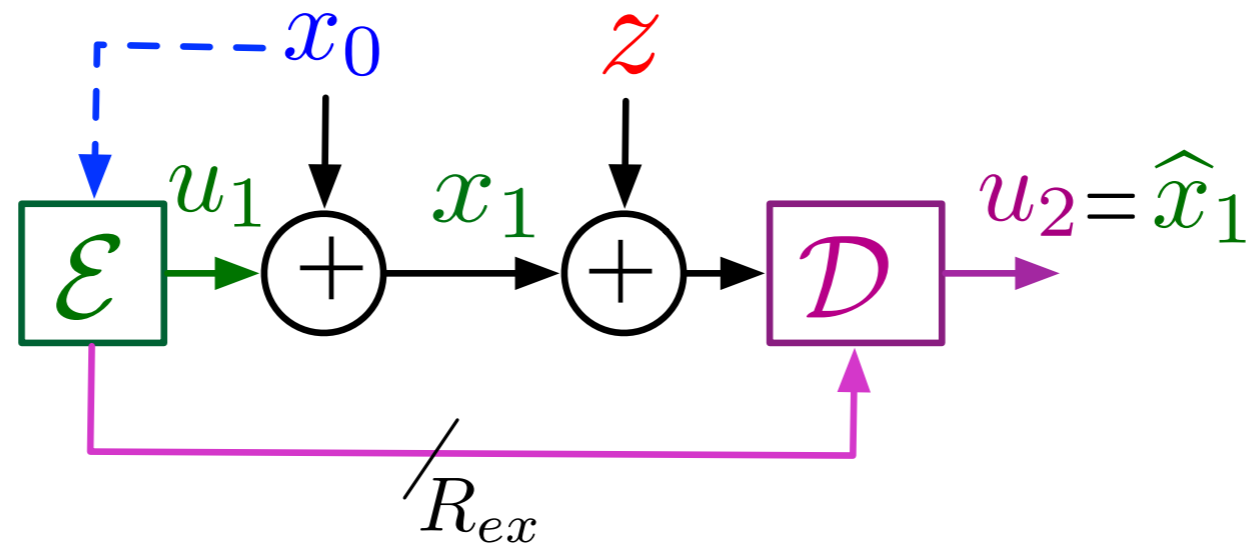
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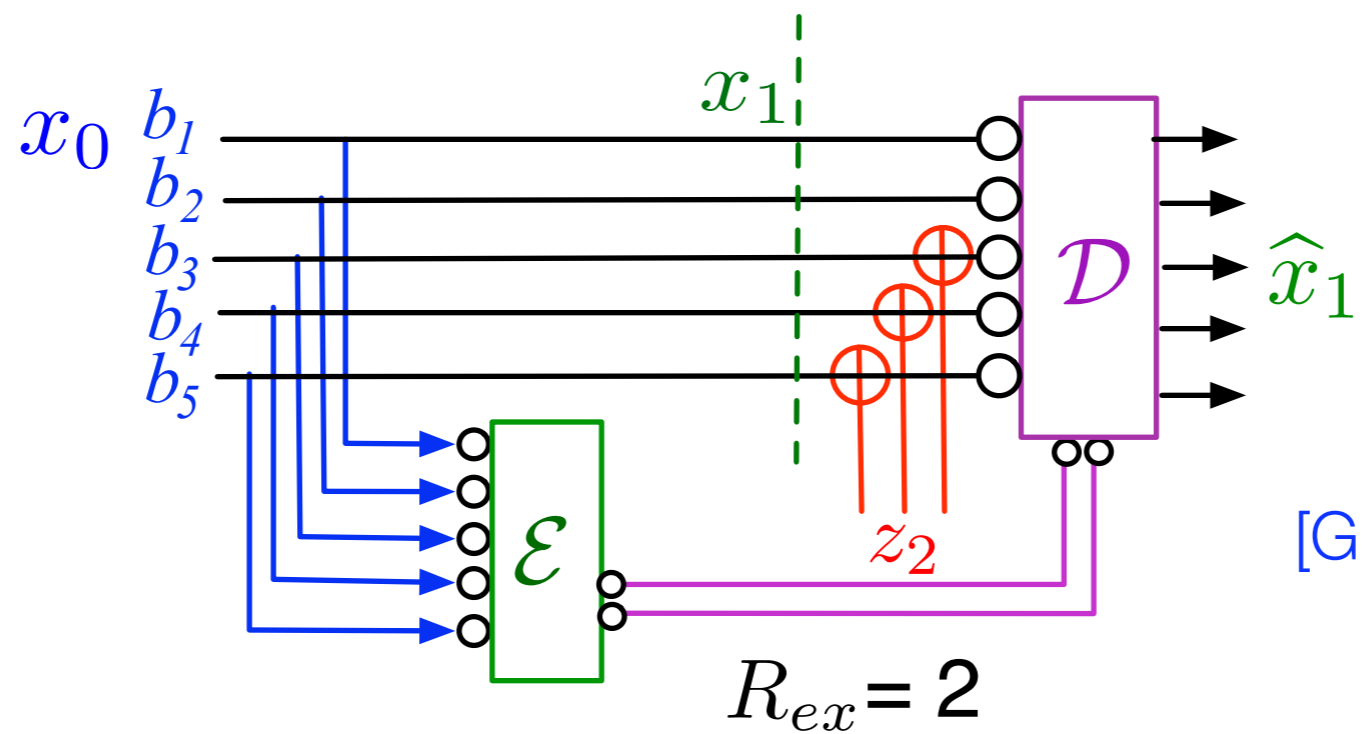
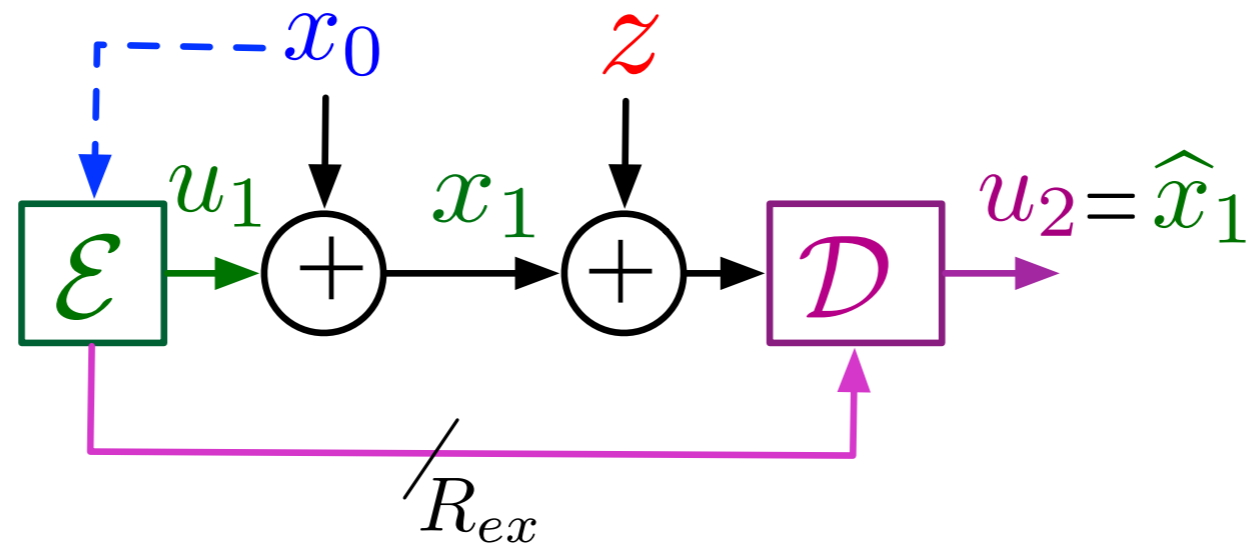
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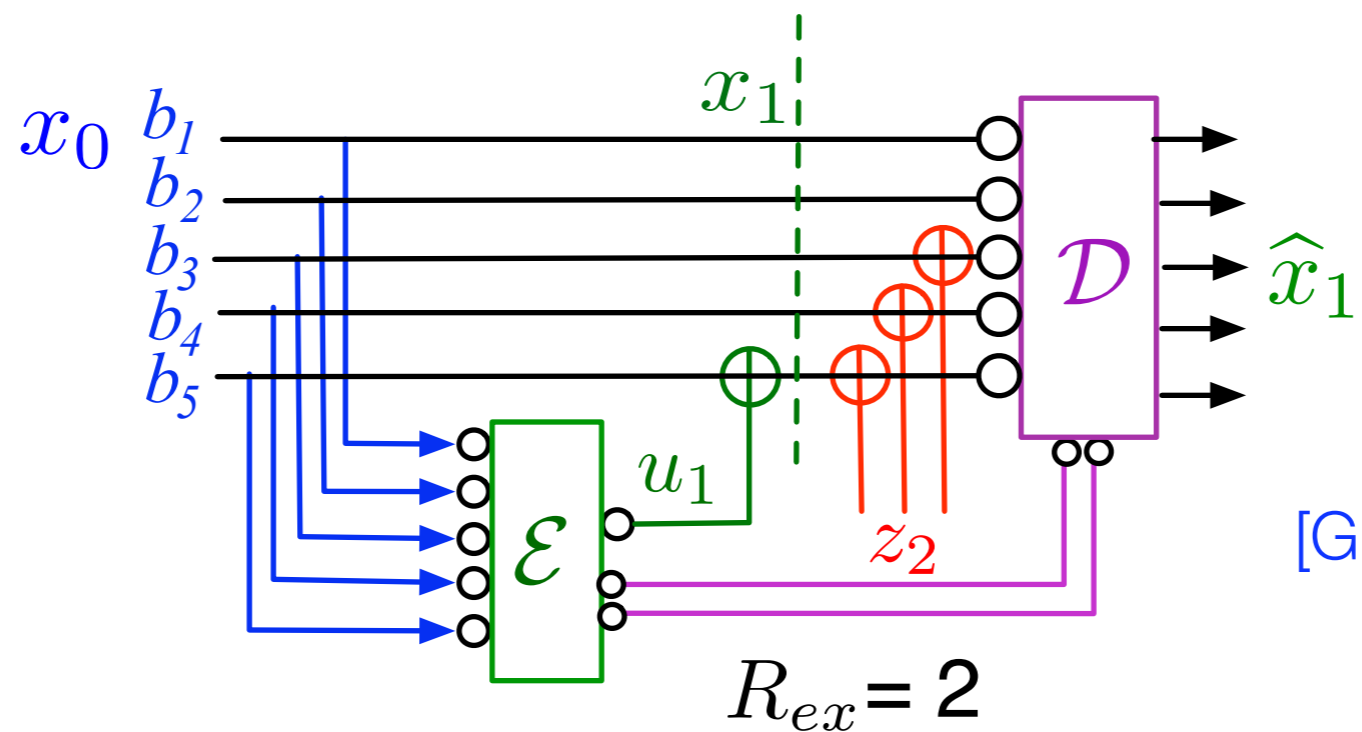
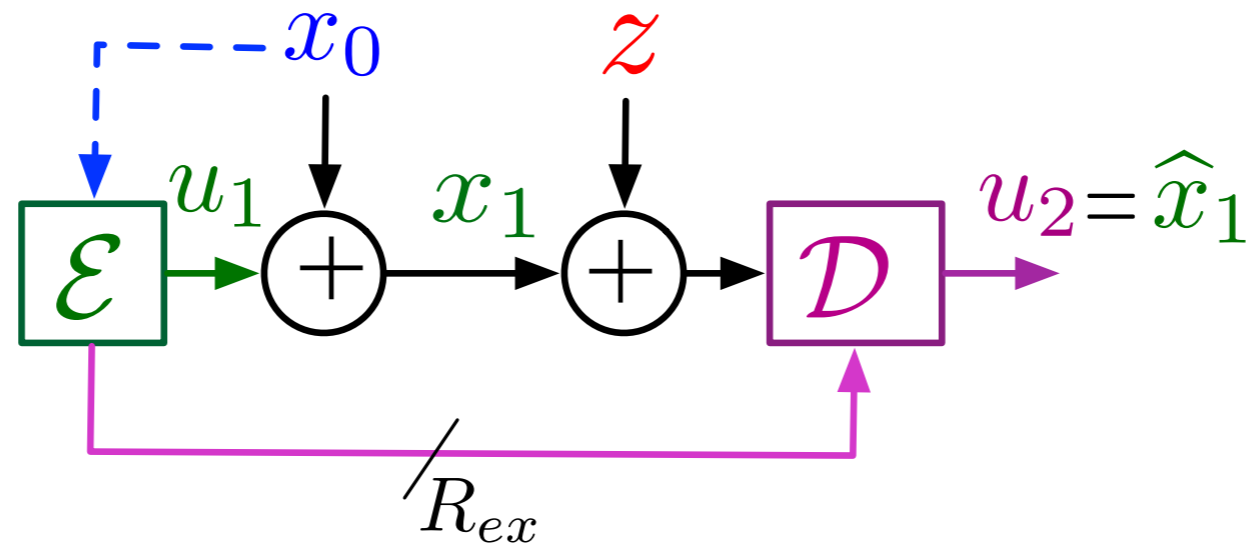
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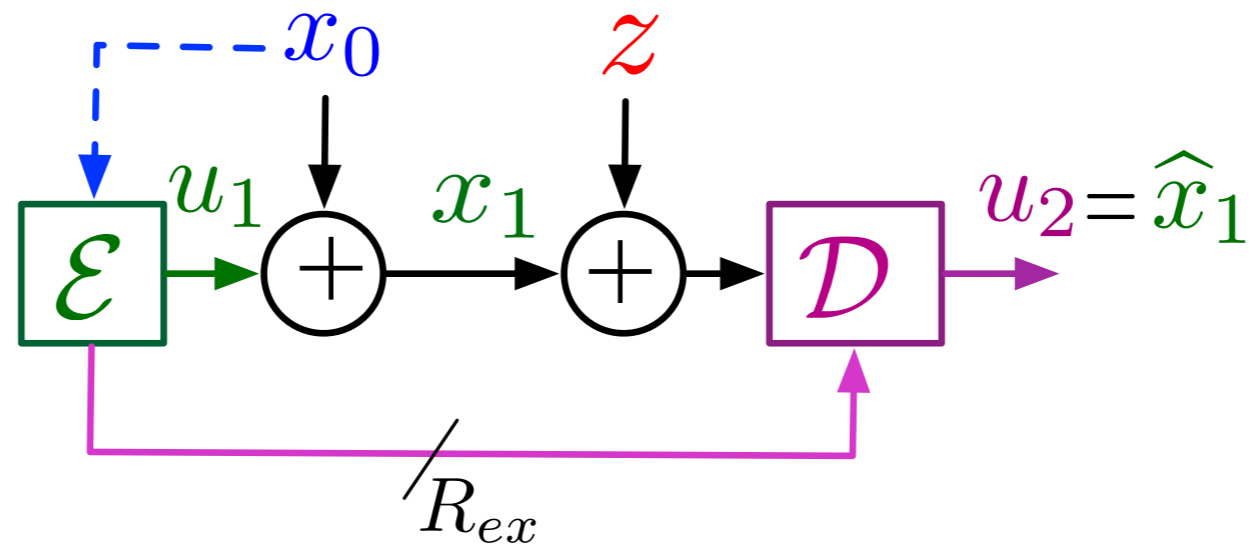


[Grover, Sahai '10]

More efficient: think “parallel channels”!

