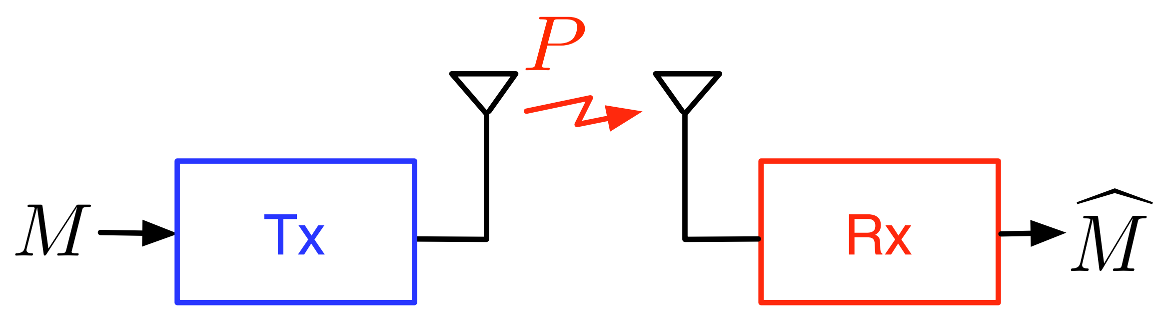
# Information theoretic tradeoffs of throughput and chip power consumption for decoding error correcting codes

Pulkit Grover, Hari Palaiyanur and Anant Sahai
UC Berkeley

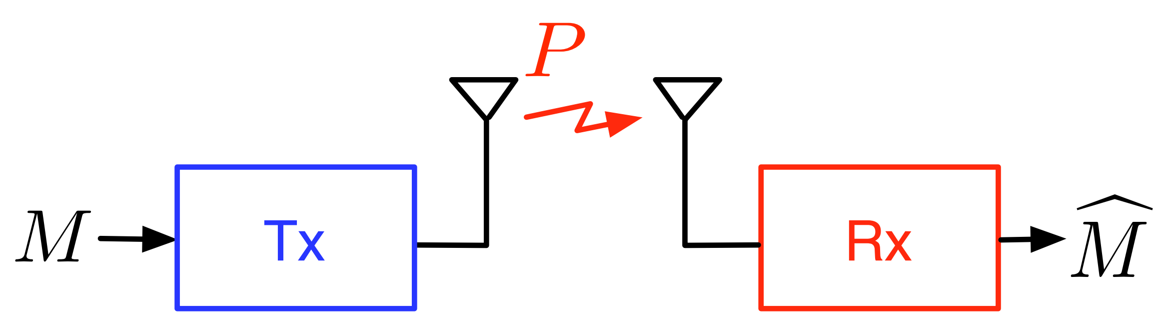
# Power in communication systems

A caricature information theorist's perspective :

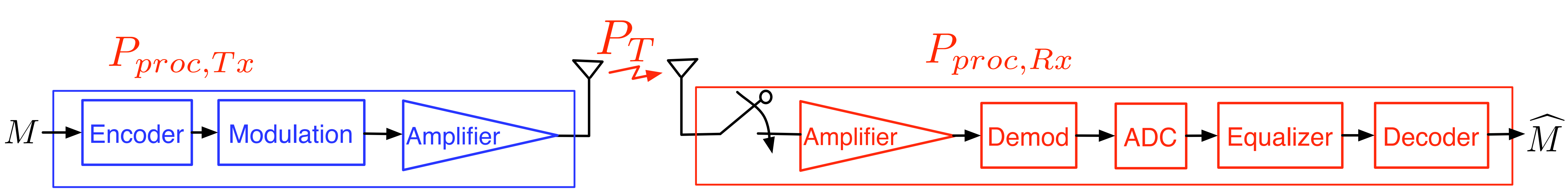


# Power in communication systems

A caricature information theorist's perspective :




A system designer's perspective :



# The disconnect

# The disconnect

"On the design of low-density parity-check codes. within 0.0045 dB of the Shannon limit."
[Chung, Forney, Richardson, Urbanke '01]

# The disconnect

"On the design of low-density parity-check codes. within 0.0045 dB of the Shannon limit."
[Chung, Forney, Richardson, Urbanke '01]

"A 47-Gbps LDPC decoder with improved low-error rate performance."
[Zheng, Anantharam, Wainwright, Nikolic '09].

# The disconnect

"On the design of low-density parity-check codes. within 0.0045 dB of the Shannon limit."
[Chung, Forney, Richardson, Urbanke '01]

| $d_l$ | 100 | | 200 | | 8000 | |
|---|---|---|---|---|---|---|
| | $x$ | $\lambda_x$ | $x$ | $\lambda_x$ | $x$ | $\lambda_x$ |
| | 2 | 0.170031 | 2 | 0.153425 | 2 | 0.096294 |
| | 3 | 0.160460 | 3 | 0.147526 | 3 | 0.095393 |
| | 6 | 0.112837 | 6 | 0.041539 | 6 | 0.033599 |
| | 7 | 0.047489 | 7 | 0.147551 | 7 | 0.091918 |
| | 10 | 0.011481 | 18 | 0.047938 | 15 | 0.031642 |
| | 11 | 0.091537 | 19 | 0.119555 | 20 | 0.086563 |
| | 26 | 0.152978 | 55 | 0.036379 | 50 | 0.093896 |
| | 27 | 0.036131 | 56 | 0.126714 | 70 | 0.006035 |
| | 100 | 0.217056 | 200 | 0.179373 | 100 | 0.018375 |
| | | | | | 150 | 0.086919 |
| | | | | | 400 | 0.089018 |
| | | | | | 900 | 0.057176 |
| | | | | | 2000 | 0.085816 |
| | | | | | 3000 | 0.006163 |
| | | | | | 6000 | 0.003028 |
| | | | | | 8000 | 0.118165 |
| $\rho_{av}$ | 10.9375 | | 12.0000 | | 18.5000 | |
| $\sigma$ | 0.97592 | | 0.97704 | | 0.9781869 | |
| $\mathrm{SNR}_{\mathrm{norm}}$ | 0.0247 | | 0.0147 | | 0.00450 | |

"A 47-Gbps LDPC decoder with improved low-error rate performance."
[Zheng, Anantharam, Wainwright, Nikolic '09].

# The disconnect

"On the design of low-density parity-check codes. within 0.0045 dB of the Shannon limit."
[Chung, Forney, Richardson, Urbanke '01]

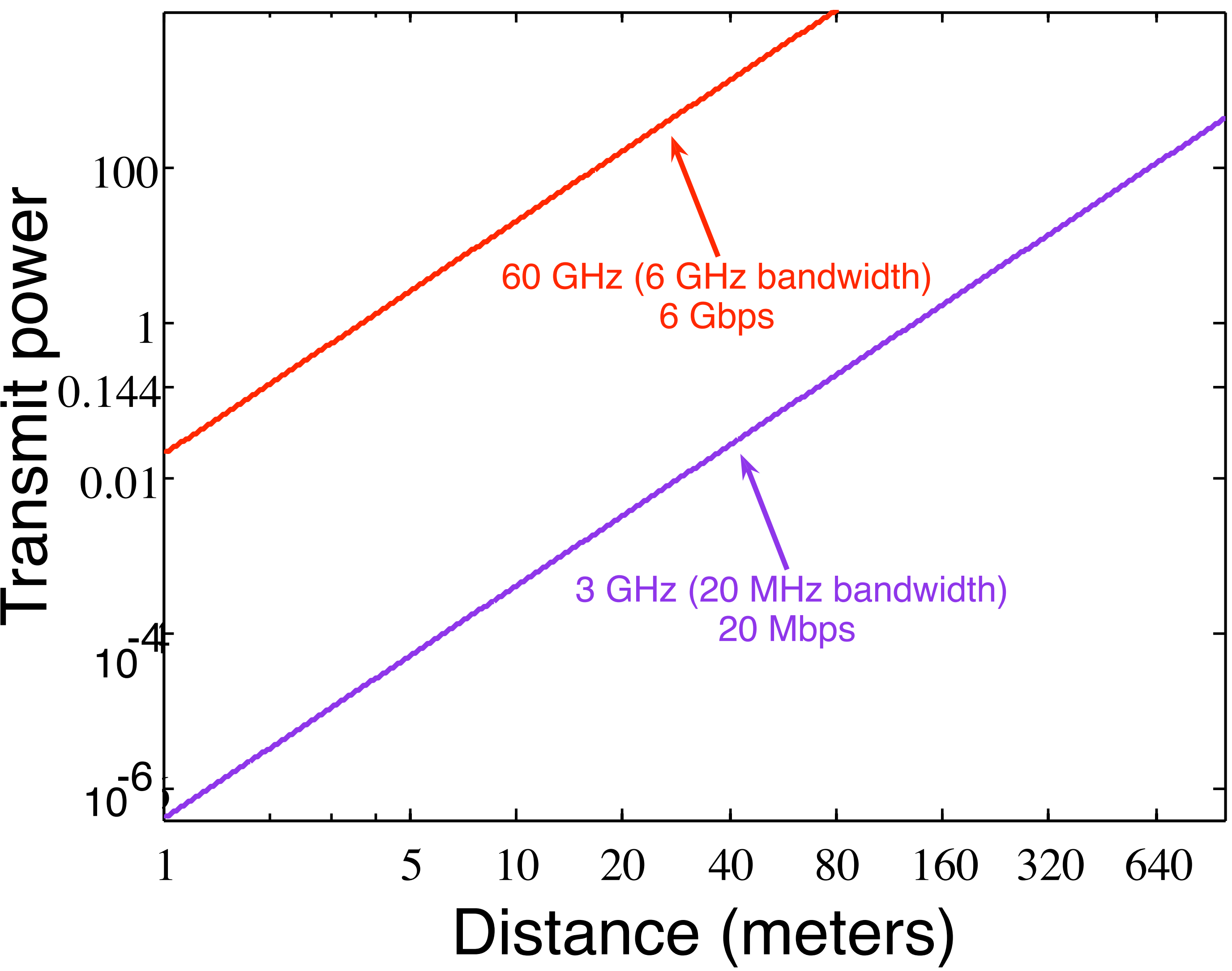
| $d_l$ | 100 | | 200 | | 8000 | |
|---|---|---|---|---|---|---|
| | $x$ | $\lambda_x$ | $x$ | $\lambda_x$ | $x$ | $\lambda_x$ |
| | 2 | 0.170031 | 2 | 0.153425 | 2 | 0.096294 |
| | 3 | 0.160460 | 3 | 0.147526 | 3 | 0.095393 |
| | 6 | 0.112837 | 6 | 0.041539 | 6 | 0.033599 |
| | 7 | 0.047489 | 7 | 0.147551 | 7 | 0.091918 |
| | 10 | 0.011481 | 18 | 0.047938 | 15 | 0.031642 |
| | 11 | 0.091537 | 19 | 0.119555 | 20 | 0.086563 |
| | 26 | 0.152978 | 55 | 0.036379 | 50 | 0.093896 |
| | 27 | 0.036131 | 56 | 0.126714 | 70 | 0.006035 |
| | 100 | 0.217056 | 200 | 0.179373 | 100 | 0.018375 |
| | | | | | 150 | 0.086919 |
| | | | | | 400 | 0.089018 |
| | | | | | 900 | 0.057176 |
| | | | | | 2000 | 0.085816 |
| | | | | | 3000 | 0.006163 |
| | | | | | 6000 | 0.003028 |
| | | | | | 8000 | 0.118165 |
| $\rho_{av}$ | 10.9375 | | 12.0000 | | 18.5000 | |
| $\sigma$ | 0.97592 | | 0.97704 | | 0.9781869 | |
| $\mathrm{SNR}_{\mathrm{norm}}$ | 0.0247 | | 0.0147 | | 0.00450 | |

"A 47-Gbps LDPC decoder with improved low-error rate performance."
[Zheng, Anantharam, Wainwright, Nikolic '09].

(6,32)-regular LDPC !

# The disconnect

"On the design of low-density parity-check codes. within 0.0045 dB of the Shannon limit."
[Chung, Forney, Richardson, Urbanke '01]

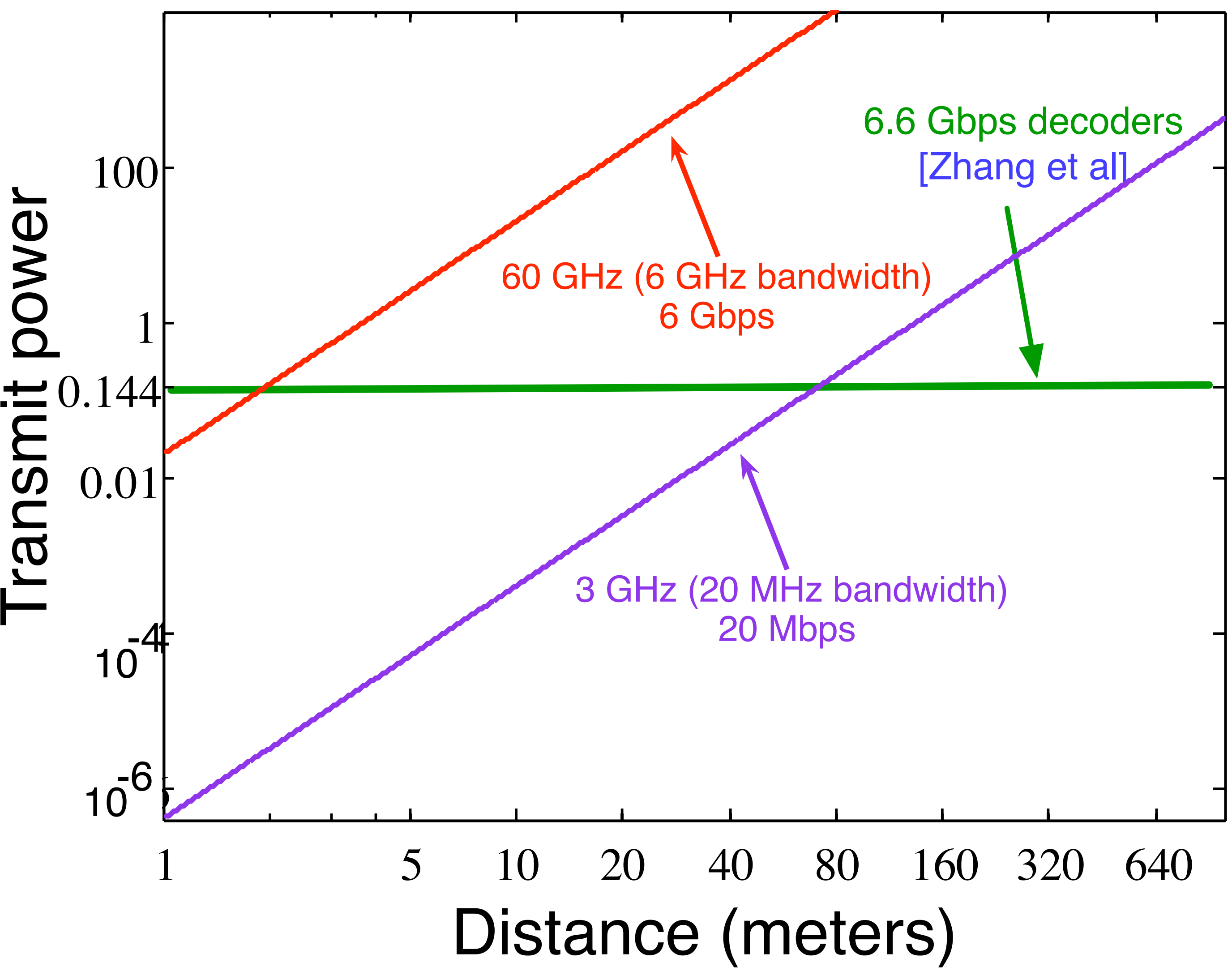



"A 47-Gbps LDPC decoder with improved low-error rate performance."
[Zheng, Anantharam, Wainwright, Nikolic '09].

(6,32)-regular LDPC !

# The disconnect

“On the design of low-density parity-check codes. within 0.0045 dB of the Shannon limit.” [Chung, Forney, Richardson, Urbanke ’01]

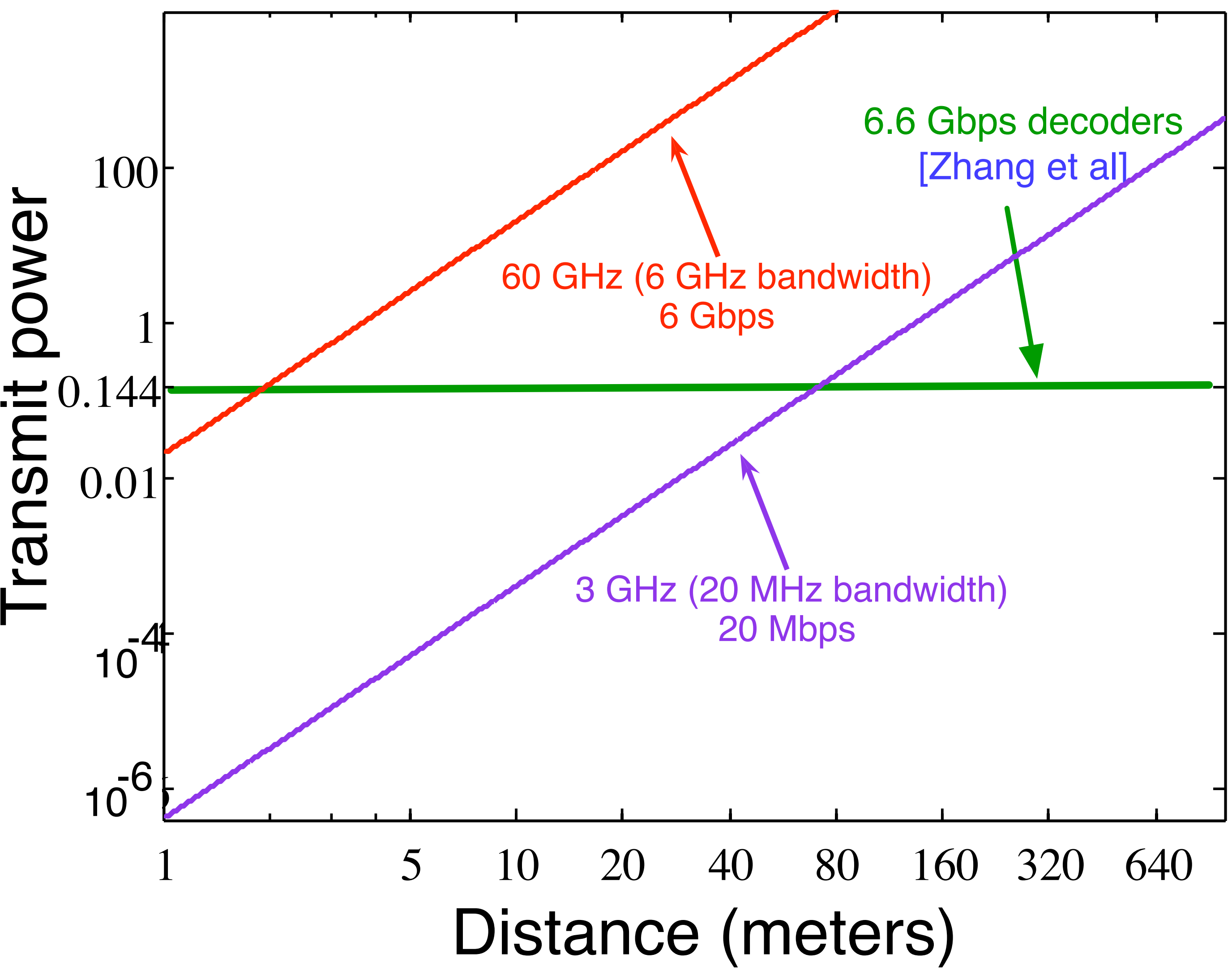



“A 47-Gbps LDPC decoder with improved low-error rate performance.” [Zheng, Anantharam, Wainwright, Nikolic ’09].

(6,32)-regular LDPC !

# The disconnect

"On the design of low-density parity-check codes. within 0.0045 dB of the Shannon limit."
[Chung, Forney, Richardson, Urbanke '01]



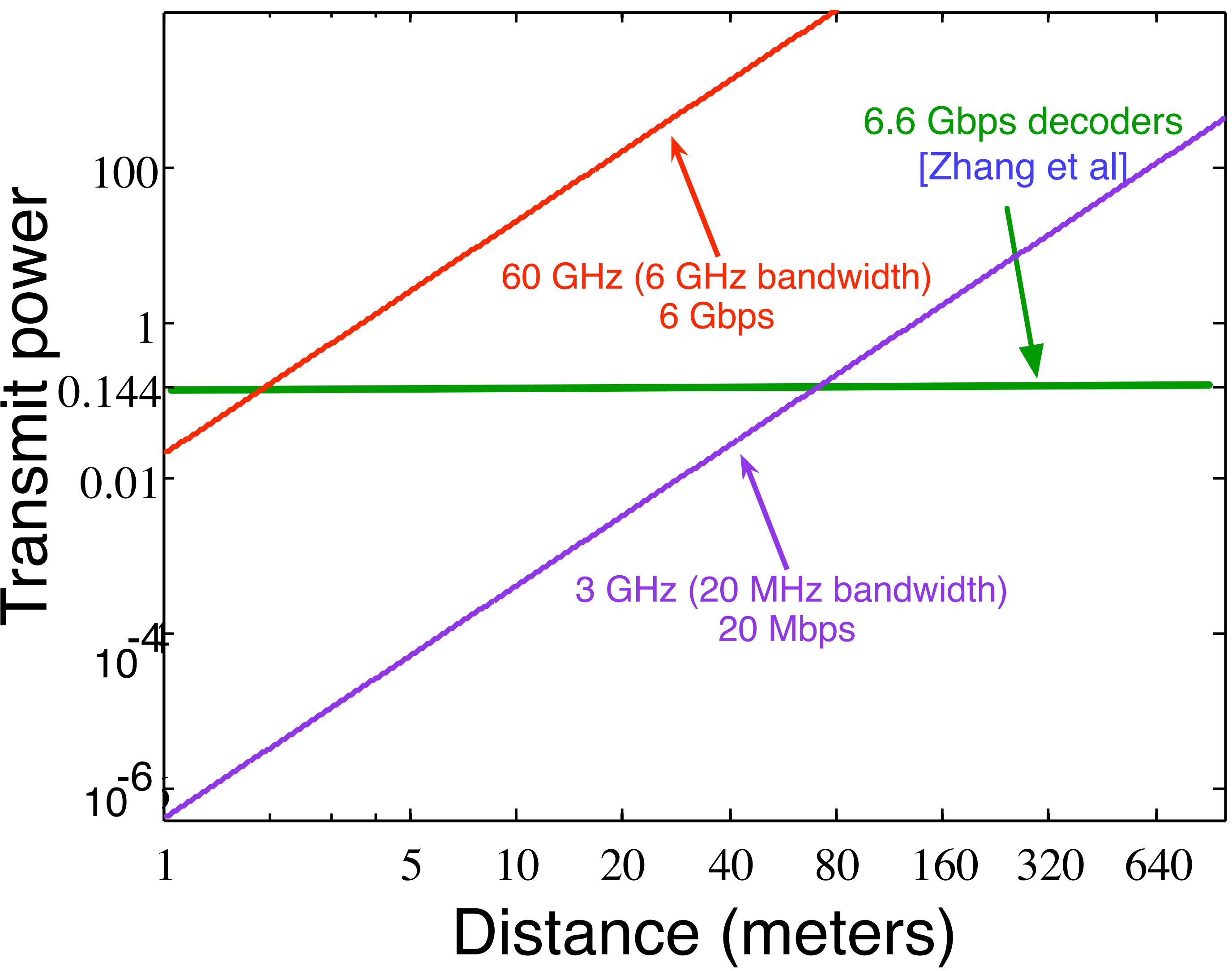

"A 47-Gbps LDPC decoder with improved low-error rate performance."
[Zheng, Anantharam, Wainwright, Nikolic '09].

(6,32)-regular LDPC !

"A 90nm CMOS Low-Power 60GHz Transceiver with Integrated Baseband Circuitry."
[Marcu et al. '09]

# The disconnect

"On the design of low-density parity-check codes. within 0.0045 dB of the Shannon limit."
[Chung, Forney, Richardson, Urbanke '01]




"A 47-Gbps LDPC decoder with improved low-error rate performance."
[Zheng, Anantharam, Wainwright, Nikolic '09].

(6,32)-regular LDPC !

"A 90nm CMOS Low-Power 60GHz Transceiver with Integrated Baseband Circuitry."
[Marcu et al. '09]

uncoded transmission!