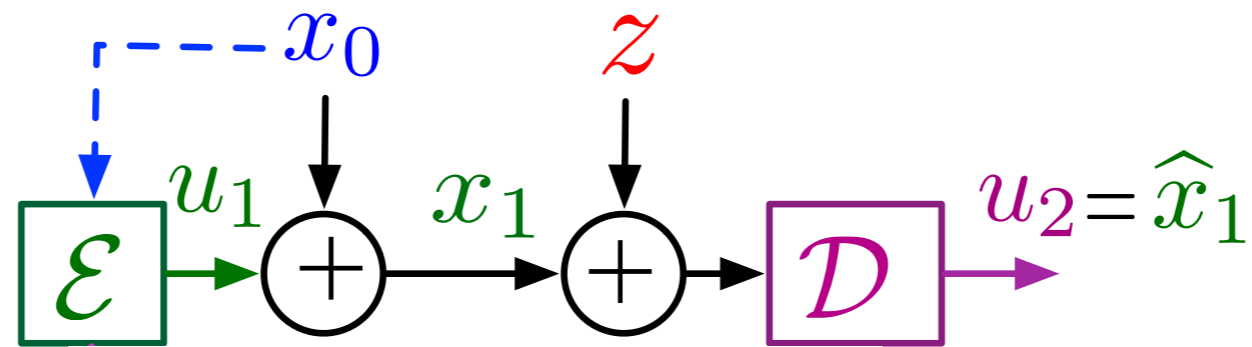
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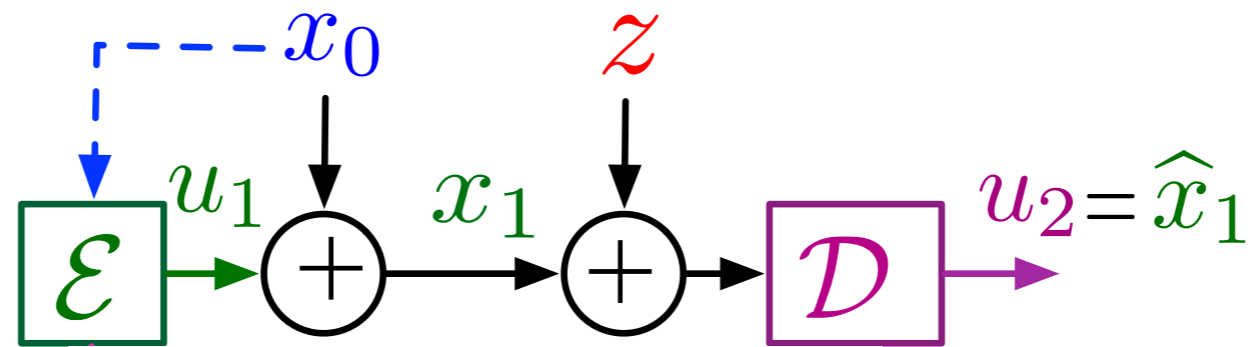
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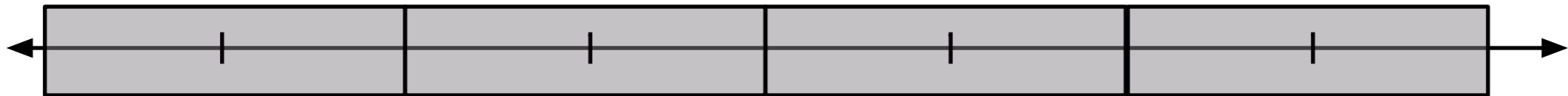


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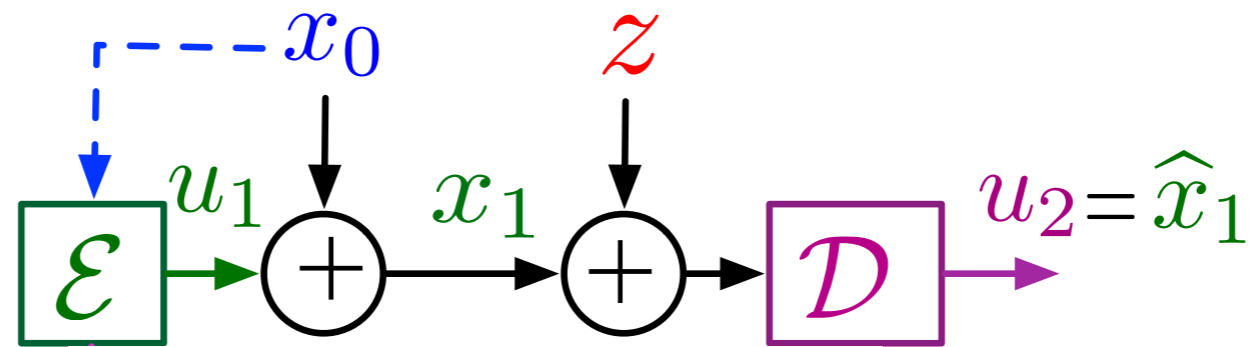
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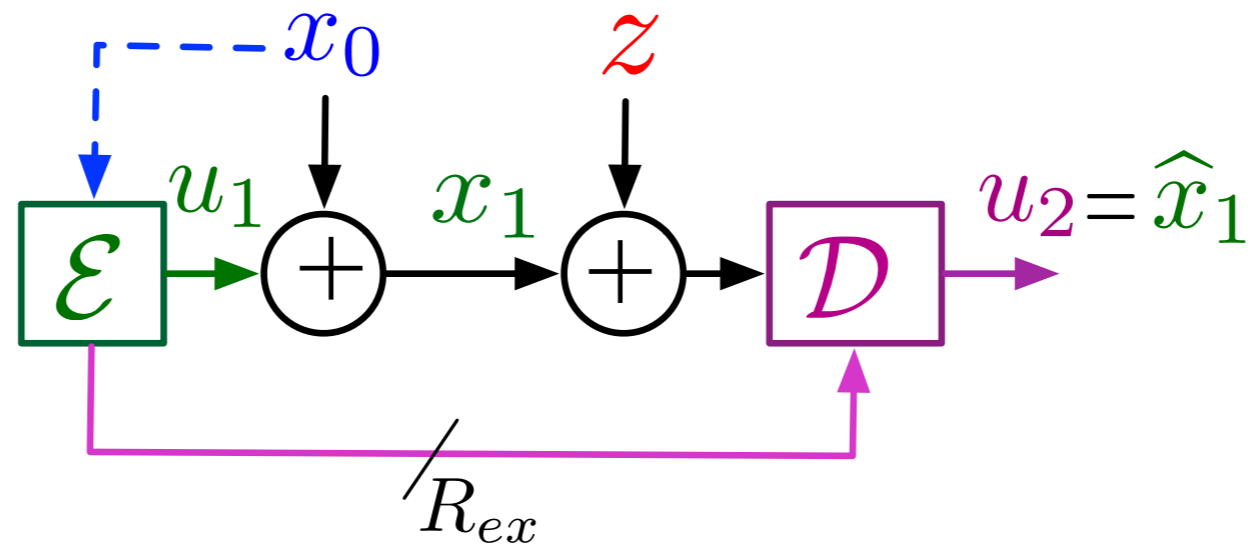
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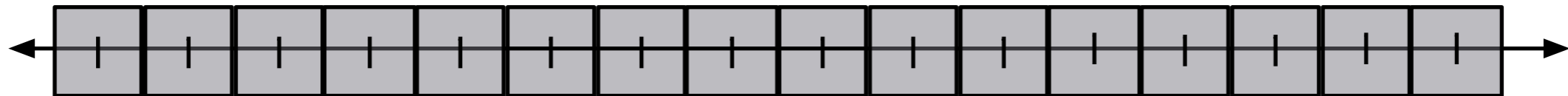
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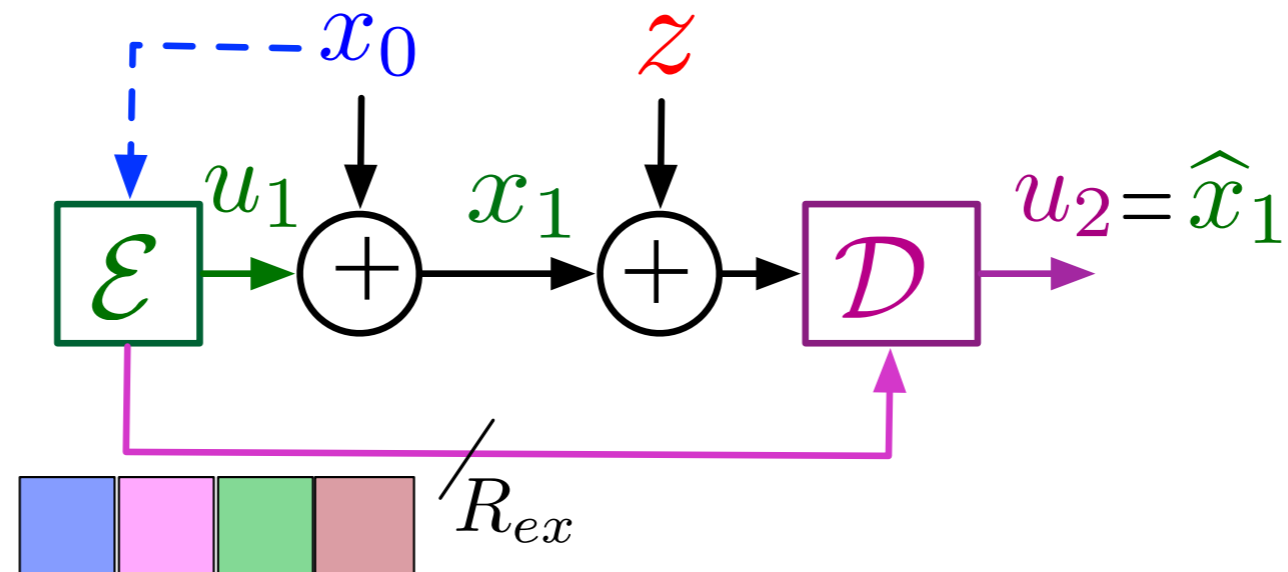
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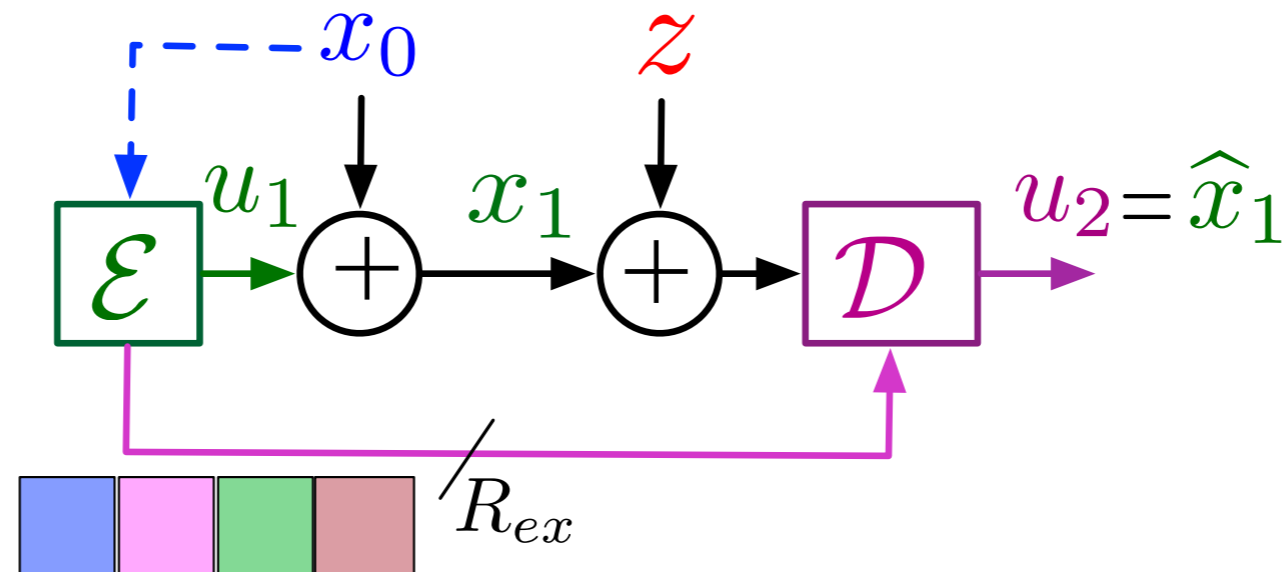
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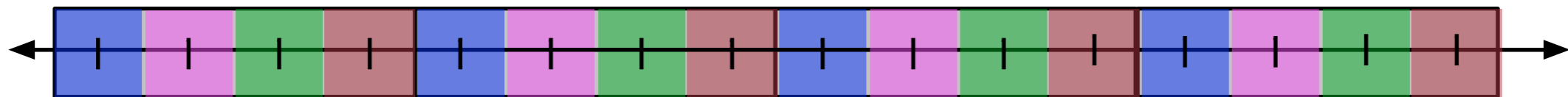
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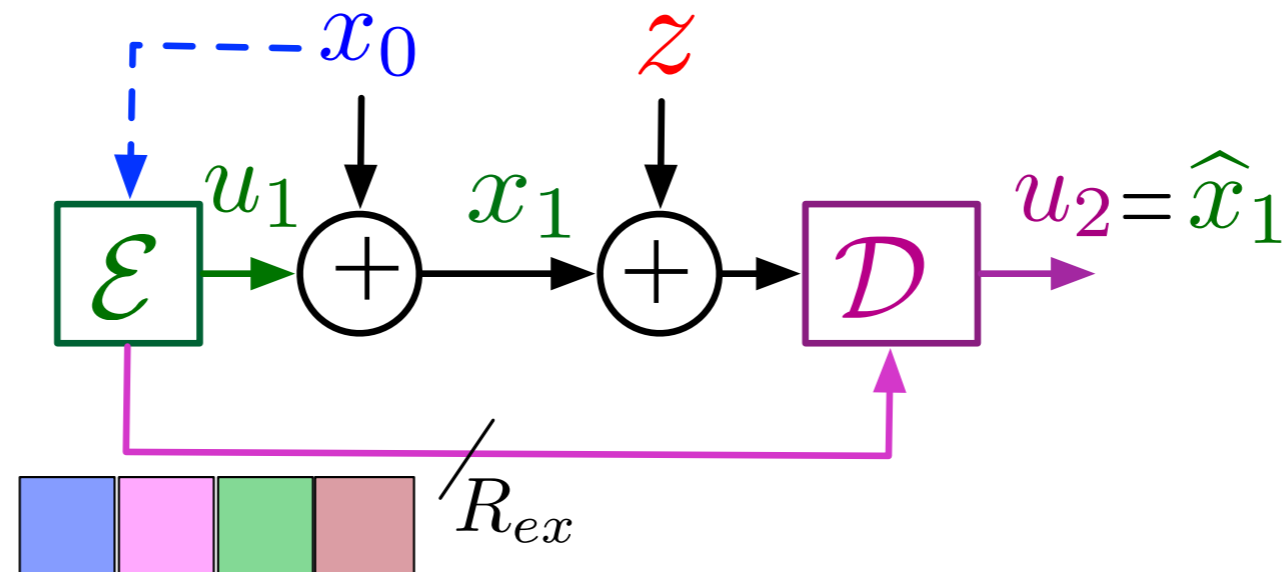
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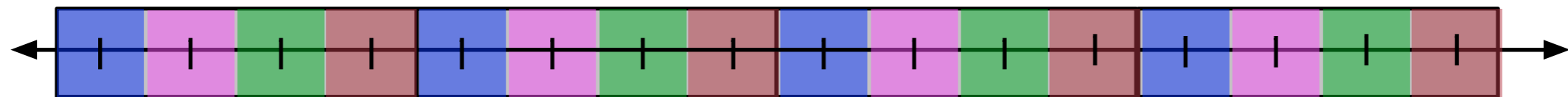
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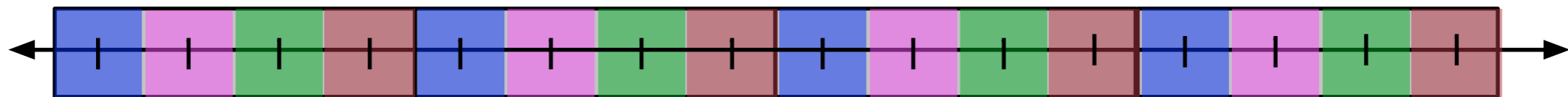
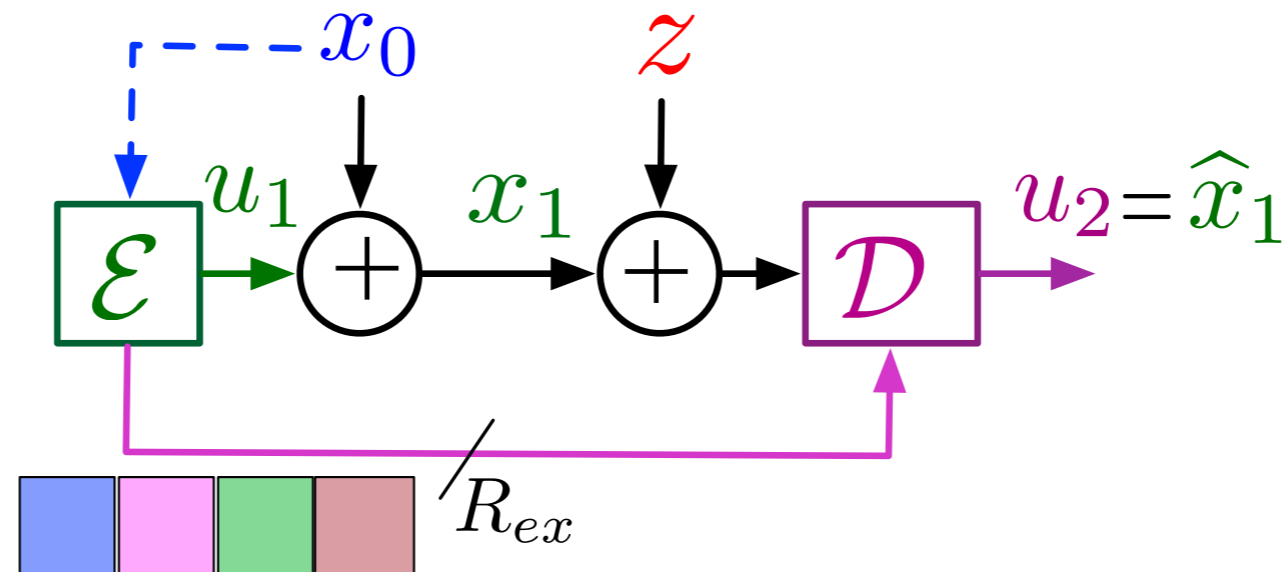