# Organization

- Complexity as power [ISIT '08, '09]
  - a lower bound on complexity of message passing decoding
  - a power consumption model
- A refined model
  - large-deviations-based bounds on power and complexity
  - obtained via understanding erroneous decoding
  - a tradeoff between transmit power, decoding power, decoding throughput, rate, and error probability

# Complexity of message-passing decoding

# Complexity of message-passing decoding

Khandekar-McEliece conjecture

# Complexity of message-passing decoding

Khandekar-McEliece conjecture

$$\#\text{iterations} \geq \Omega\left(\frac{1}{C(P_T) - R}\right) + \Omega\left(\frac{1}{P_e}\right)$$

or

$$\Omega\left(\frac{1}{C(P_T) - R} \log \frac{1}{P_e}\right)$$

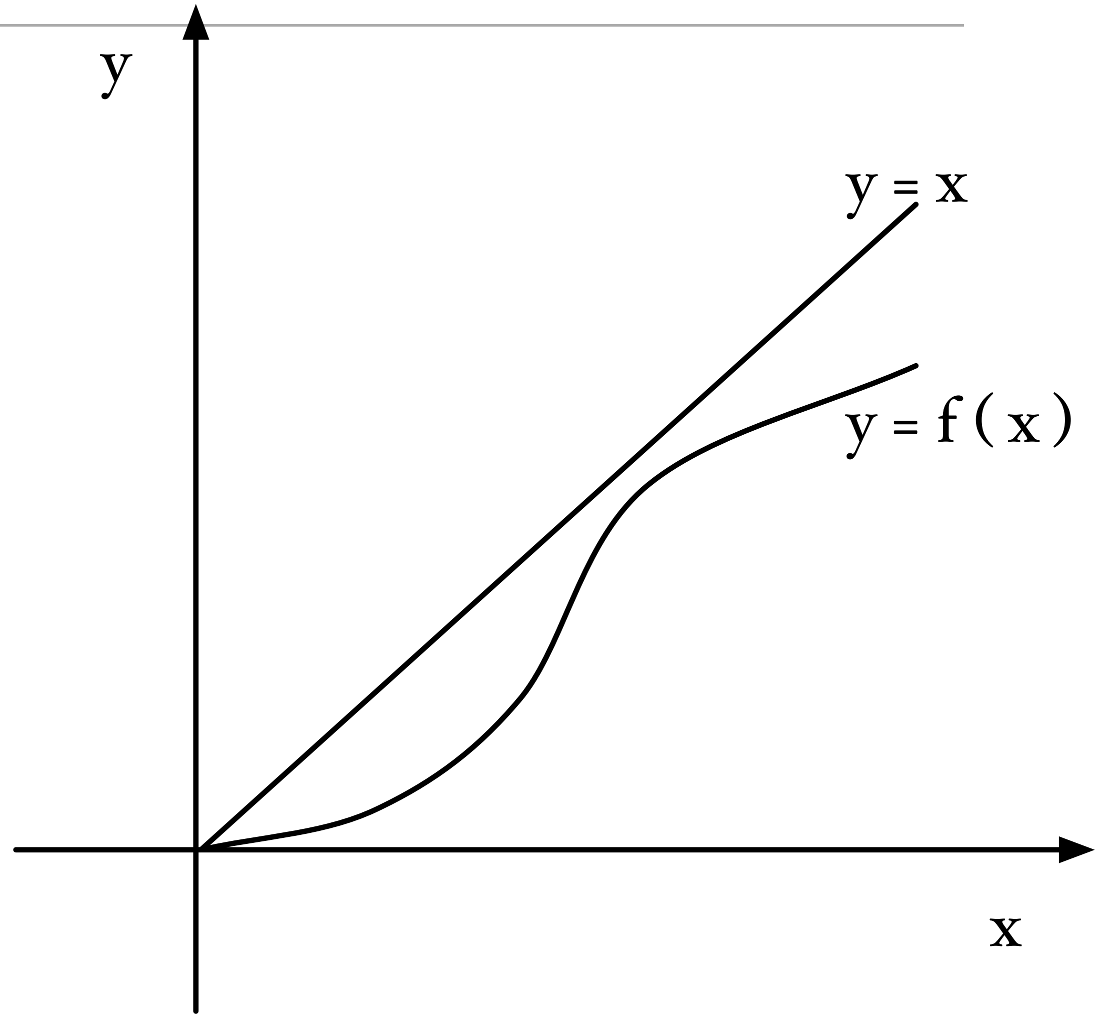
# Complexity of message-passing decoding

Khandekar-McEliece conjecture

$$\#\text{iterations} \geq \Omega\left(\frac{1}{C(P_T) - R}\right) + \Omega\left(\frac{1}{P_e}\right)$$

or

$$\Omega\left(\frac{1}{C(P_T) - R}\log\frac{1}{P_e}\right)$$



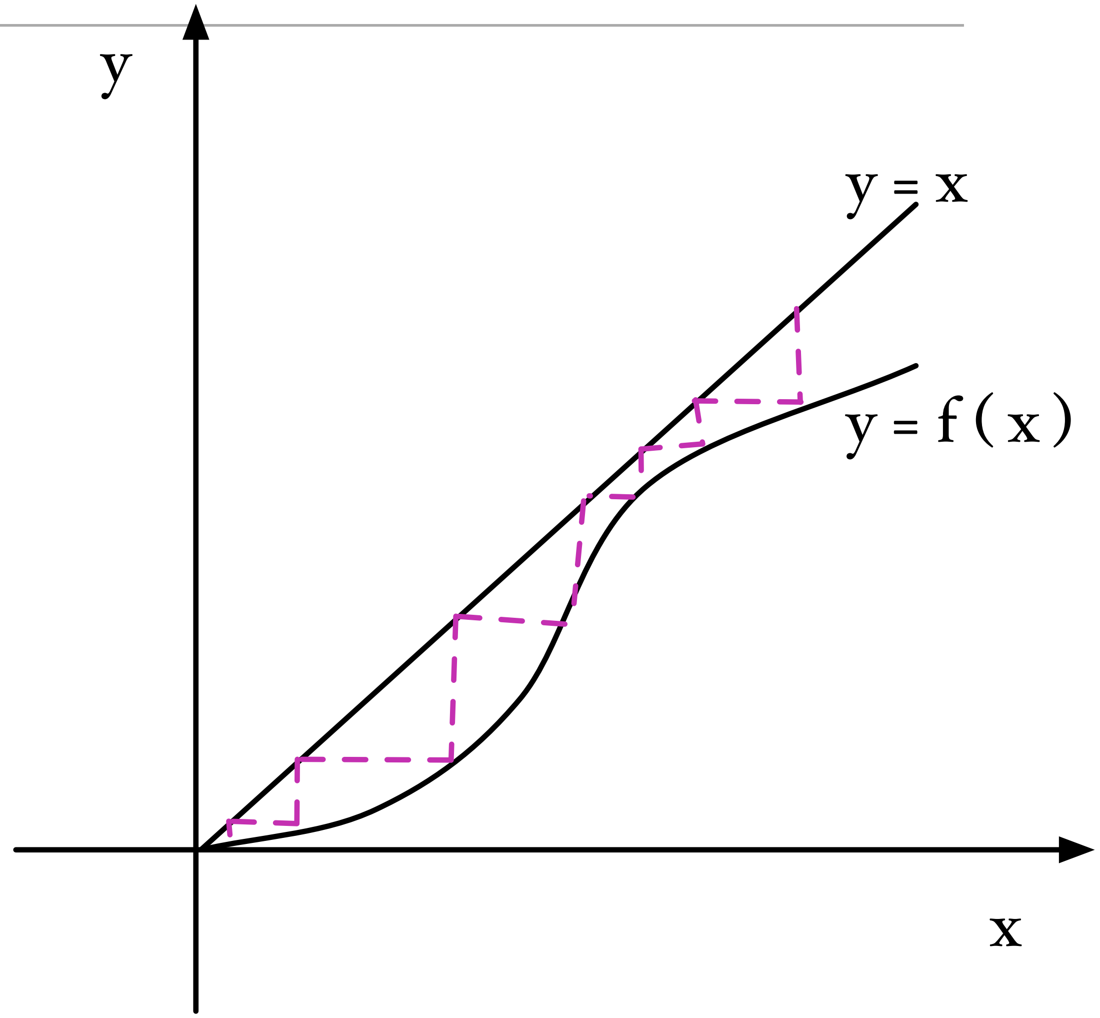
# Complexity of message-passing decoding

Khandekar-McEliece conjecture

$$\#\text{iterations} \geq \Omega\left(\frac{1}{C(P_T) - R}\right) + \Omega\left(\frac{1}{P_e}\right)$$

or

$$\Omega\left(\frac{1}{C(P_T) - R}\log\frac{1}{P_e}\right)$$



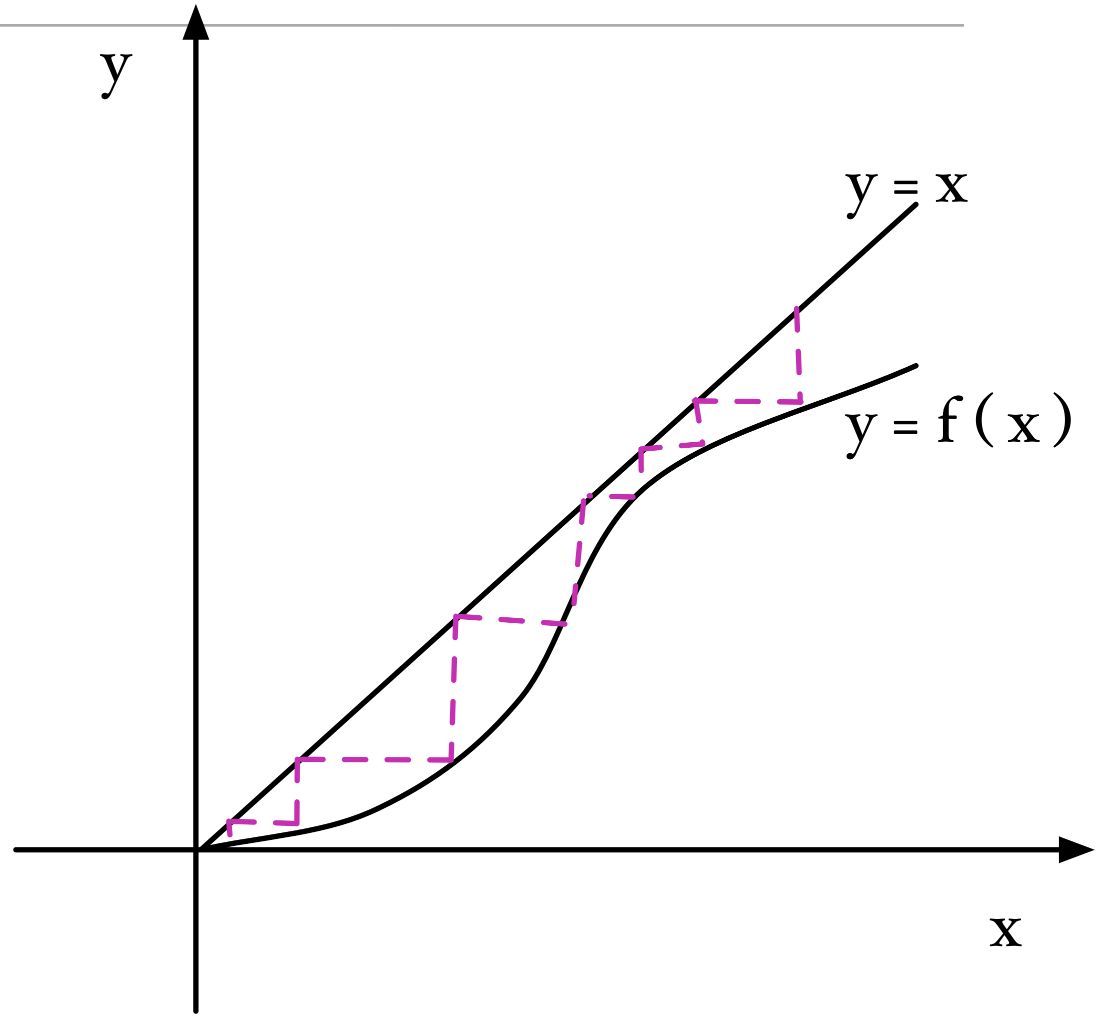
# Complexity of message-passing decoding

Khandekar-McEliece conjecture

$$\#\text{iterations} \geq \Omega\left(\frac{1}{C(P_T) - R}\right) + \Omega\left(\frac{1}{P_e}\right)$$