[Grover, Sahai '10]



outperform Martins's strategies by an unbounded factor

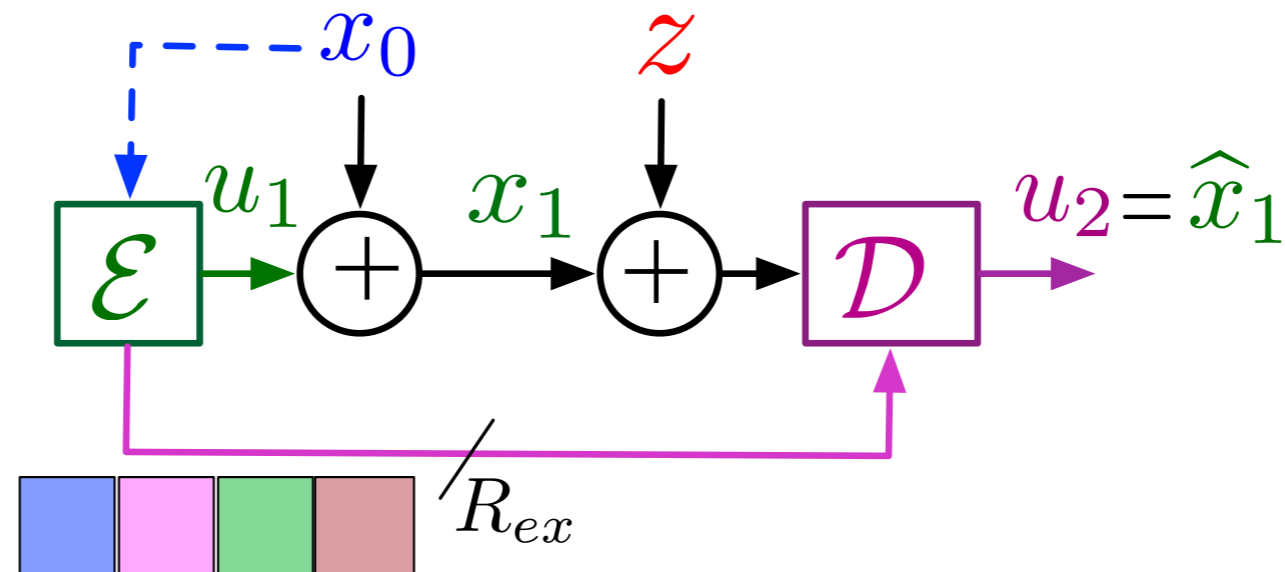
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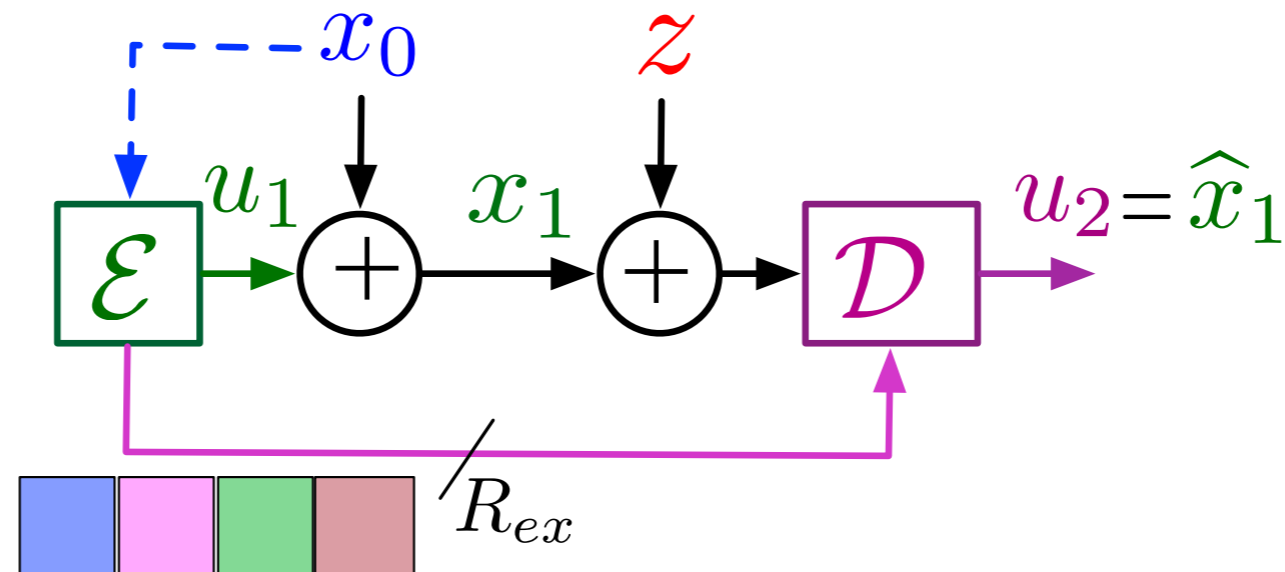


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Moral:

More efficient: think “parallel channels”!



[Grover, Sahai '10]

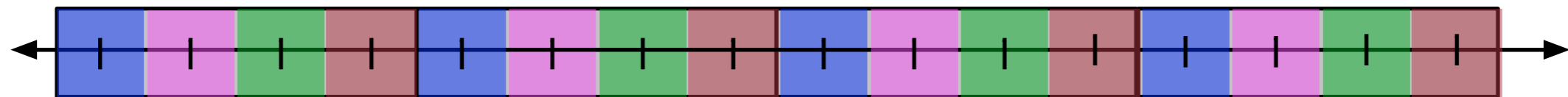
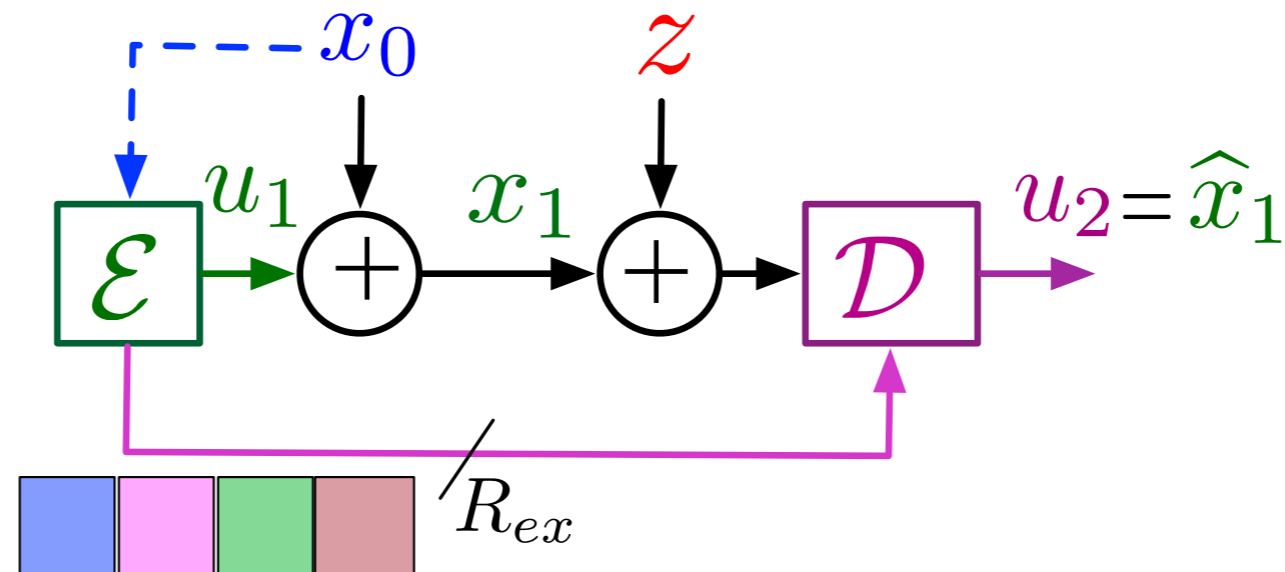


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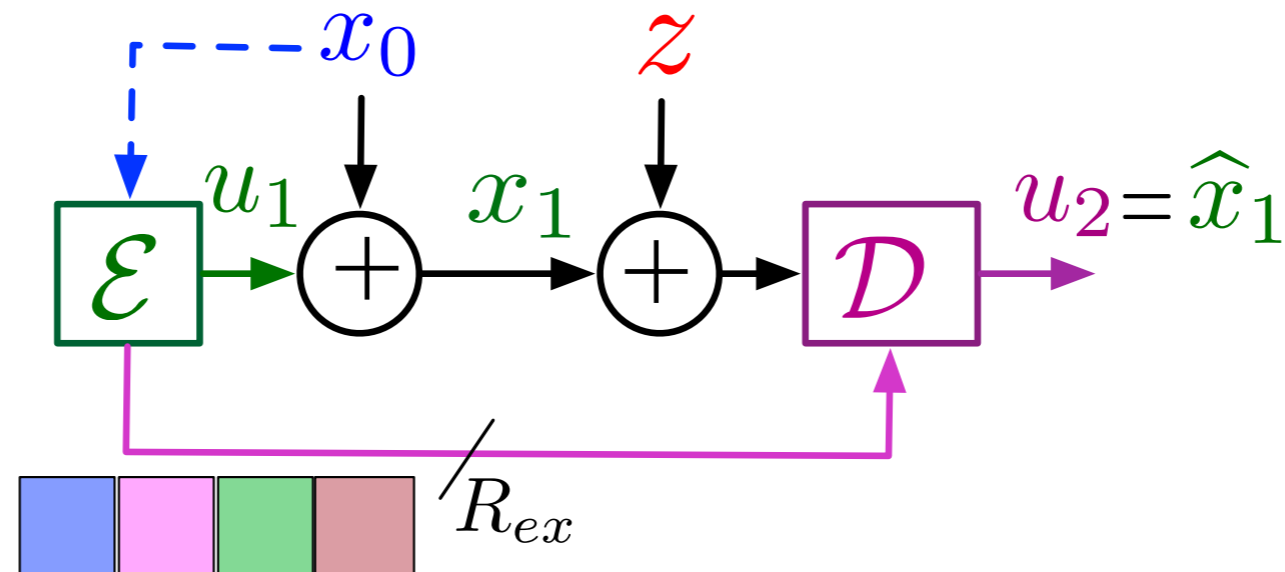
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signal most significant bits through plant

More efficient: think “parallel channels”!