or

$$\Omega\left(\frac{1}{C(P_T) - R} \log \frac{1}{P_e}\right)$$




Relation with gap proved by Sason for LDPC, IRA, ARA

# how low can this complexity be?

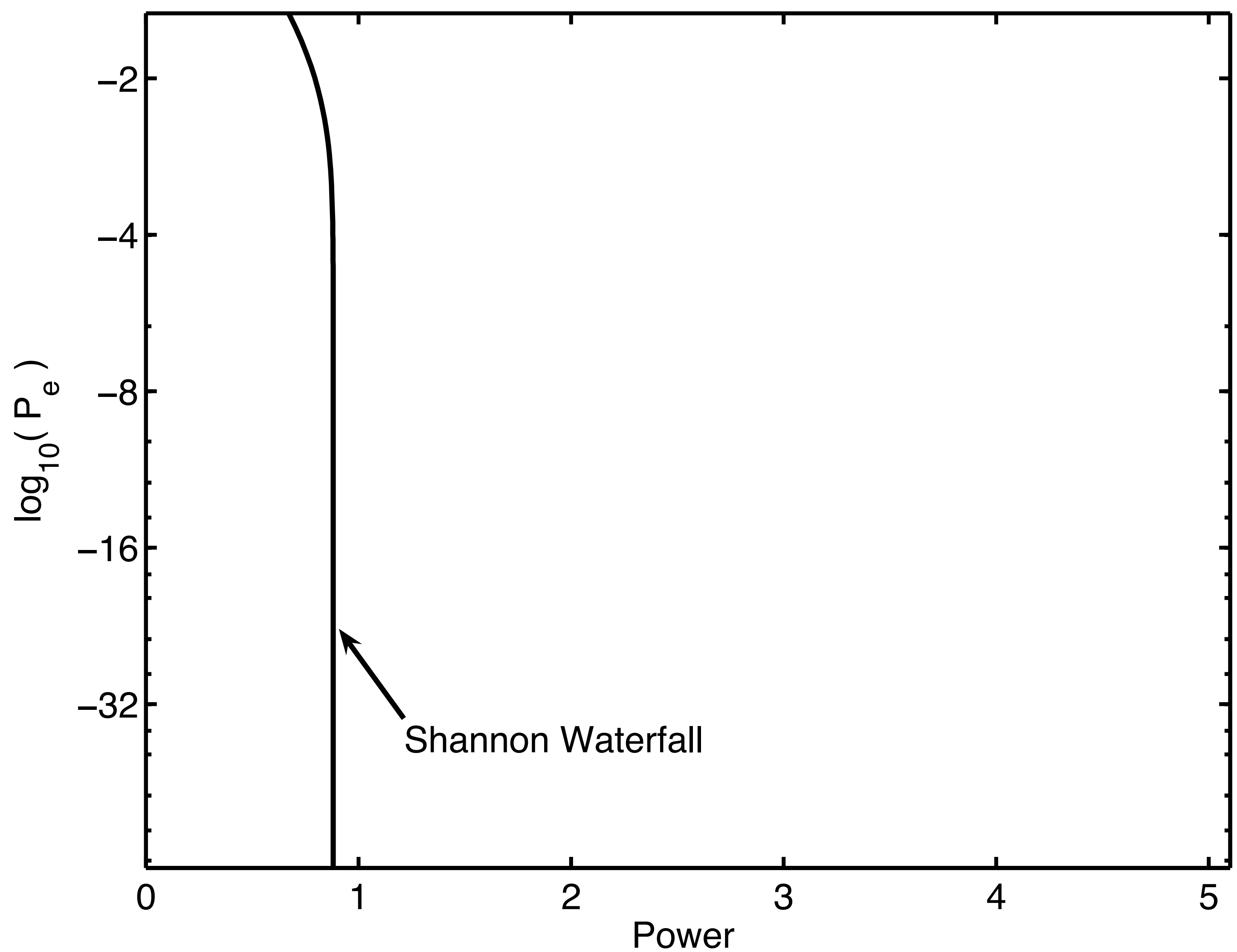
**Theorem** [ISIT '08]

$$l = \#\ \text{iterations} \gtrsim \frac{1}{\log(\alpha - 1)} \log\left(\frac{\log \frac{1}{P_e}}{(C(P_T) - R)^2}\right)$$

. . . for any code, and any message-passing algorithm

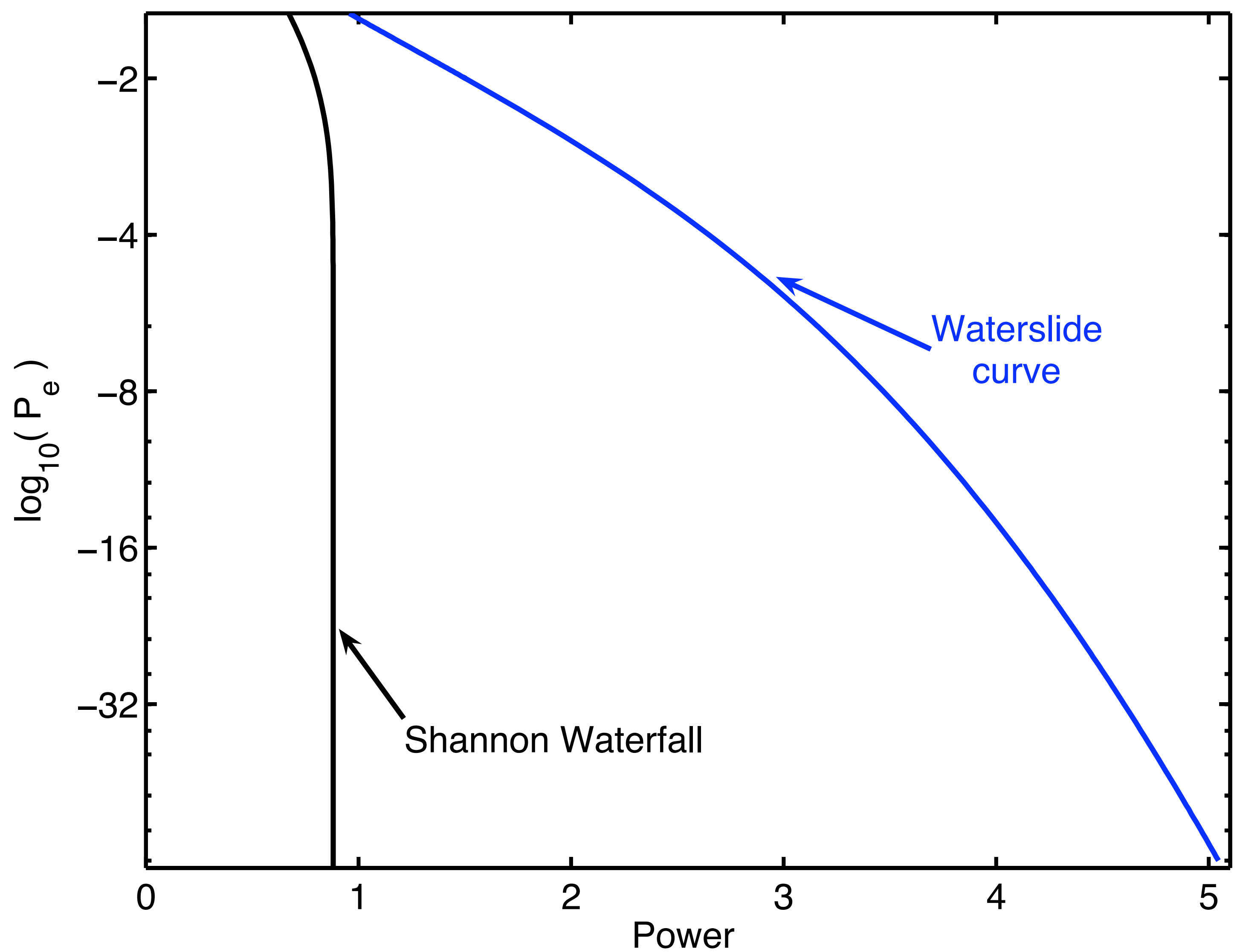
# Waterfall or Waterslide?

$$P_{\text{total}} \geq P_T + \frac{\gamma}{\log(\alpha)} \log\left(\frac{\log \frac{1}{P_e}}{(C(P_T) - R)^2}\right)$$



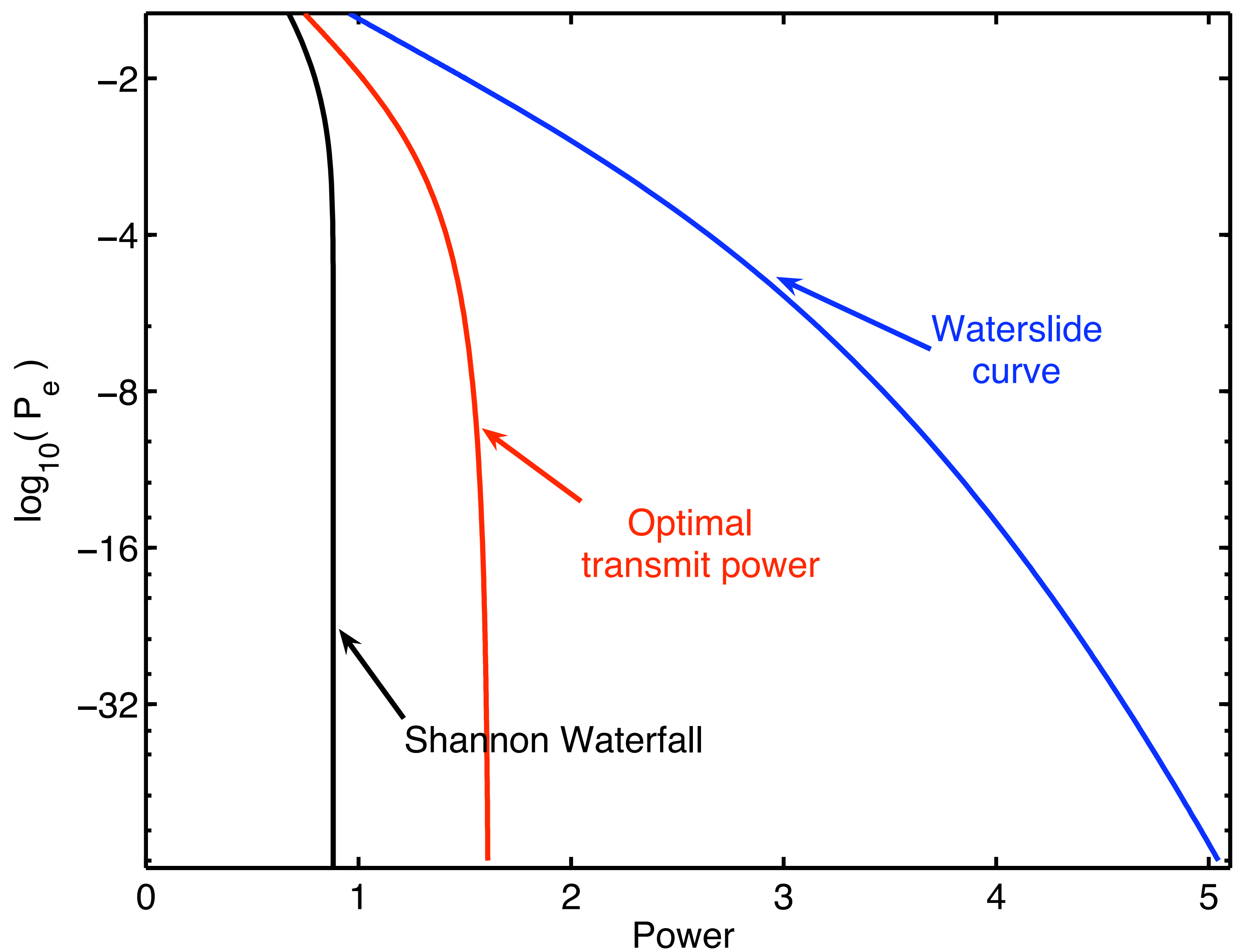
# Waterfall or Waterslide?

$$P_{\text{total}} \geq P_T + \frac{\gamma}{\log(\alpha)} \log\left(\frac{\log \frac{1}{P_e}}{(C(P_T) - R)^2}\right)$$



# Waterfall or Waterslide?

$$P_{\text{total}} \geq P_T + \frac{\gamma}{\log(\alpha)} \log\left(\frac{\log \frac{1}{P_e}}{(C(P_T) - R)^2}\right)$$

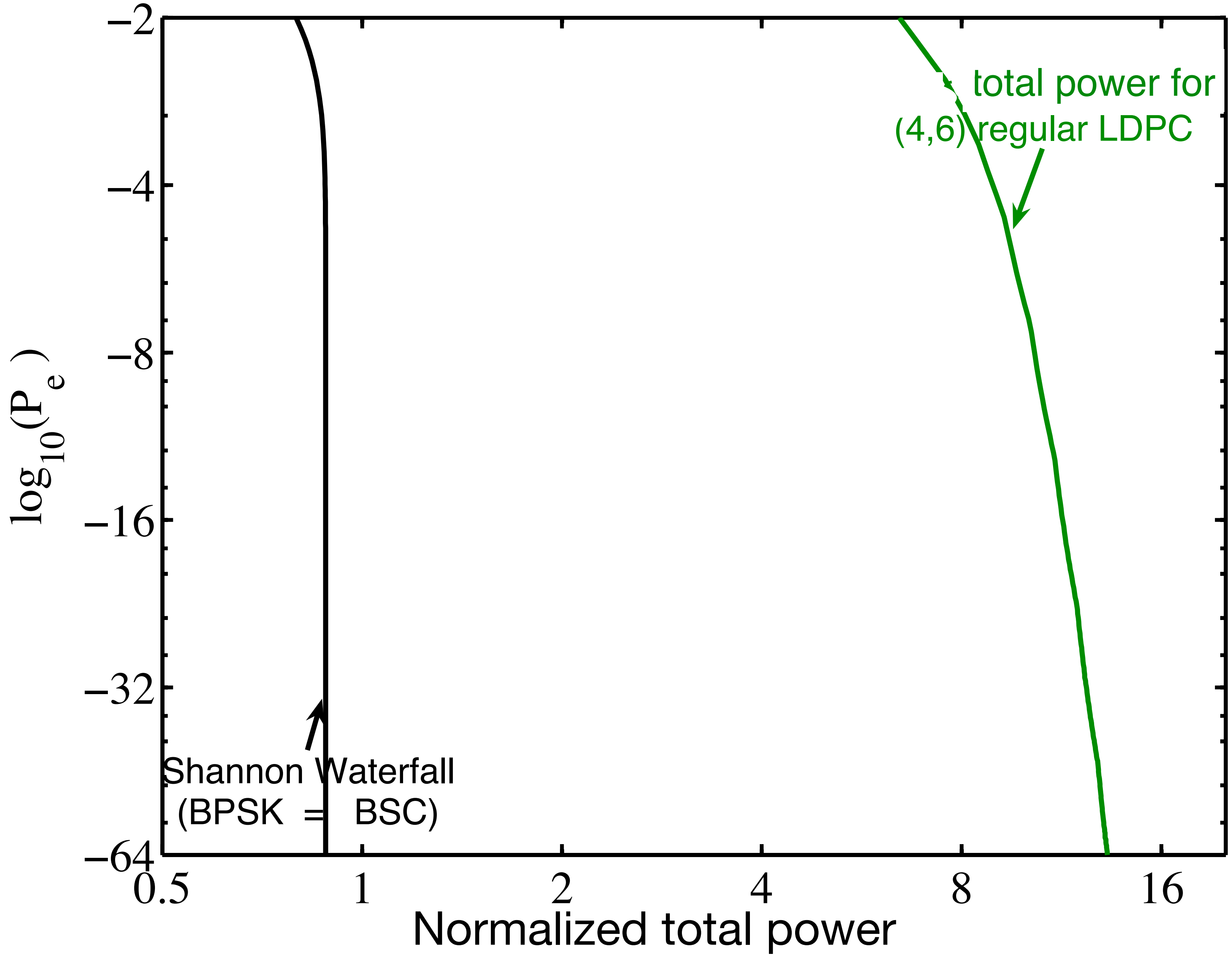


# What codes are good?

energy per operation E = 1 pJ,
distance = 17 m,
f = 3 GHz
path-loss exponent = 3,
maximum node connectivity= 4
T = 300 K
Rate = 1/3

[Grover, Sahai '09]

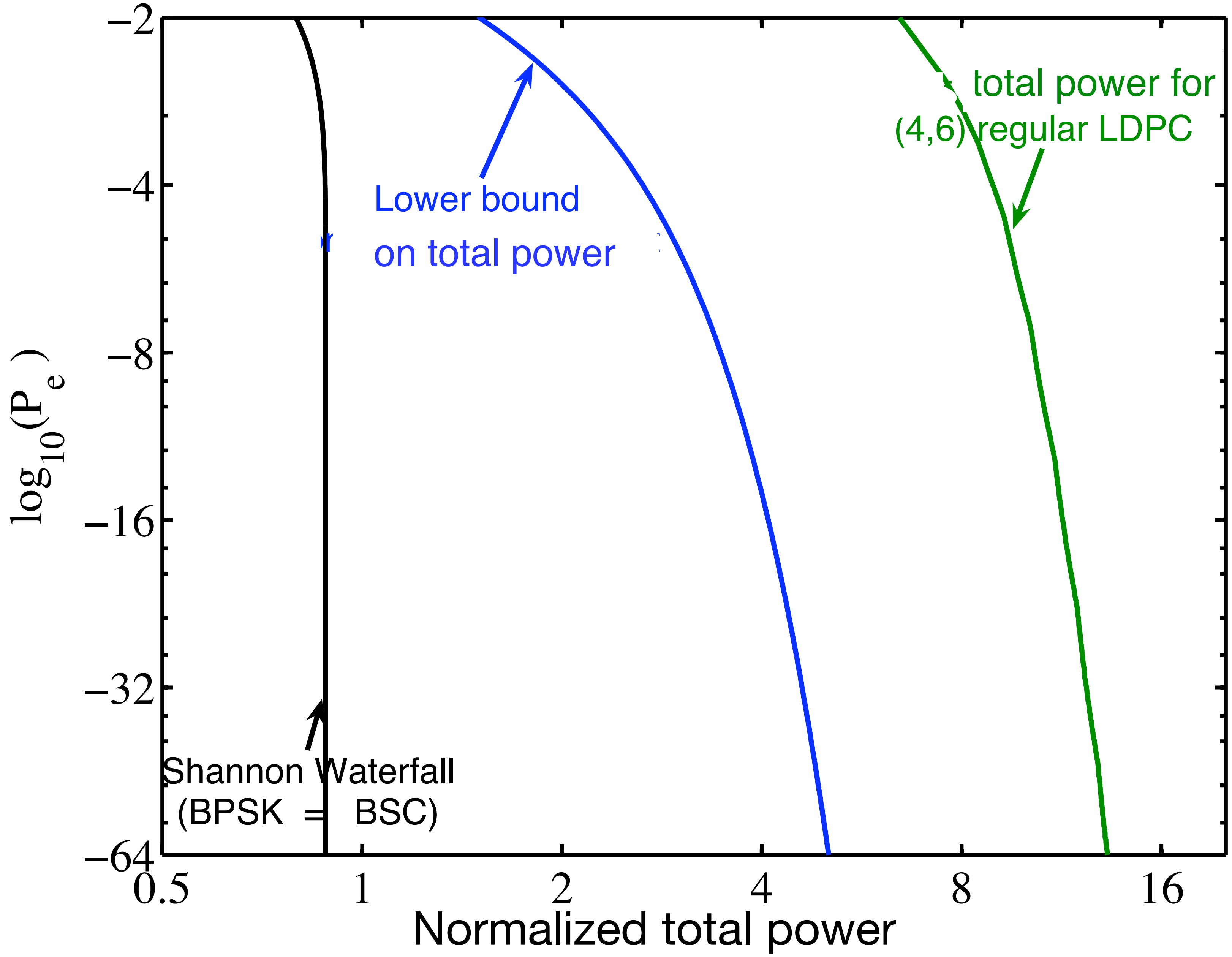
# What codes are good?




energy per operation E = 1 pJ,
distance = 17 m,
f = 3 GHz
path-loss exponent = 3,
maximum node connectivity= 4
T = 300 K
Rate = 1/3

[Grover, Sahai '09]

# What codes are good?



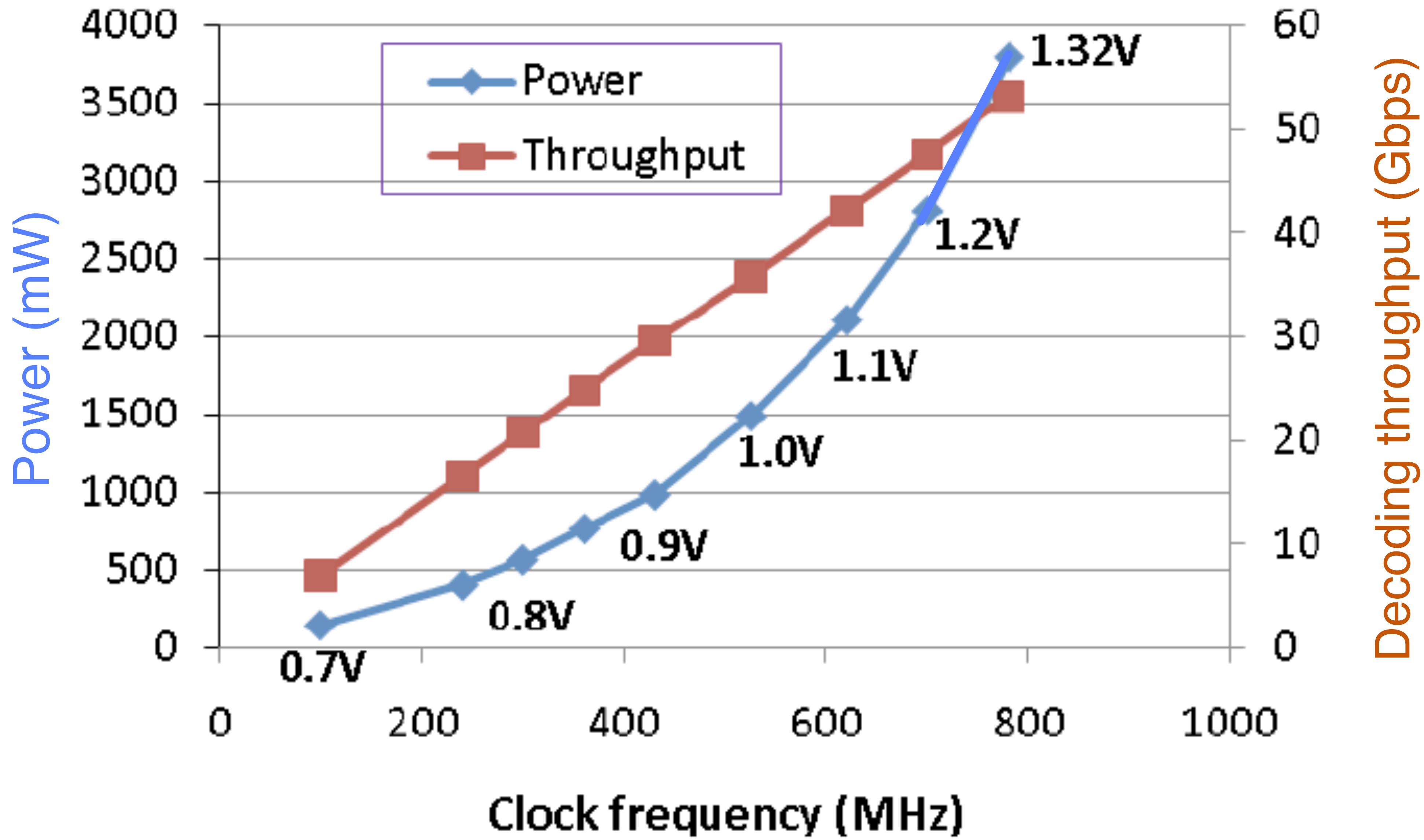

energy per operation E = 1 pJ,
distance = 17 m,
f = 3 GHz
path-loss exponent = 3,
maximum node connectivity= 4
T = 300 K
Rate = 1/3

[Grover, Sahai ’09]

# Organization

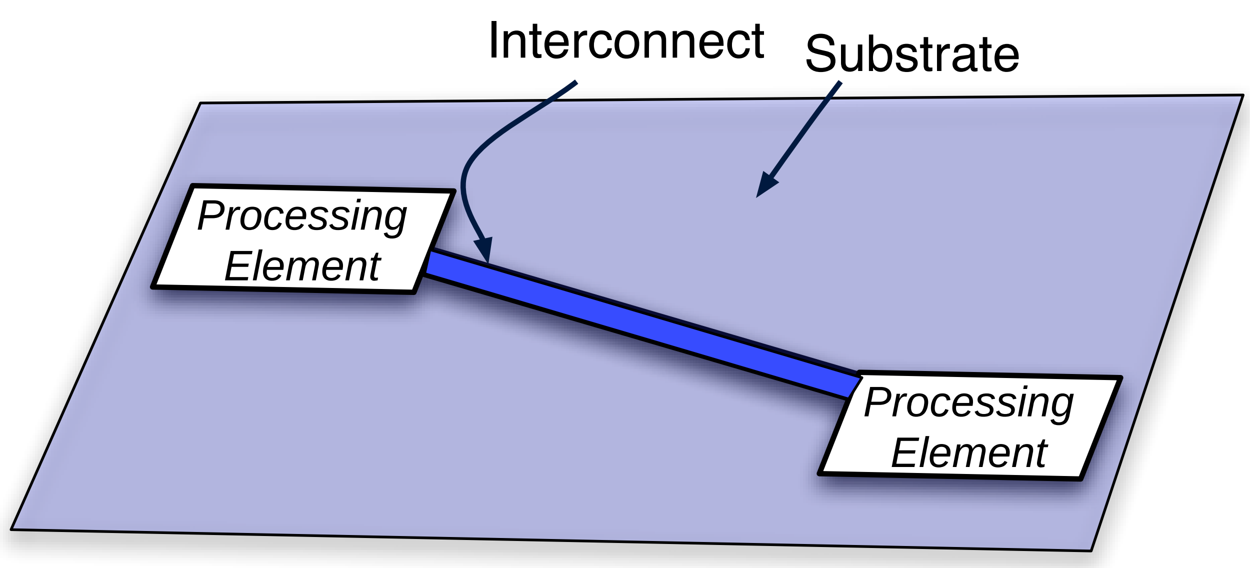
- Complexity as power [ISIT ’08, ’09]
  - a lower bound on complexity of message passing decoding
  - a power consumption model
- A refined model
  - large-deviations-based bounds on power and complexity
  - obtained via understanding erroneous decoding
  - a tradeoff between transmit power, decoding power, decoding throughput, rate, and error probability