[Grover, Sahai '10]



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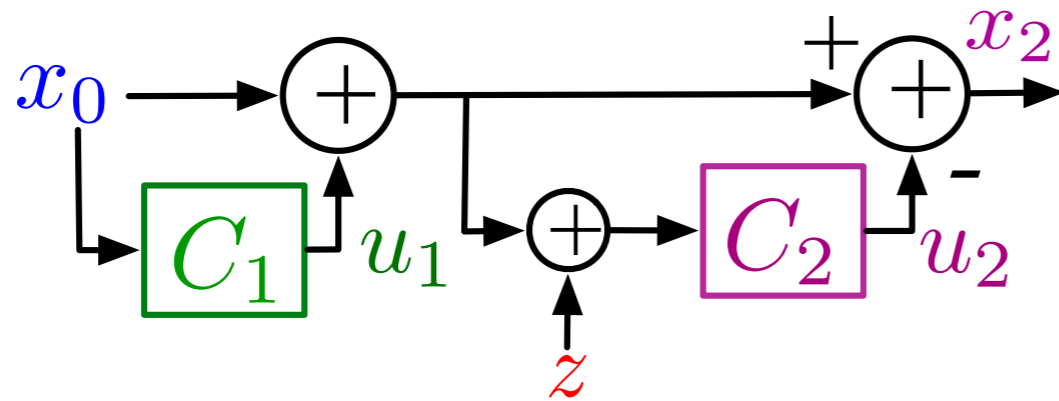
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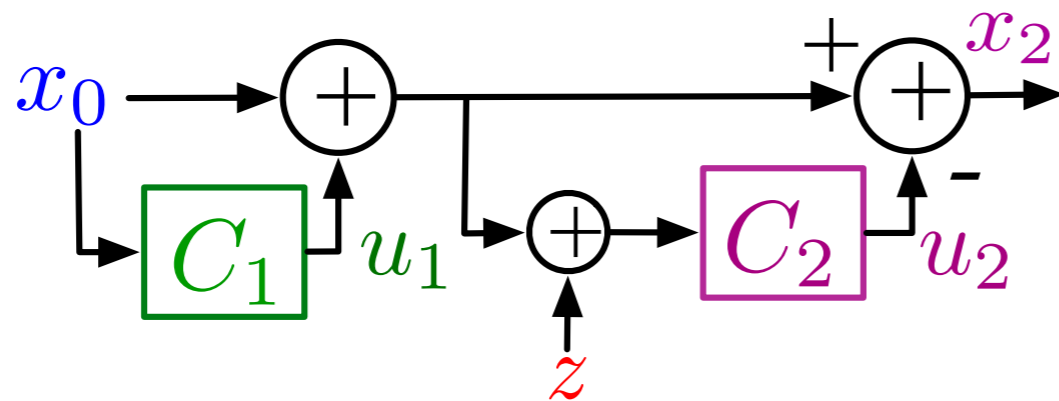
signal most significant bits through plant

Signal even when communication channel is present

Is the counterexample **relevant**? Part II

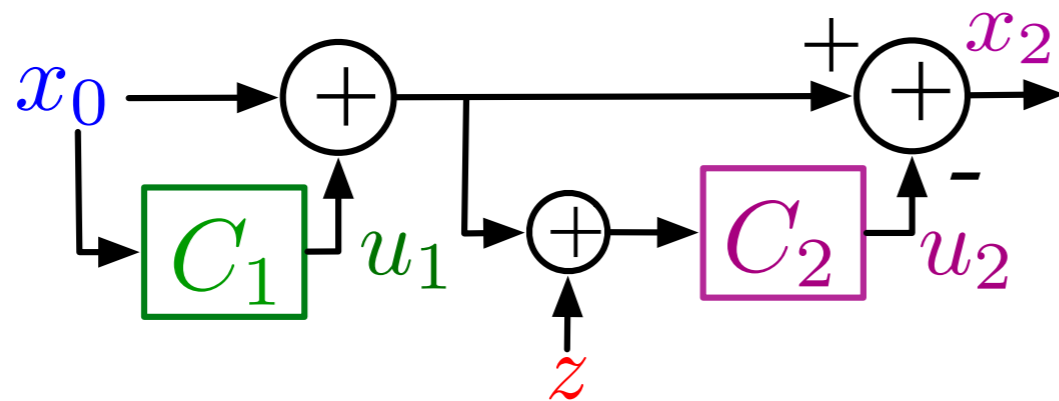


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$$\min_{x_0, z} \max \frac{k^2 u_1^2 + x_2^2}{x_0^2 + z^2}$$

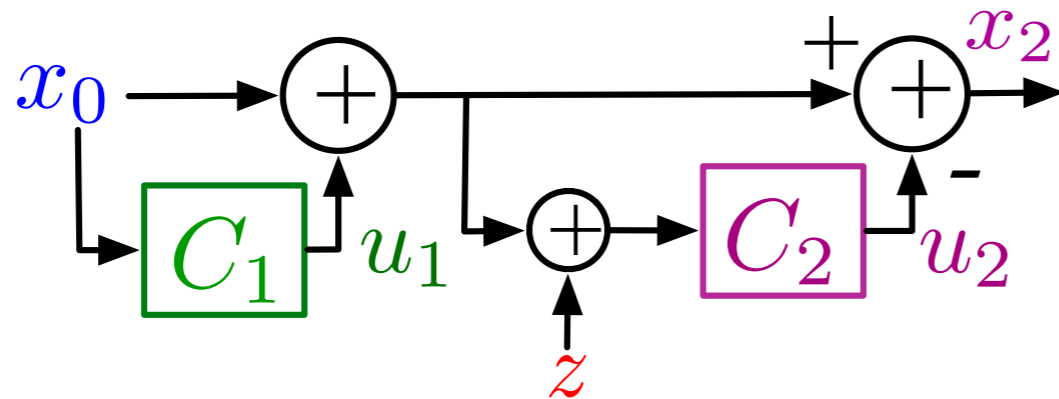
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Induced norm: linear strategies optimal! [Rotkowitz '06]

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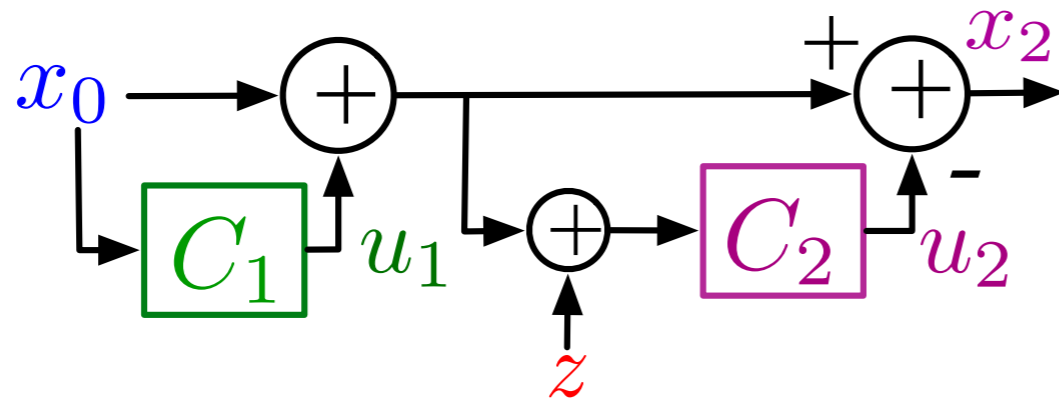


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Witsenhausen's LQG formulation:

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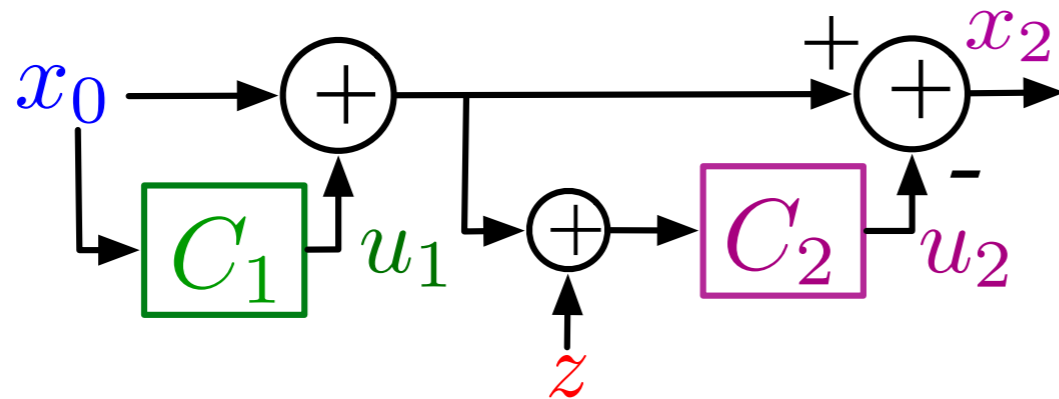


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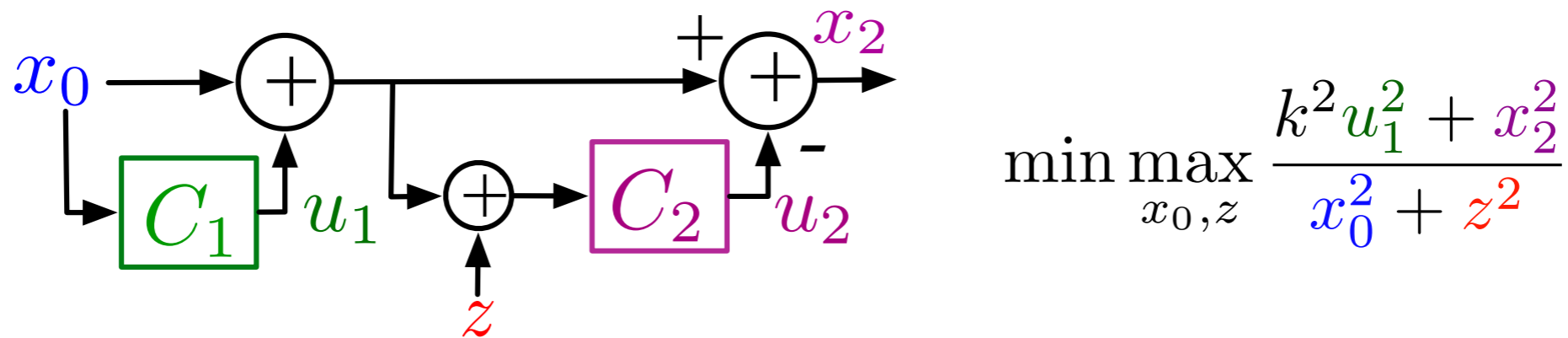


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harder!

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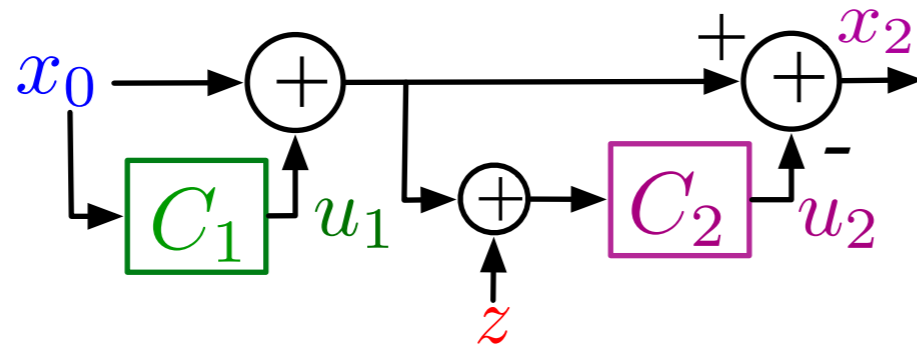
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Q2. Why consider the quadratic norm?

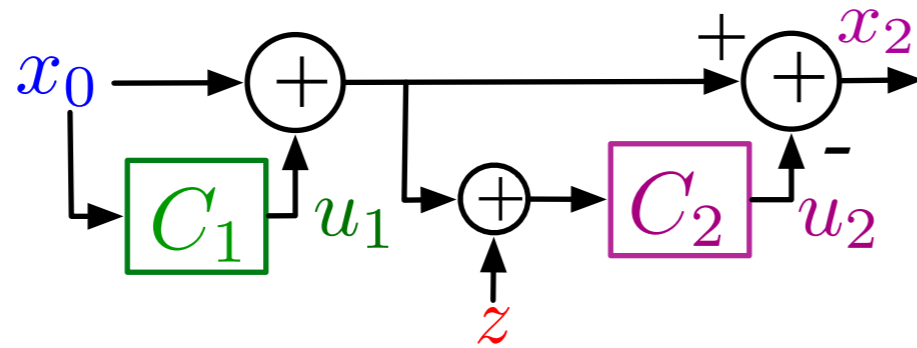
[John Doyle, "Paths ahead", MIT, 2009]

Induced norm formulation too conservative?



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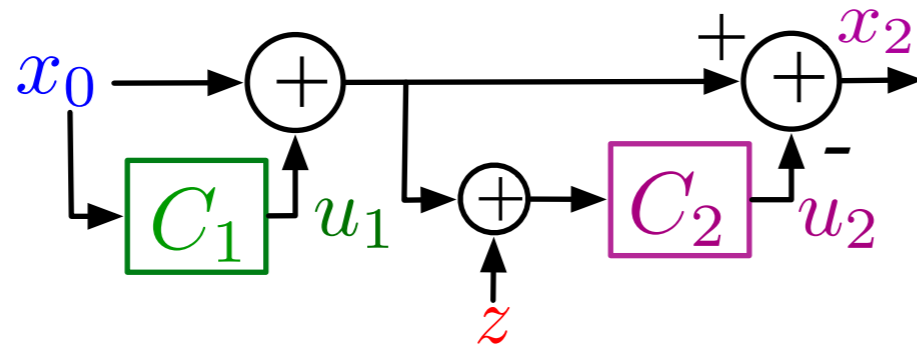
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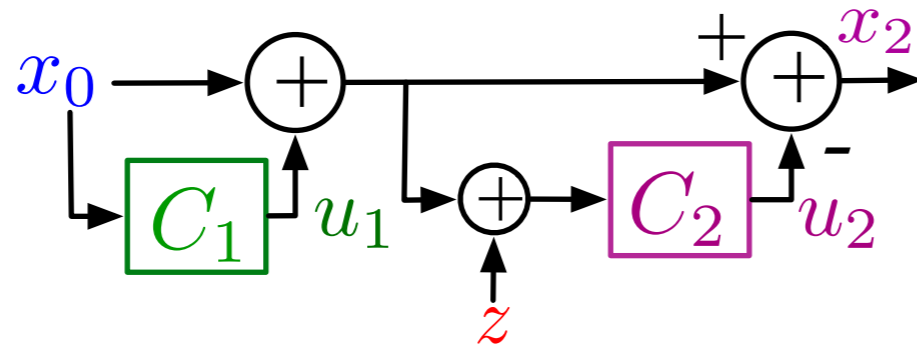


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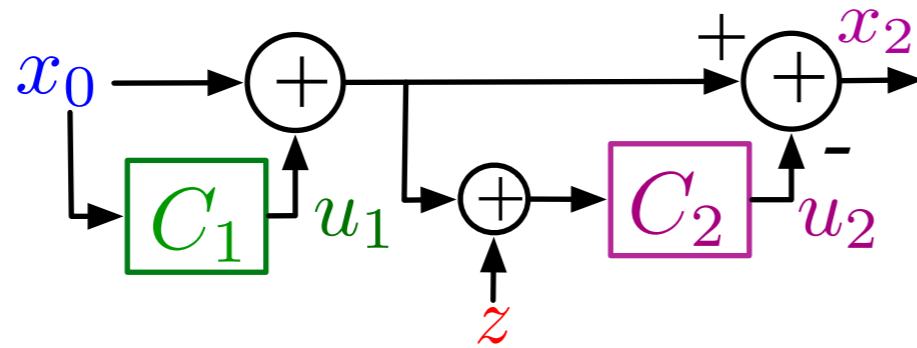
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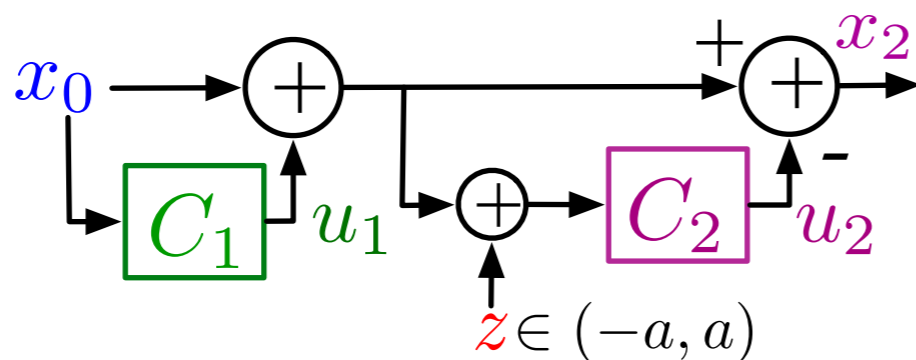