# Power vs throughput

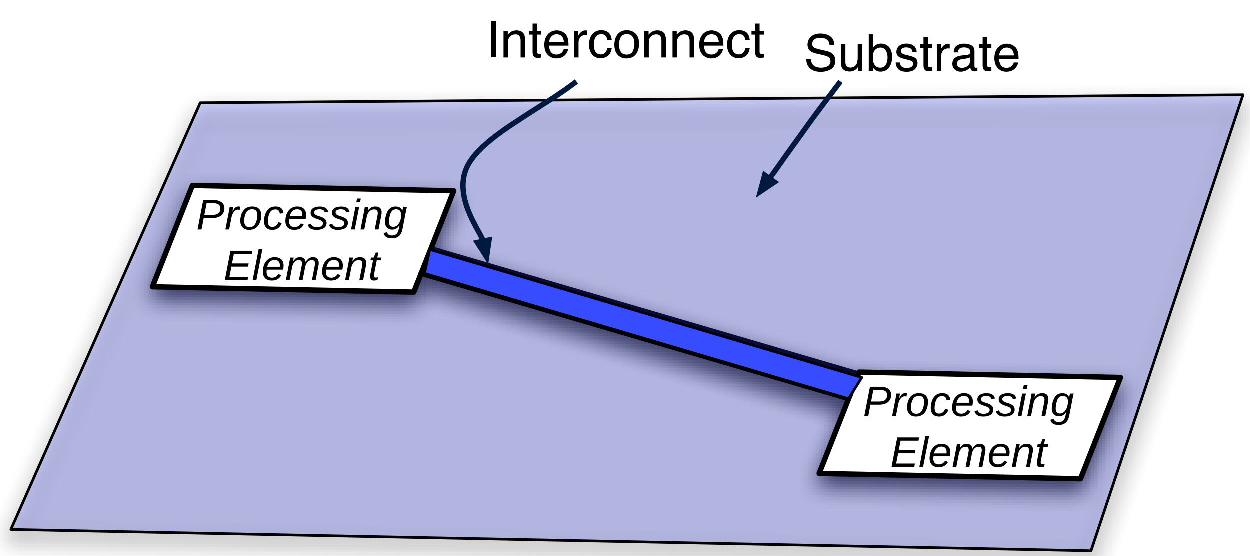


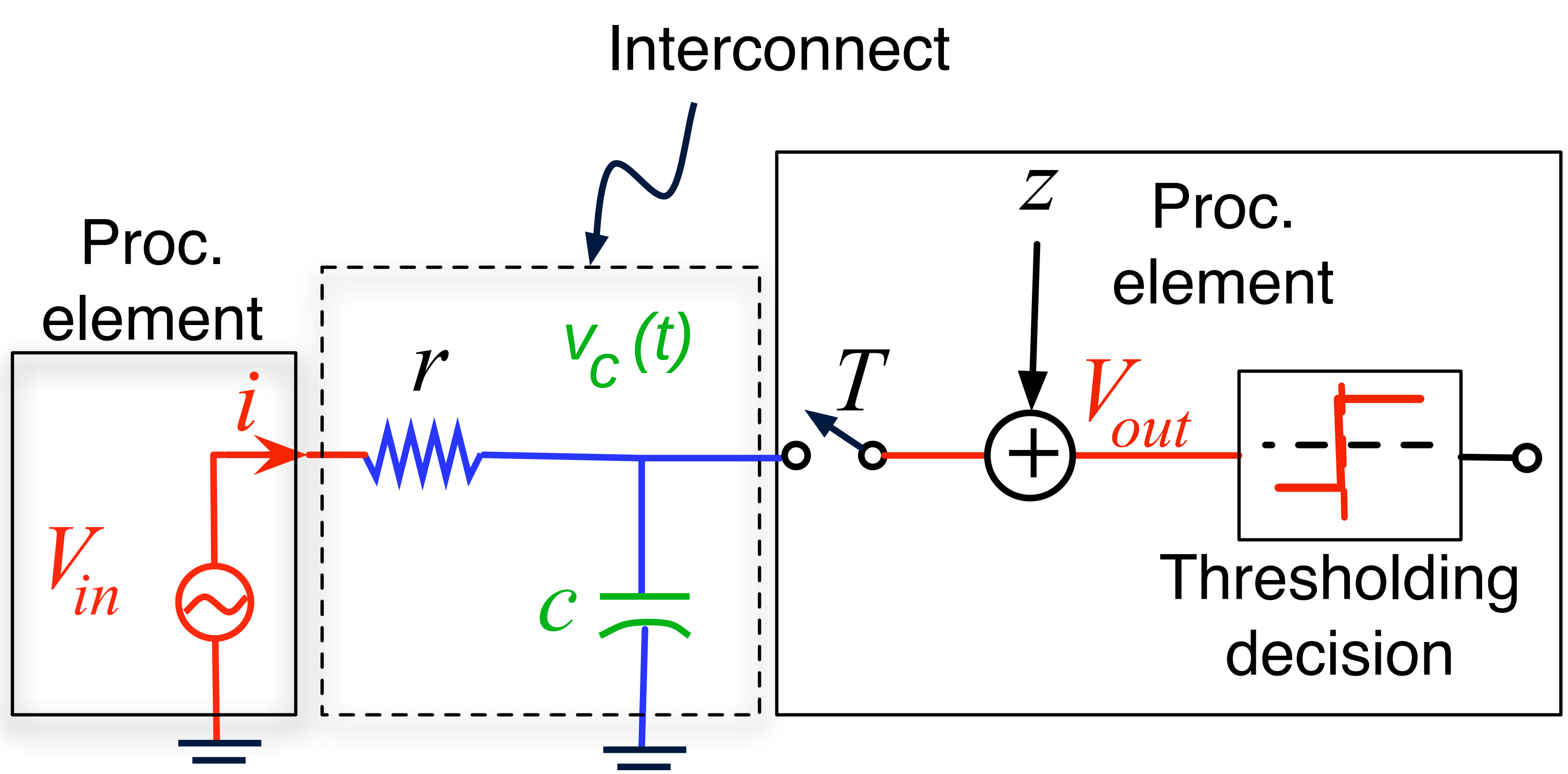

“A 47-Gbps LDPC decoder with improved low-error rate performance.”
[Zheng, Anantharam, Wainwright, Nikolic].

# Circuit capacitance

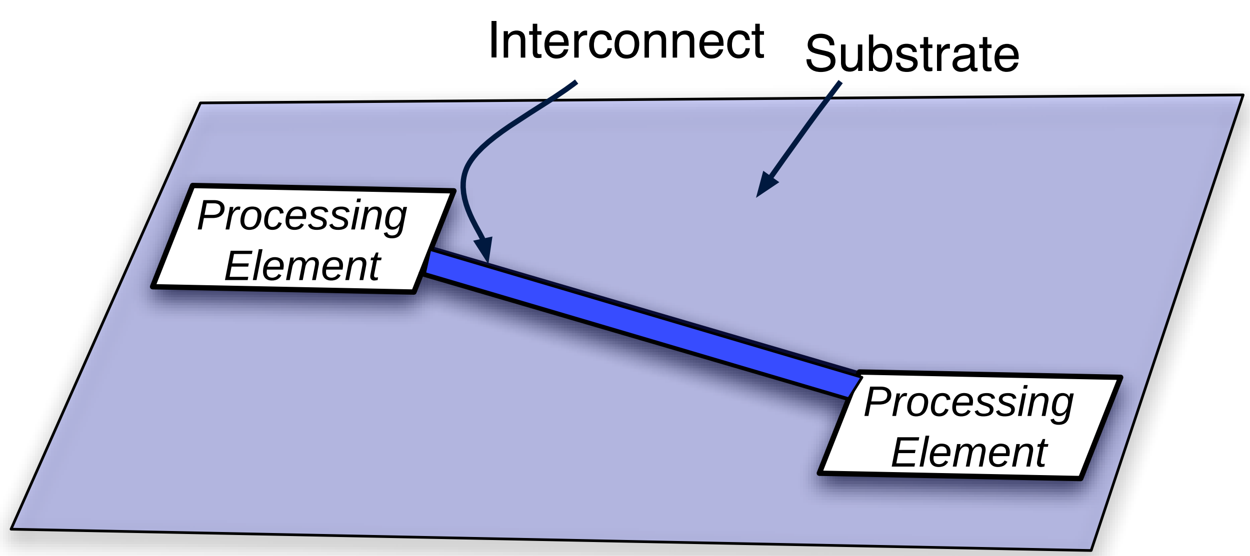


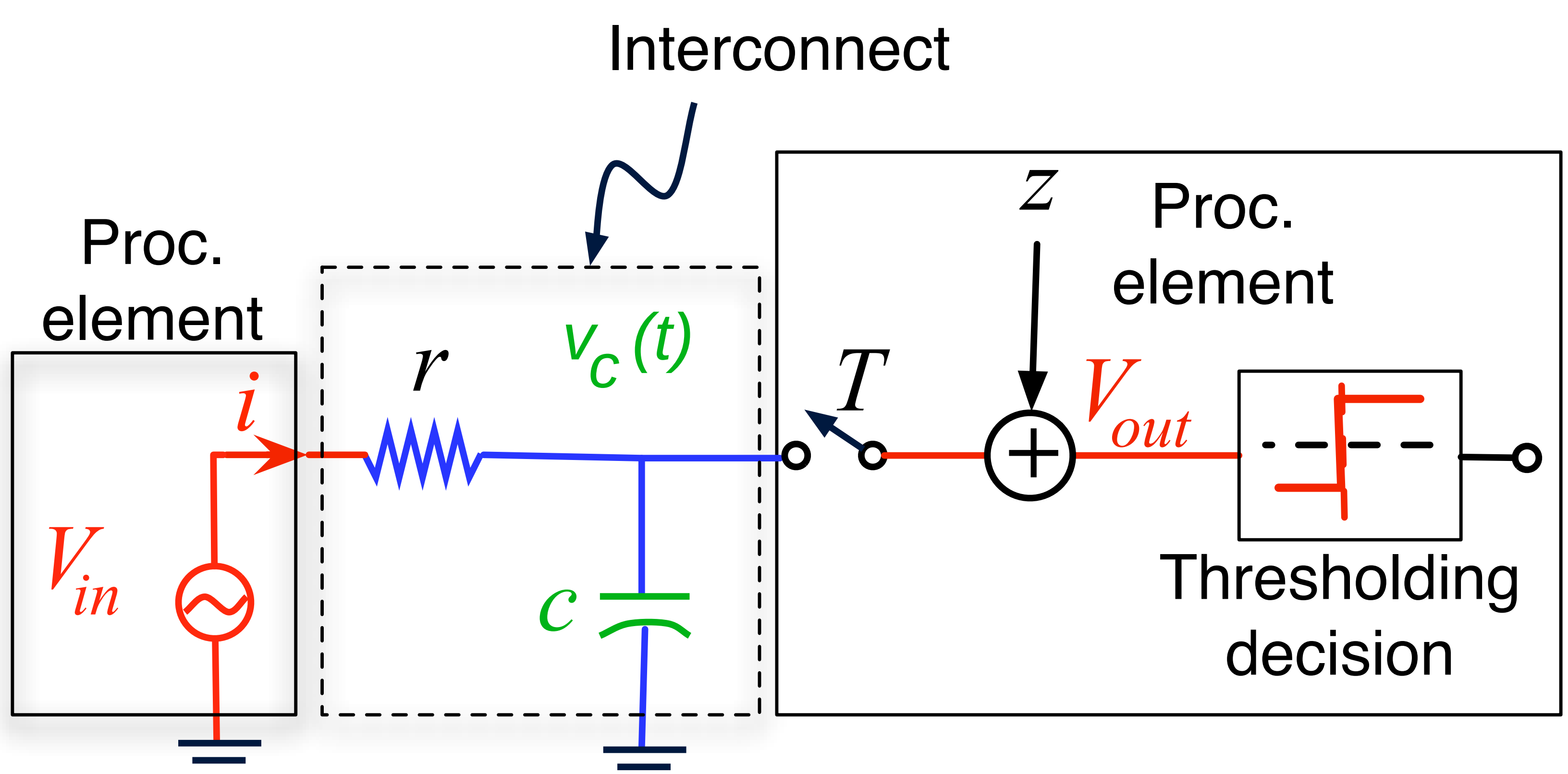
# Circuit capacitance

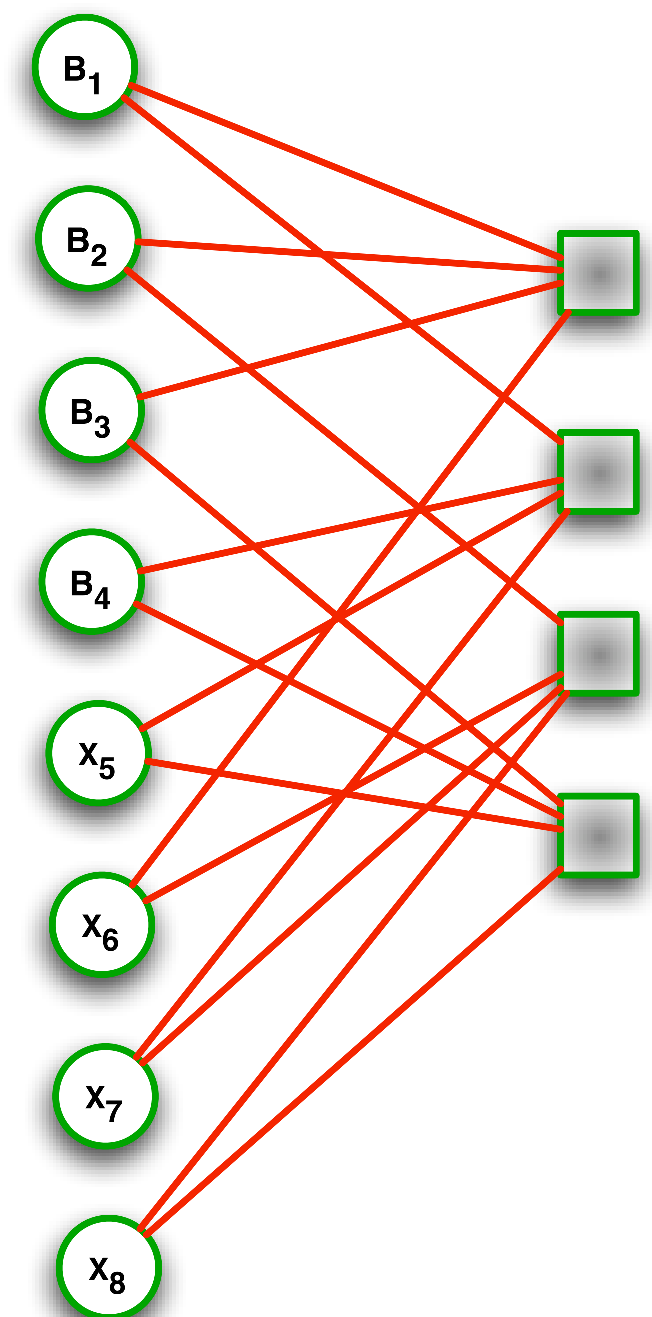


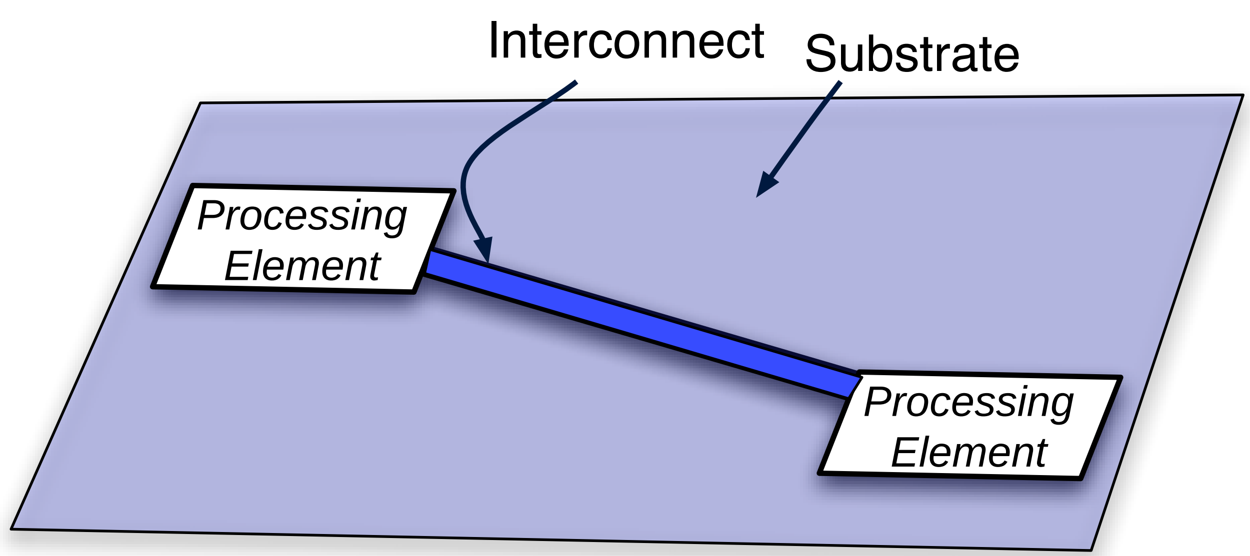
# Circuit capacitance

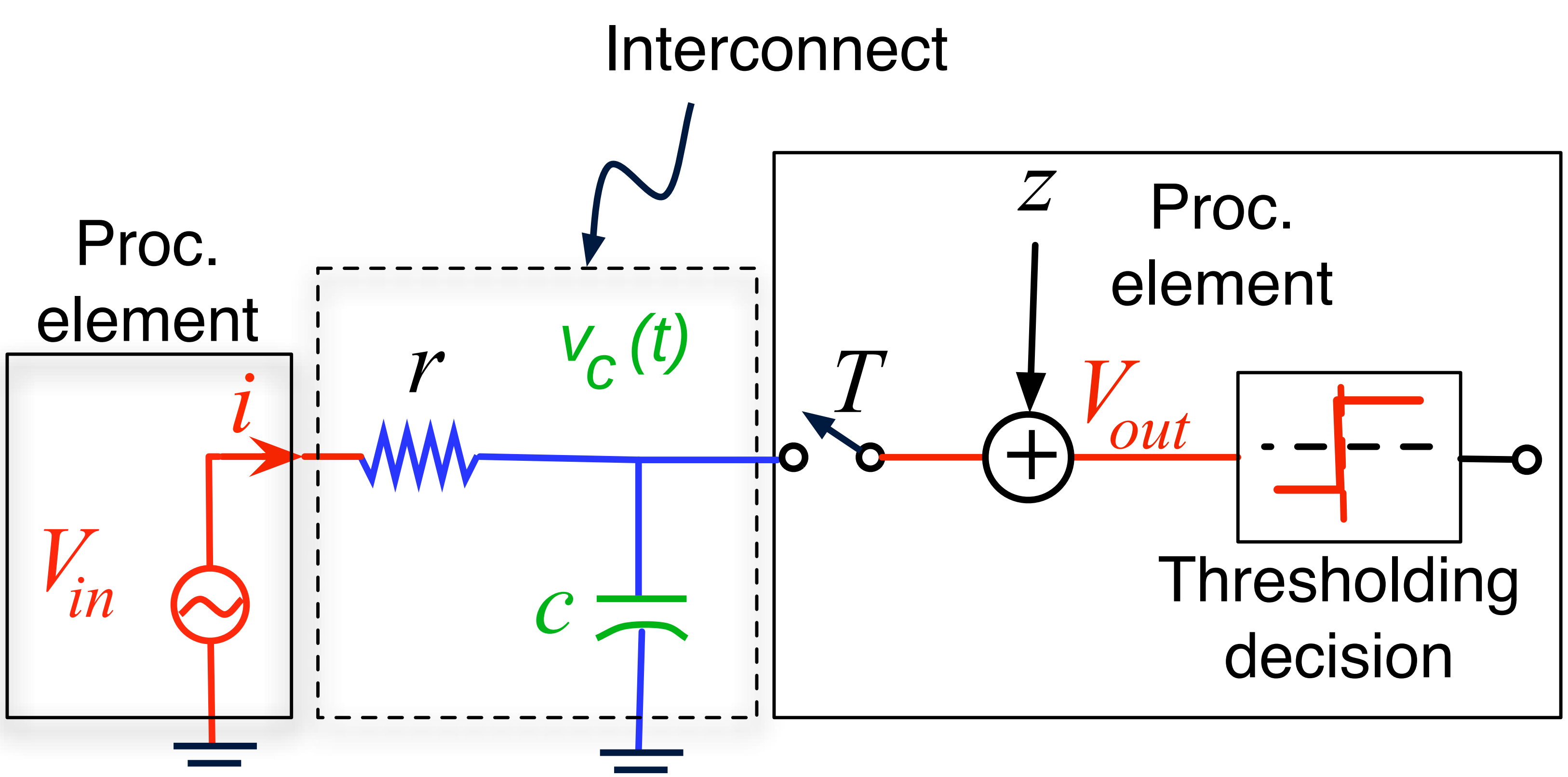


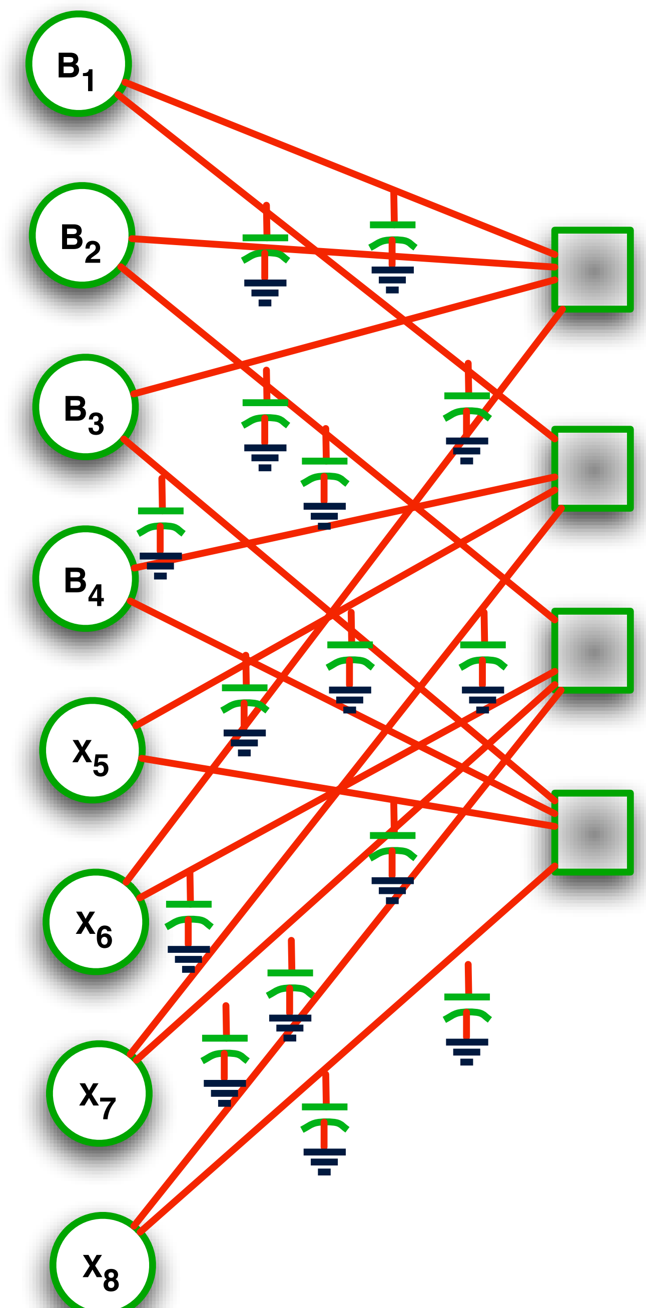



# Circuit capacitance

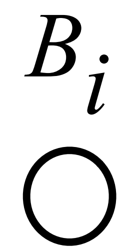