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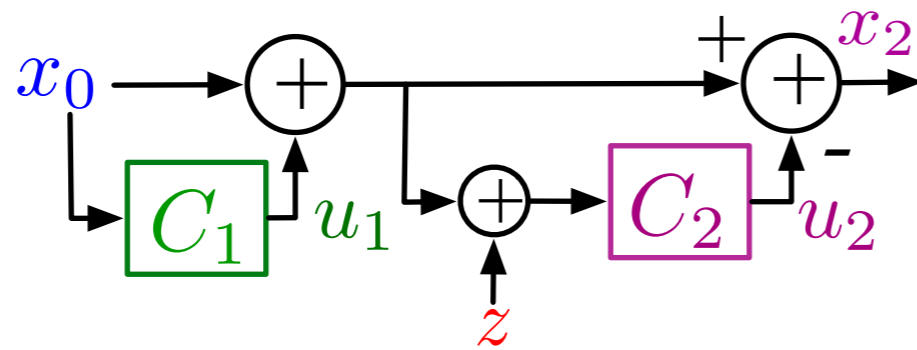
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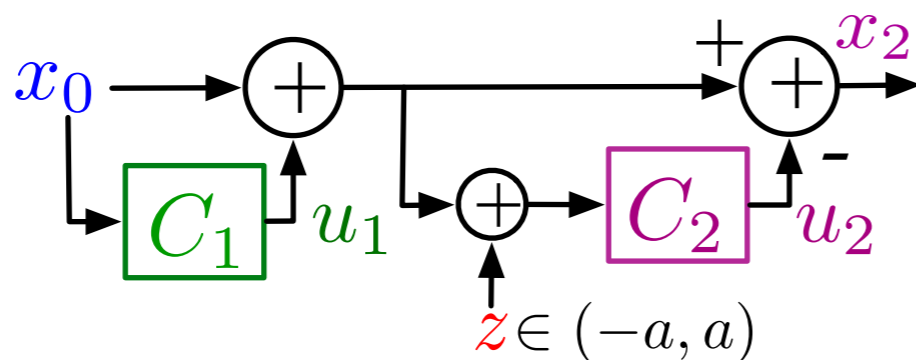
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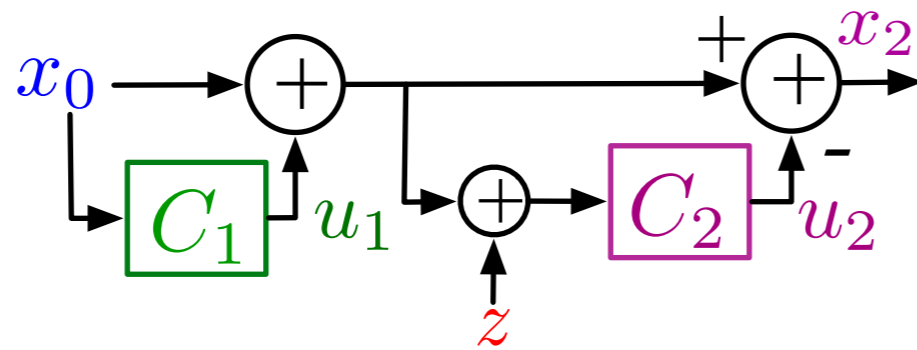
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A quadratic cost

$$\min_{x_0, z \in (-a, a)} \max k^2 u_1^2 + x_2^2$$

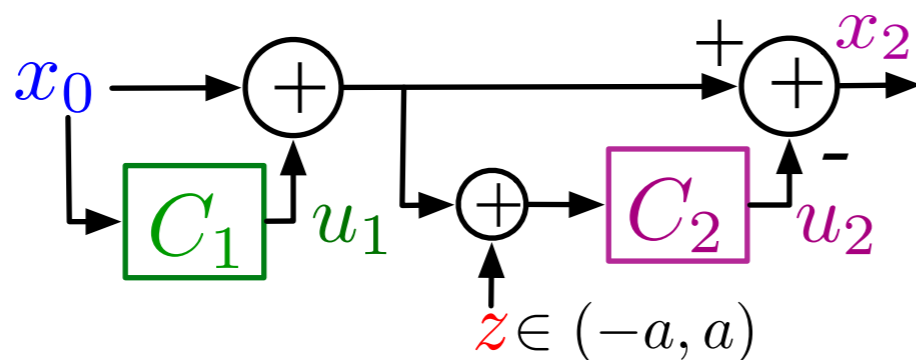
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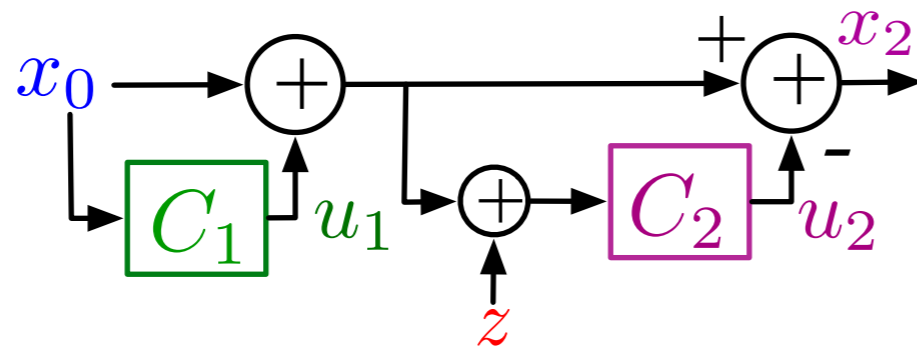


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Quantization (**signaling**) :

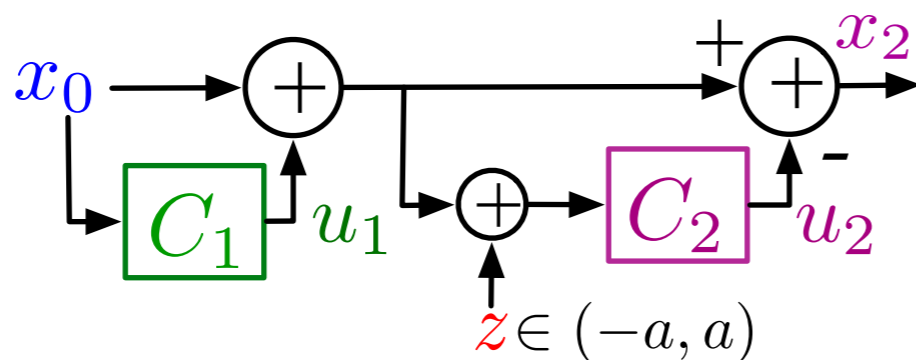
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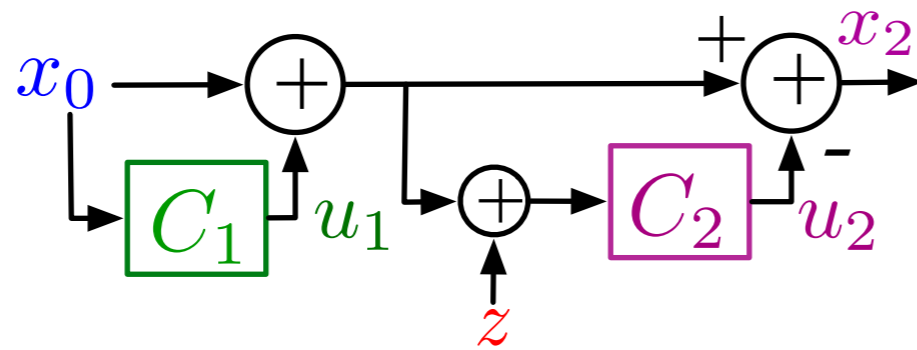


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Quantization (**signaling**): can outperform linear by an **unbounded** factor

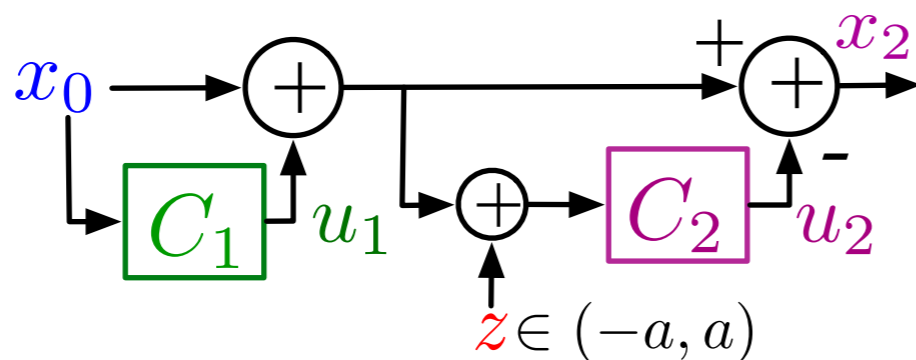
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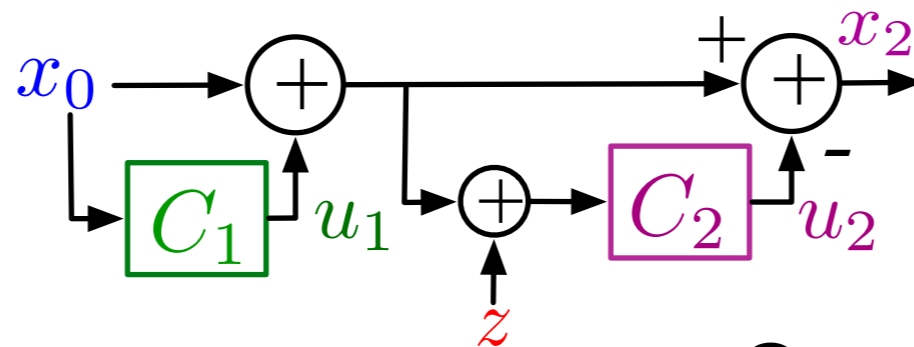


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Quantization (**signaling**): can outperform linear by an **unbounded** factor
is **approximately optimal!**

Which is a “right” norm?



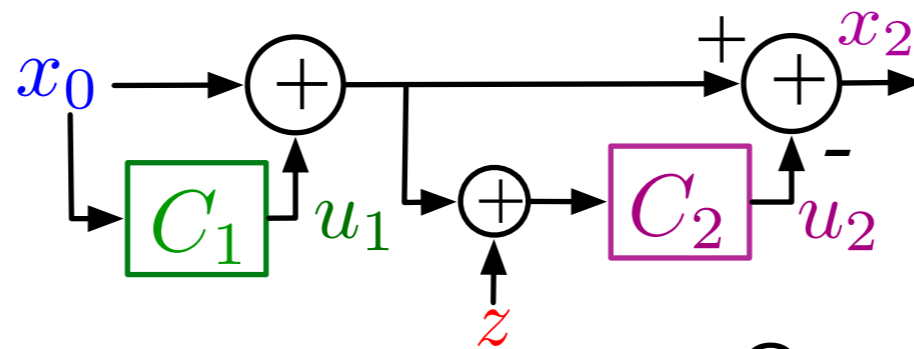
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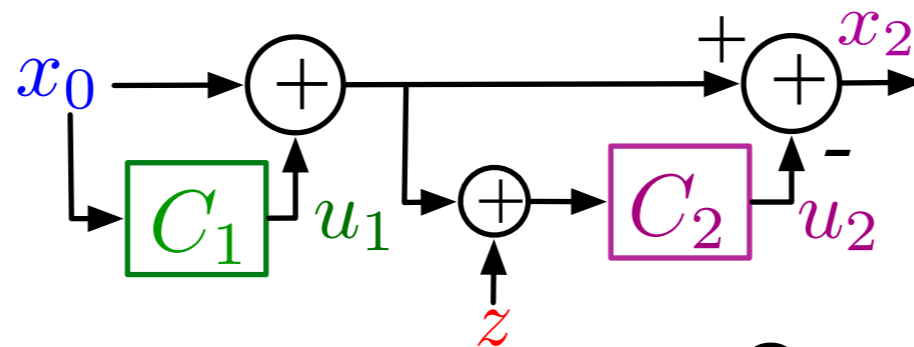
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Induced norm: cost normalized by **size of state and noise**.

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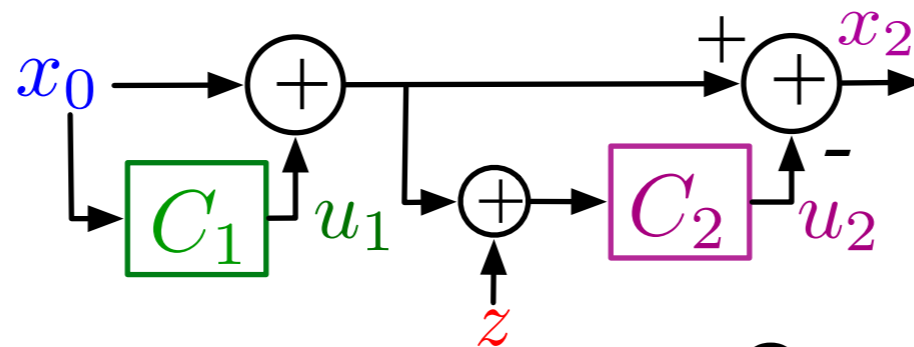
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In many practical cases, small perturbations and small noises **do cost less!**

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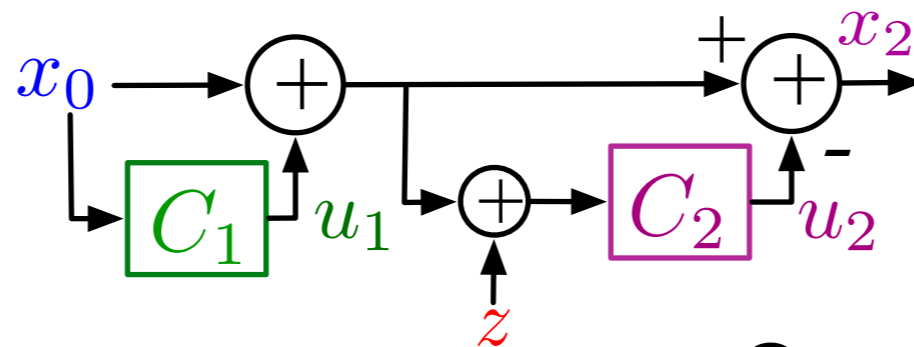
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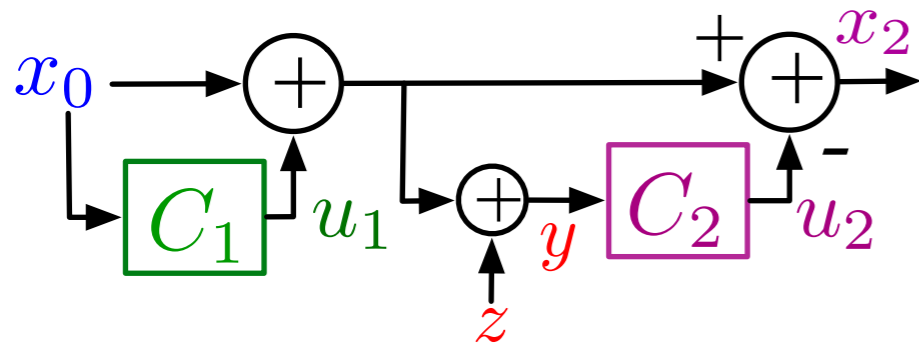
Moral: **signal** even in **adversarial** situations

What problems could we address now?

Goal: control a system using decentralized connected controllers

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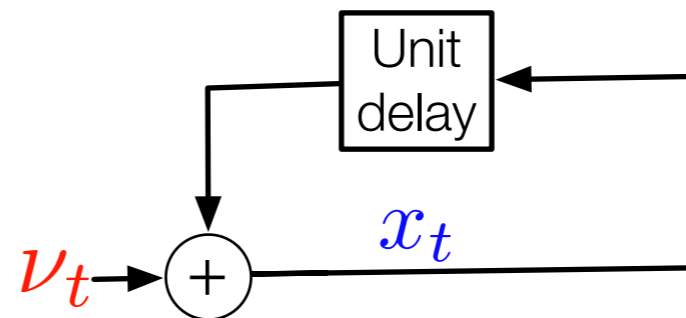
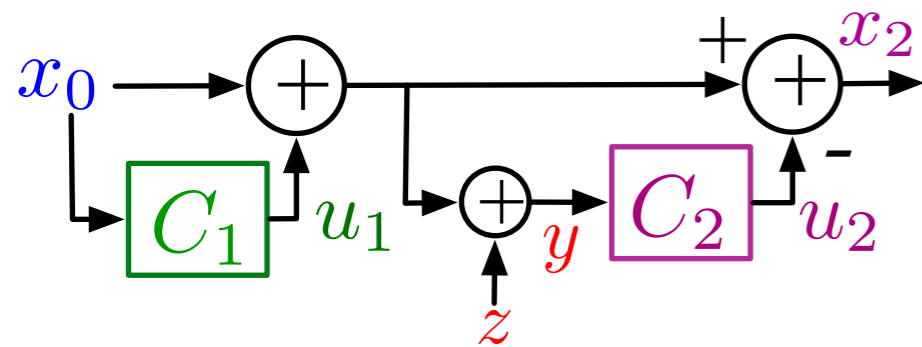
Goal: control a system using decentralized connected controllers



Witsenhausen's counterexample

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Goal: control a system using decentralized connected controllers



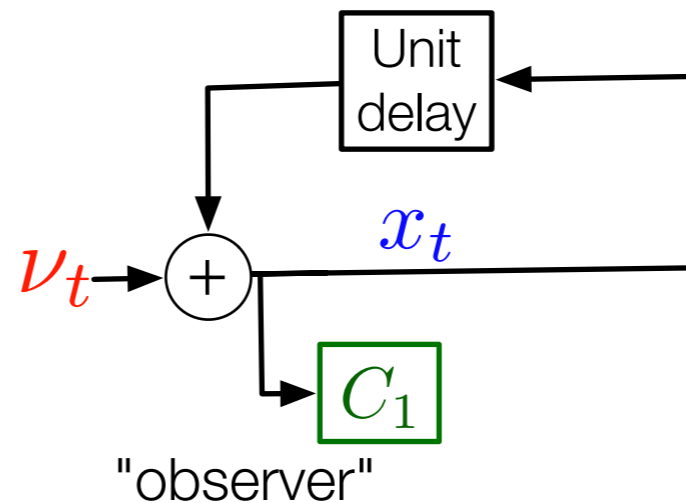
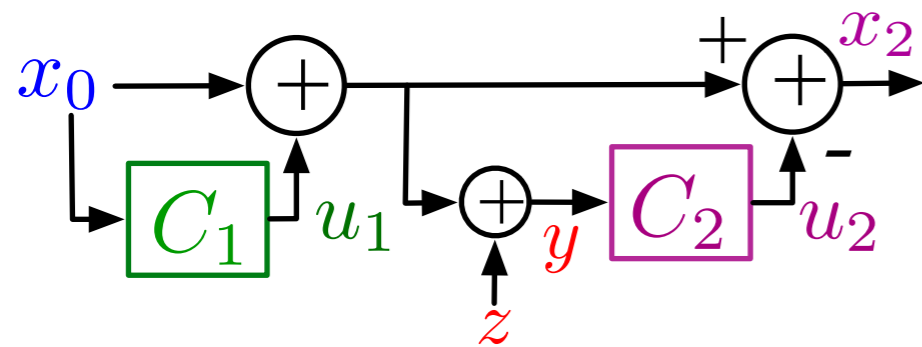
Witsenhausen's counterexample

communicating an evolving source

- [Wong, Brockett '90]
- [Borkar, Mitter '95]
- [Tatikonda, Mitter '01]
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What problems could we address now?

Goal: control a system using decentralized connected controllers



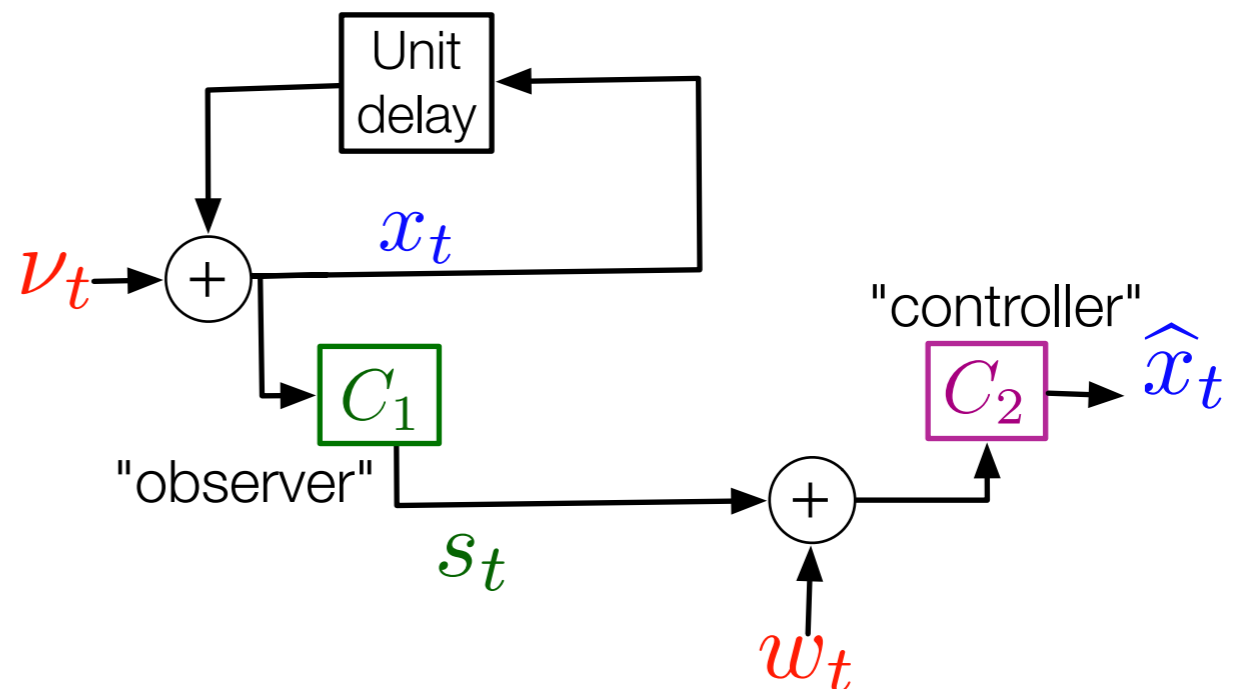
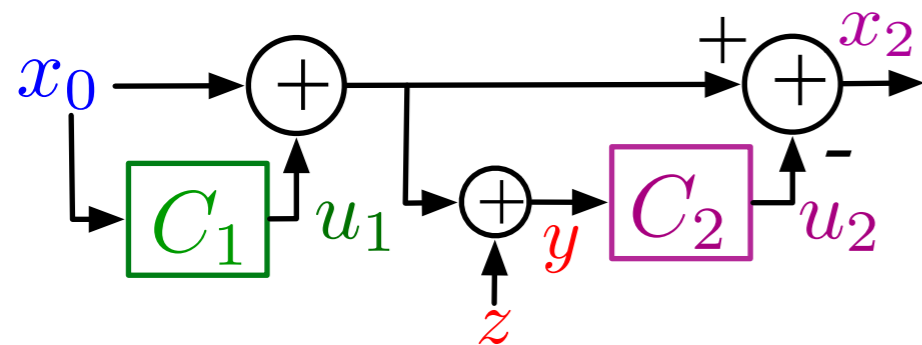
Witsenhausen's counterexample

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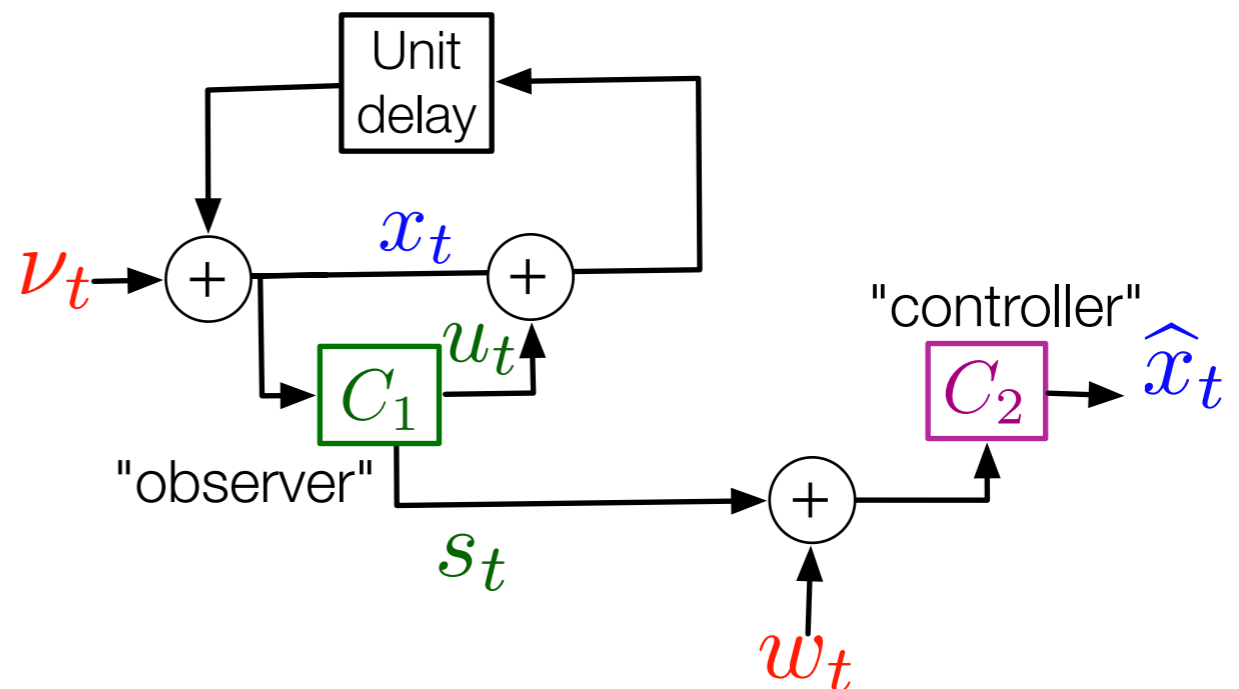
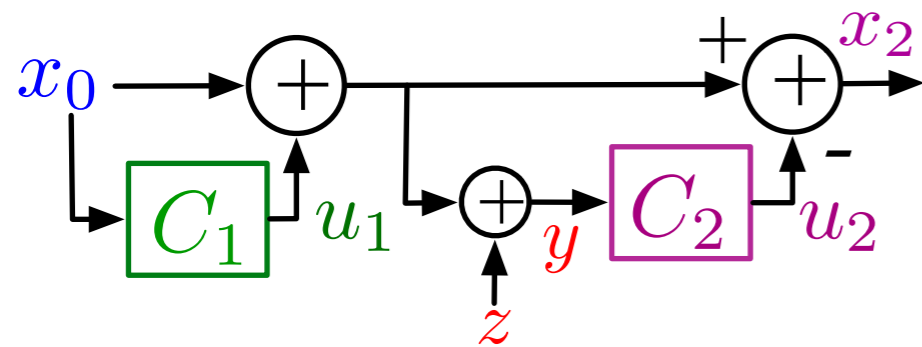
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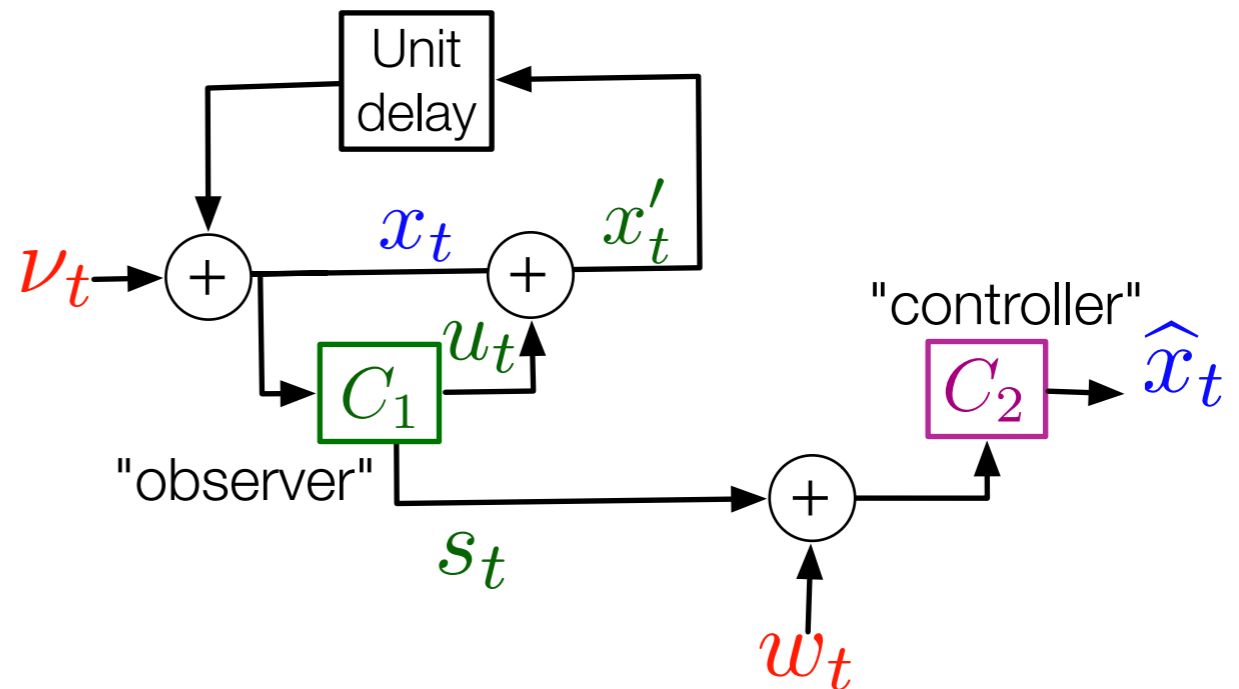
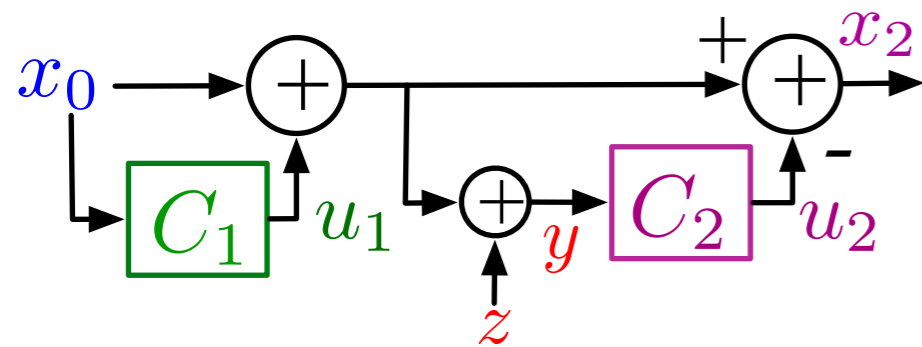
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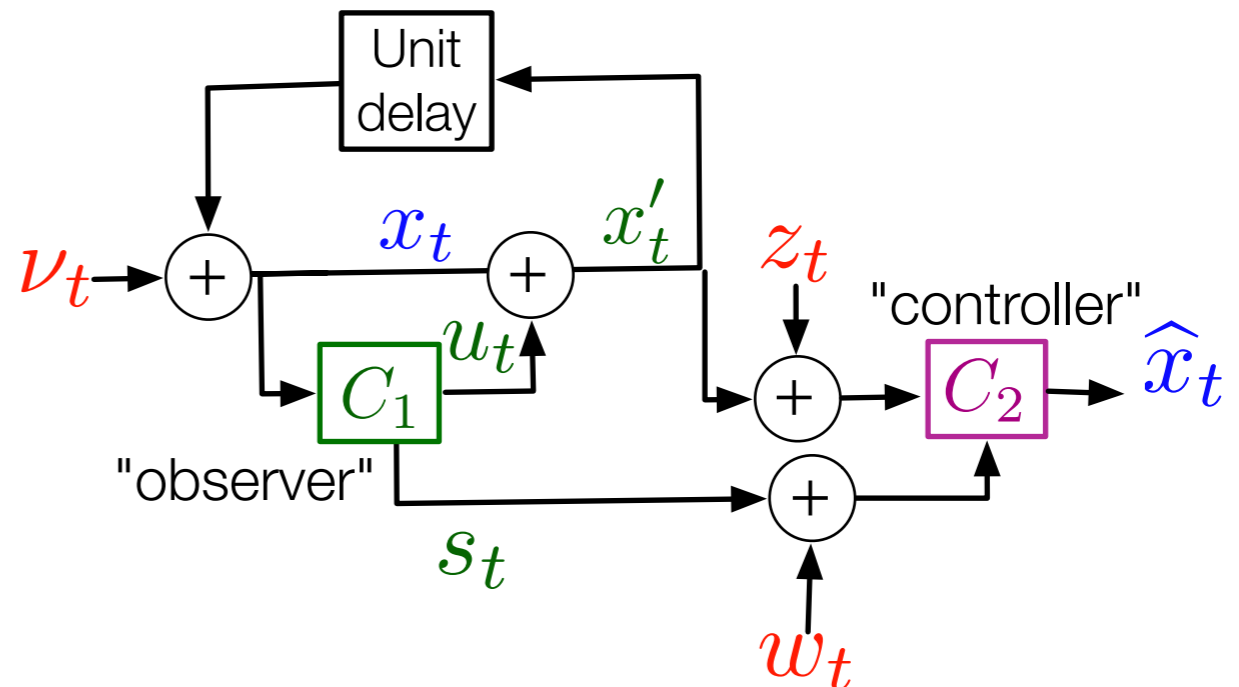
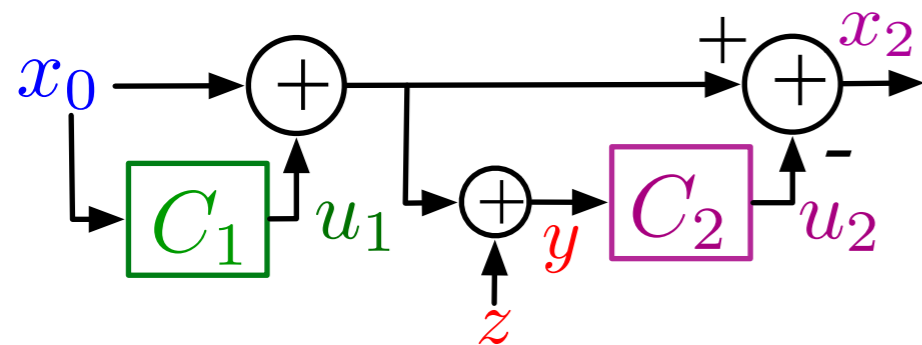
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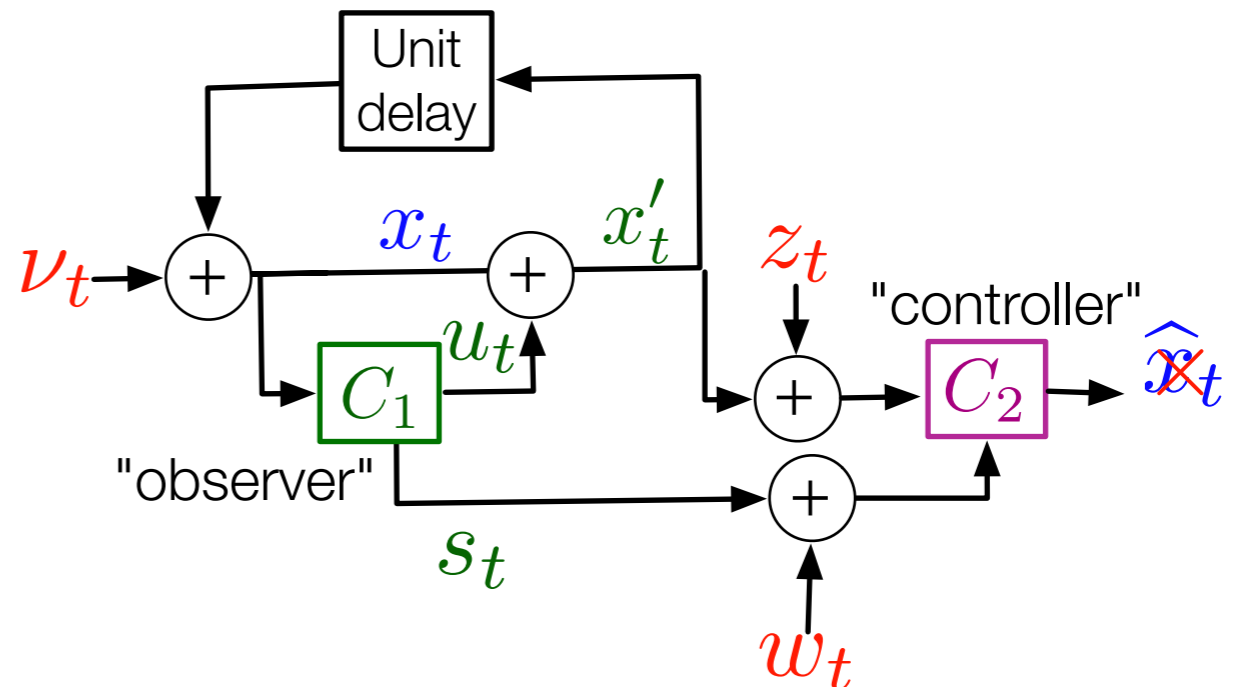
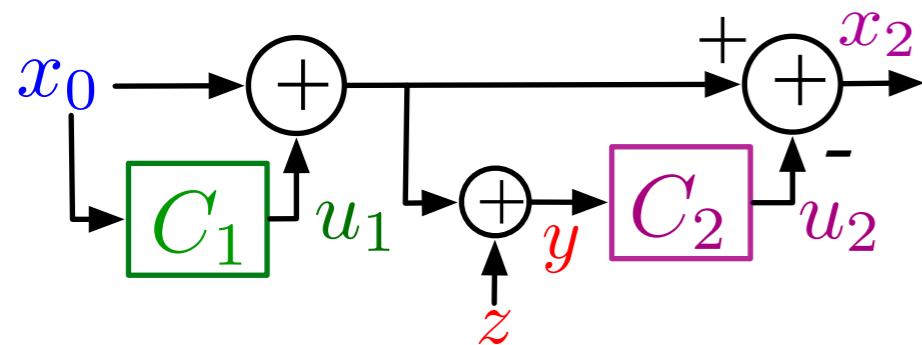
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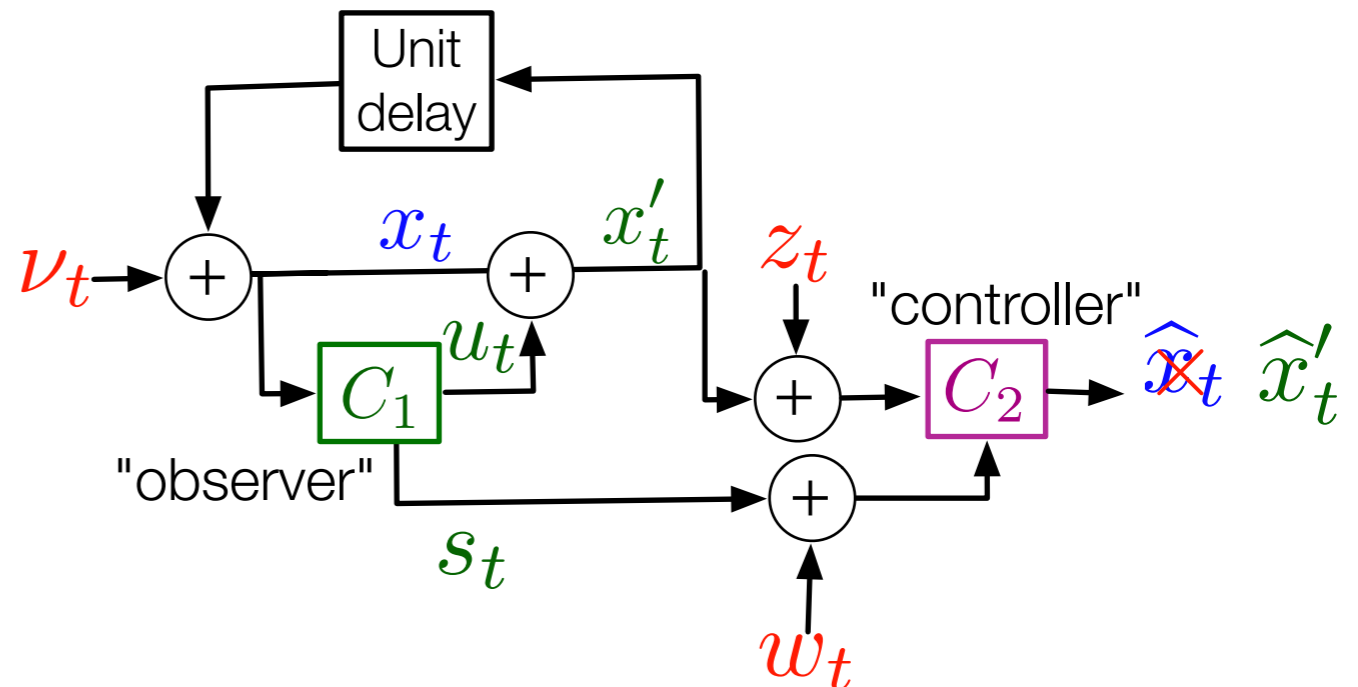
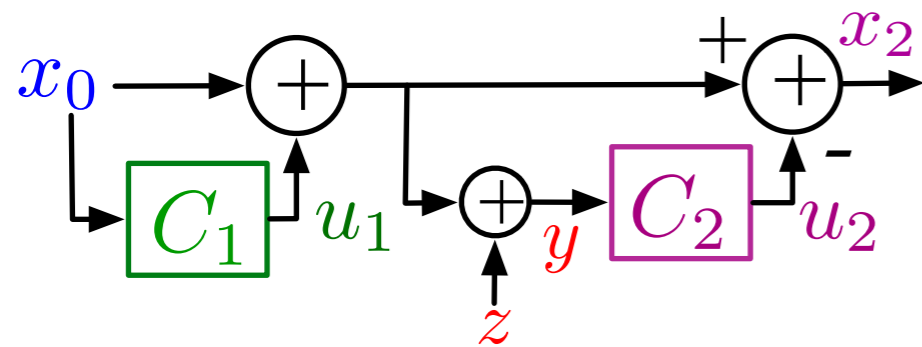
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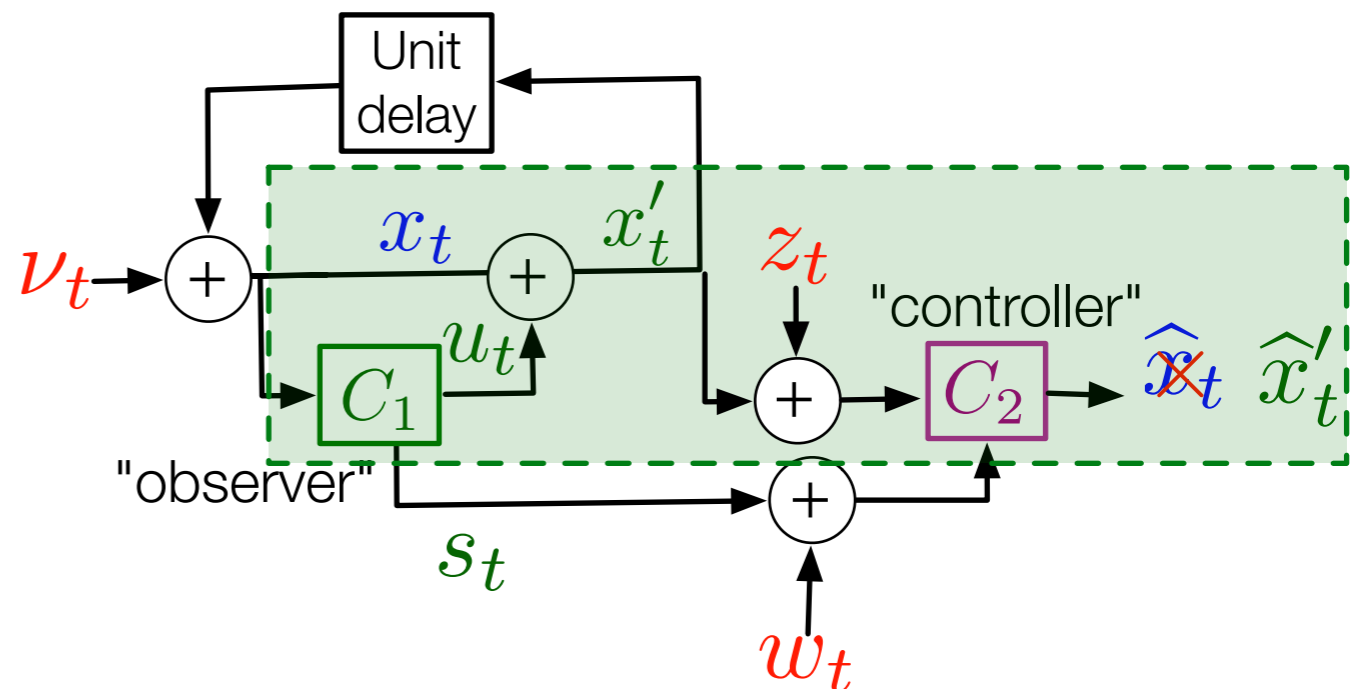
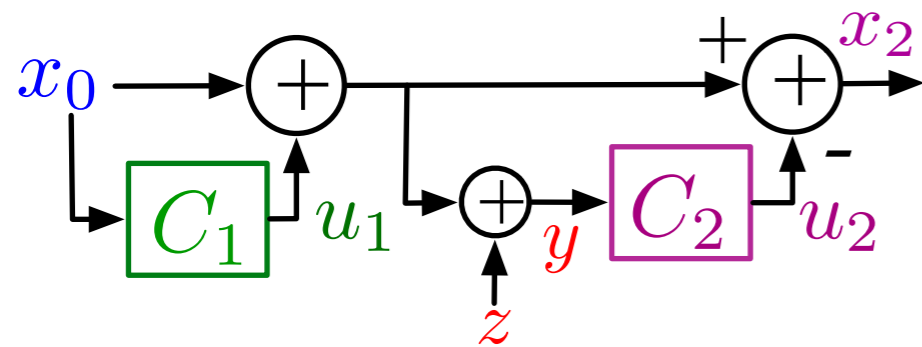
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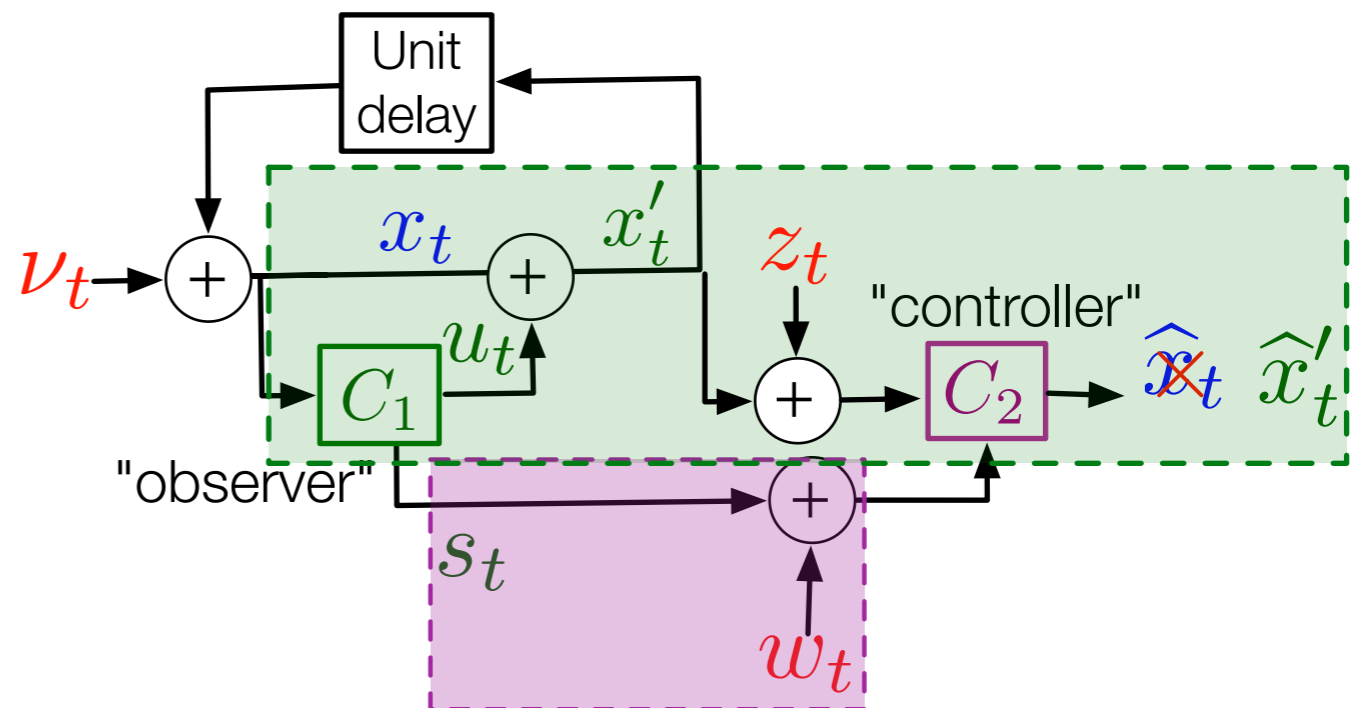
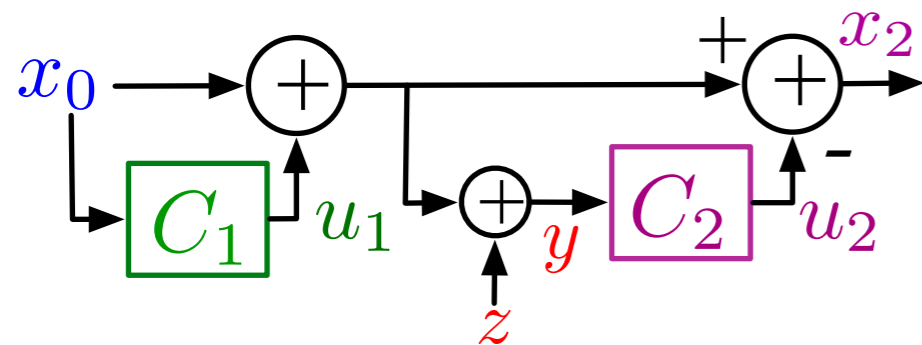
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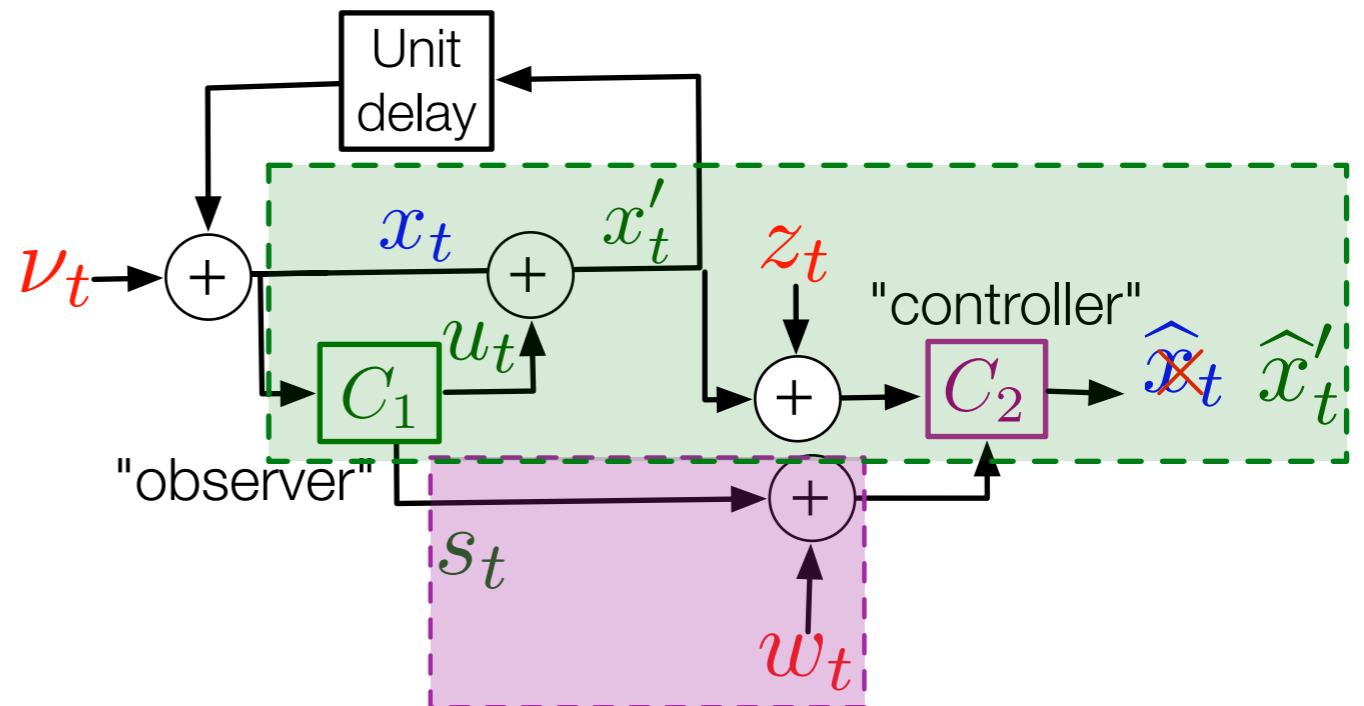
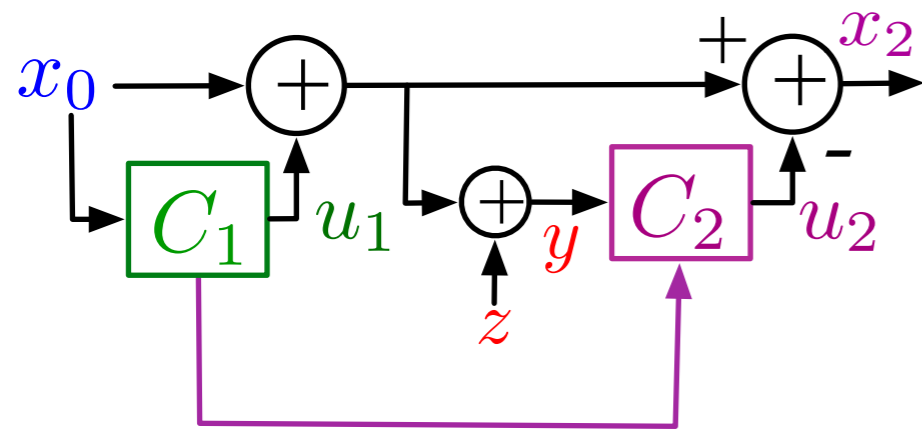
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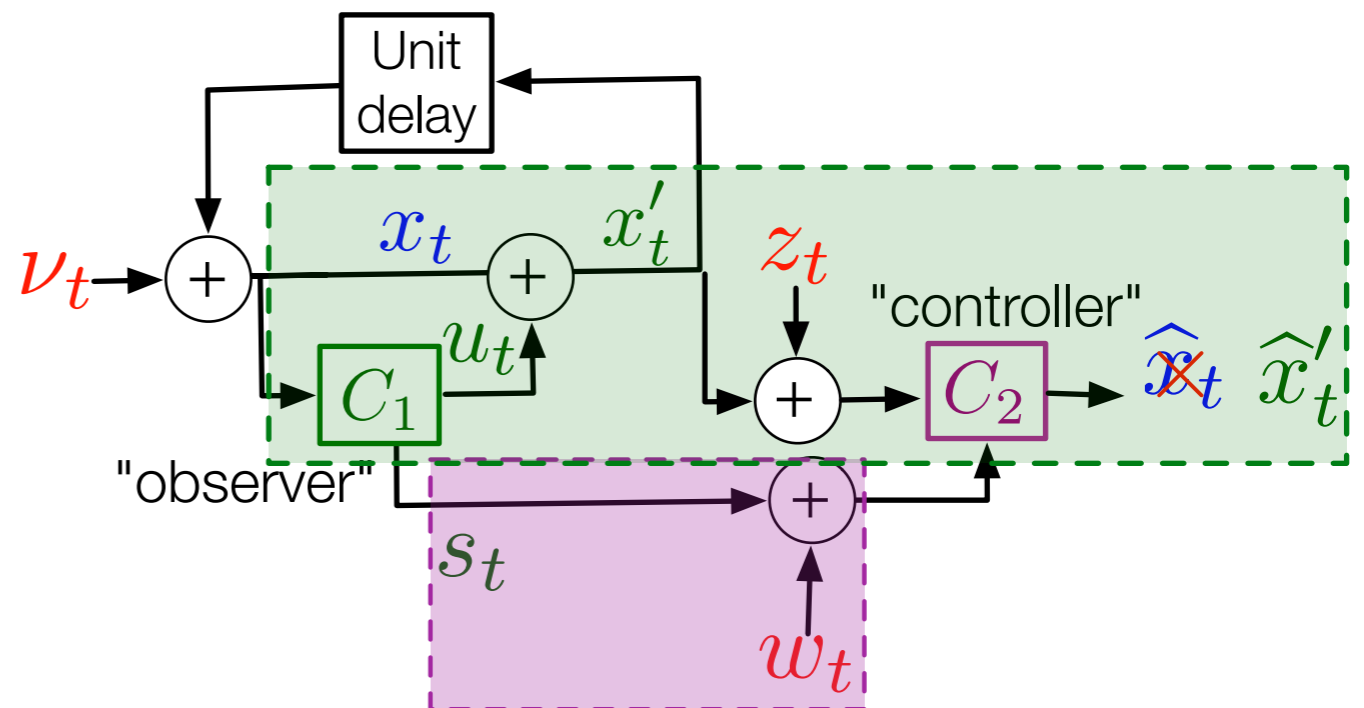
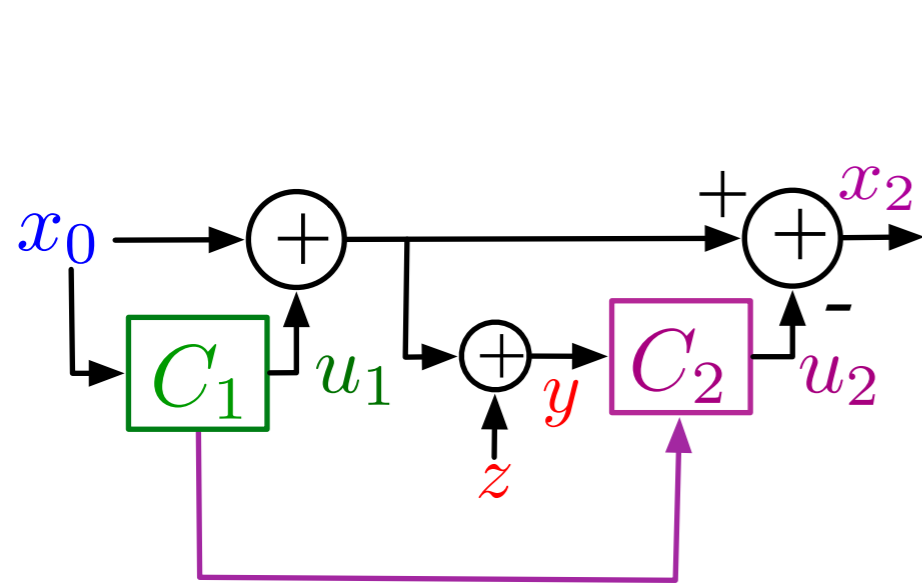
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Witsenhausen's counterexample "actively" communicating an evolving source

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Witsenhausen's Counterexample can help understand these larger problems