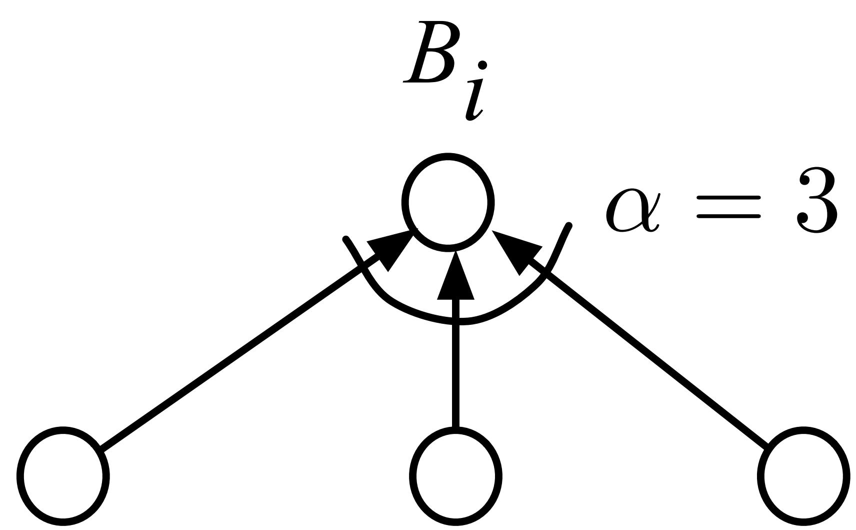



# Decoding neighborhood

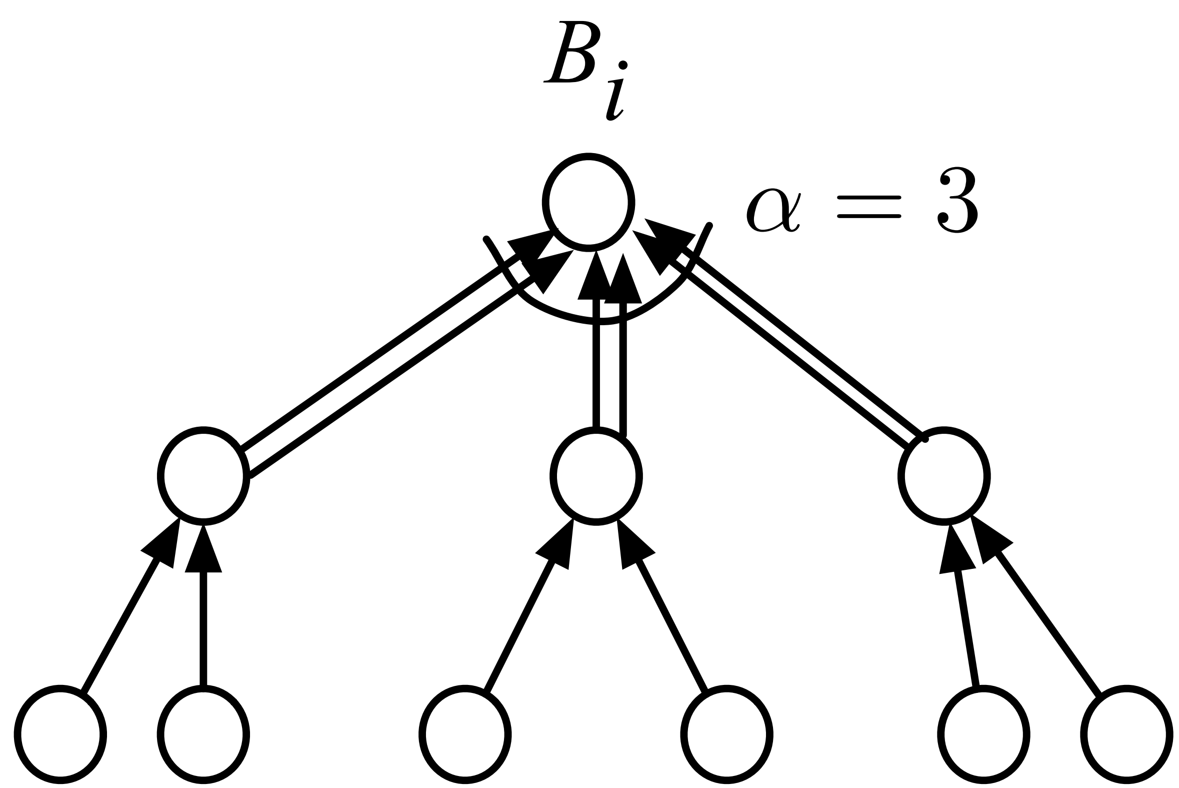
# Decoding neighborhood



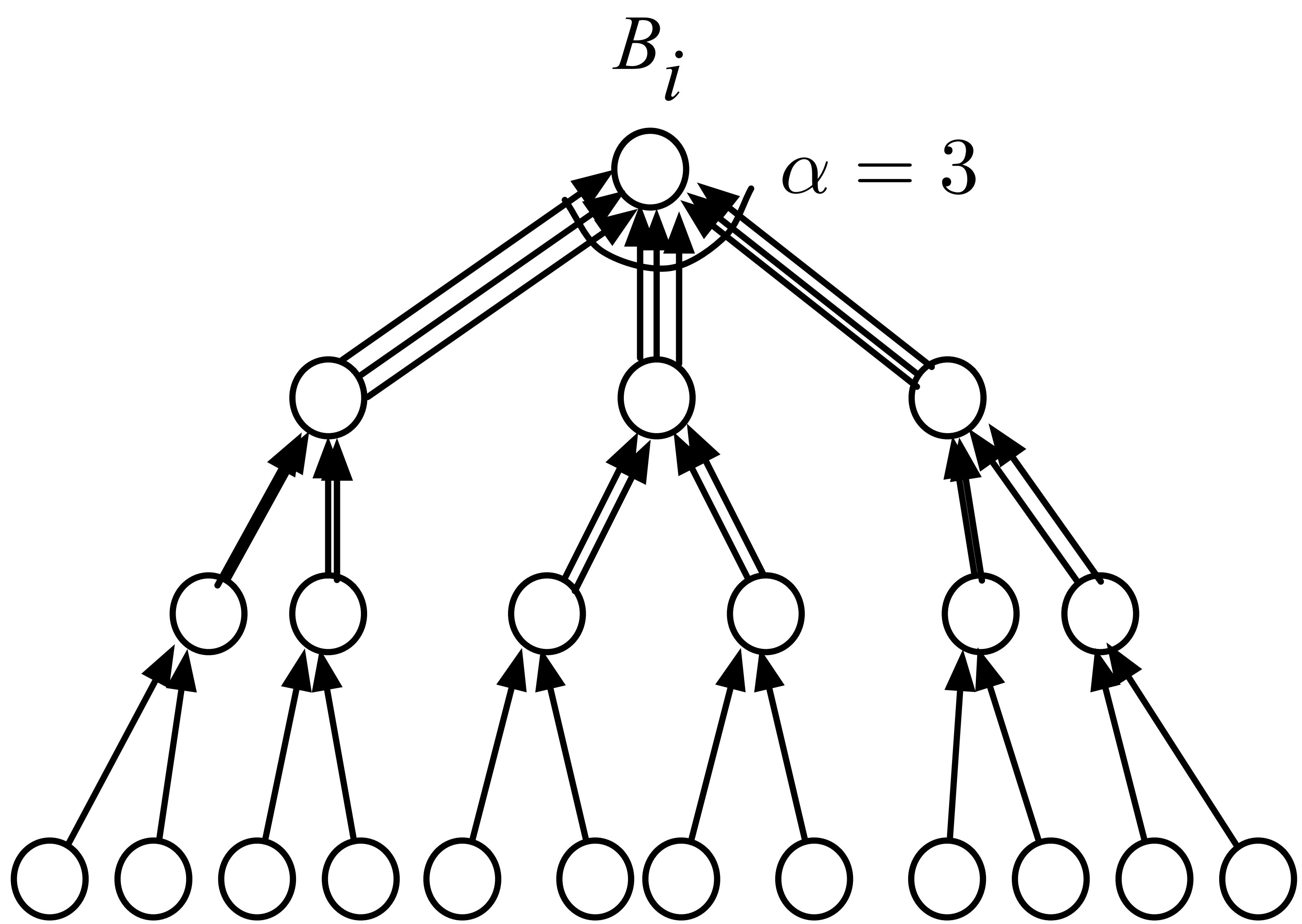
# Decoding neighborhood



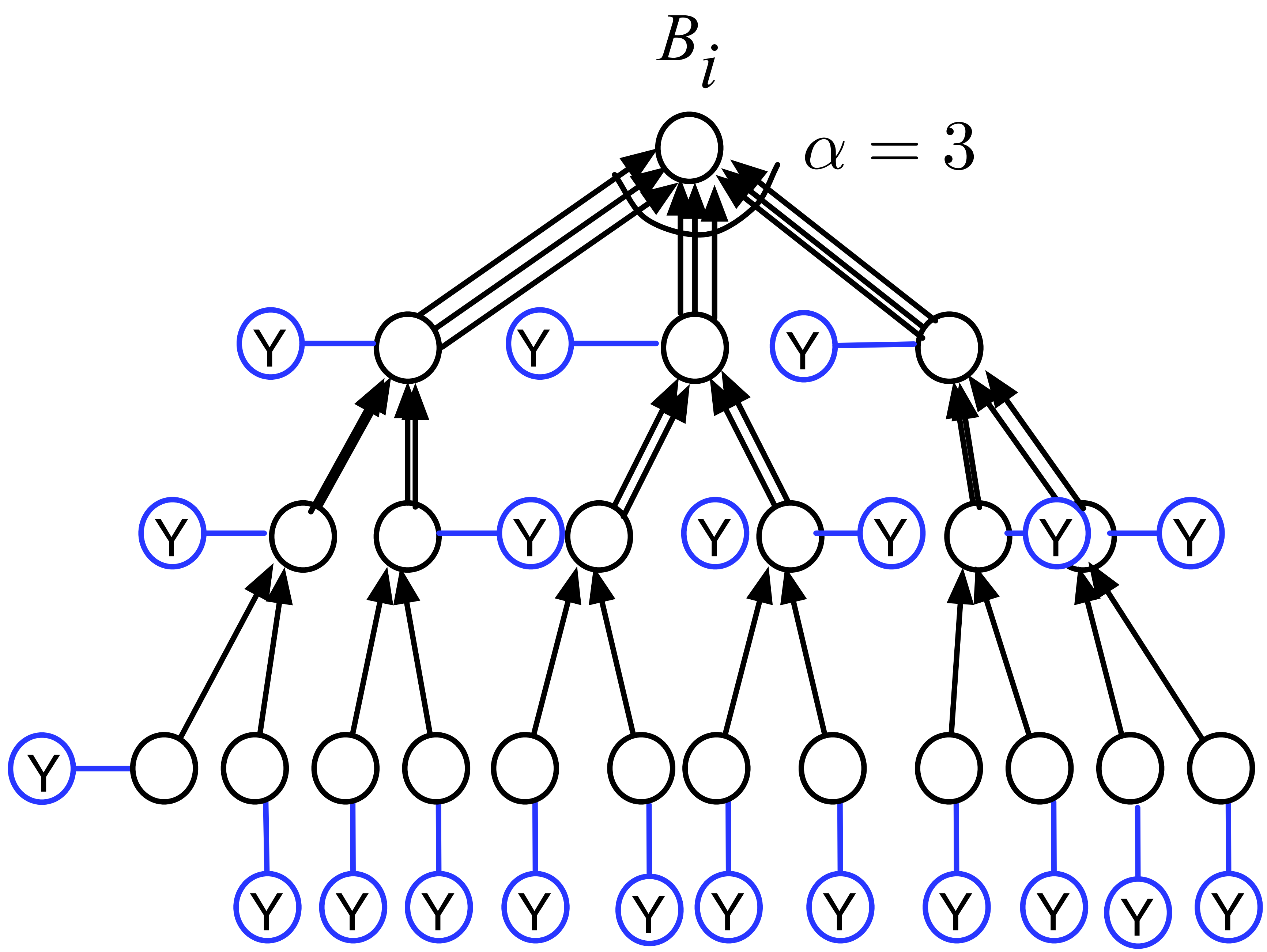
# Decoding neighborhood



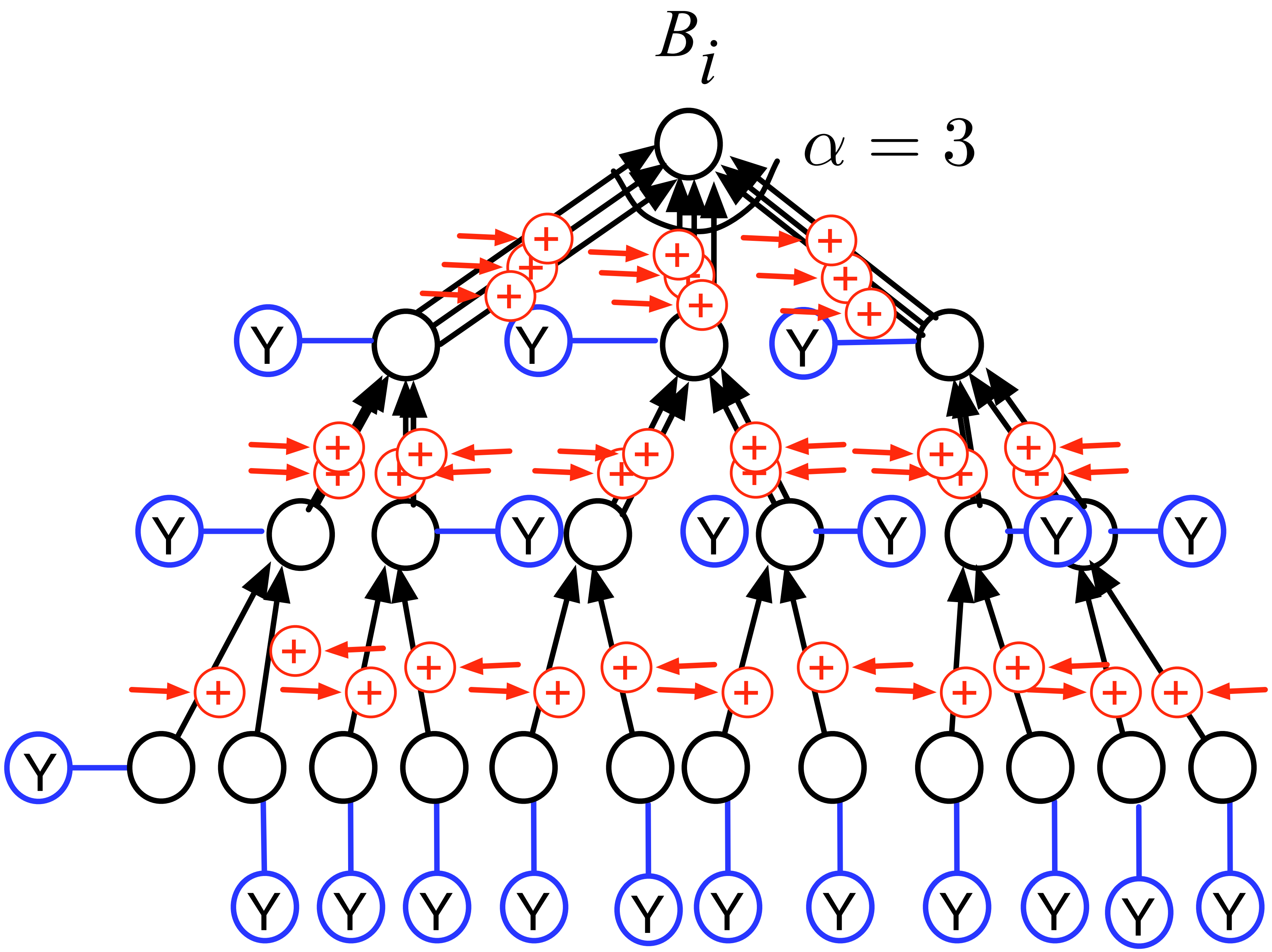
# Decoding neighborhood



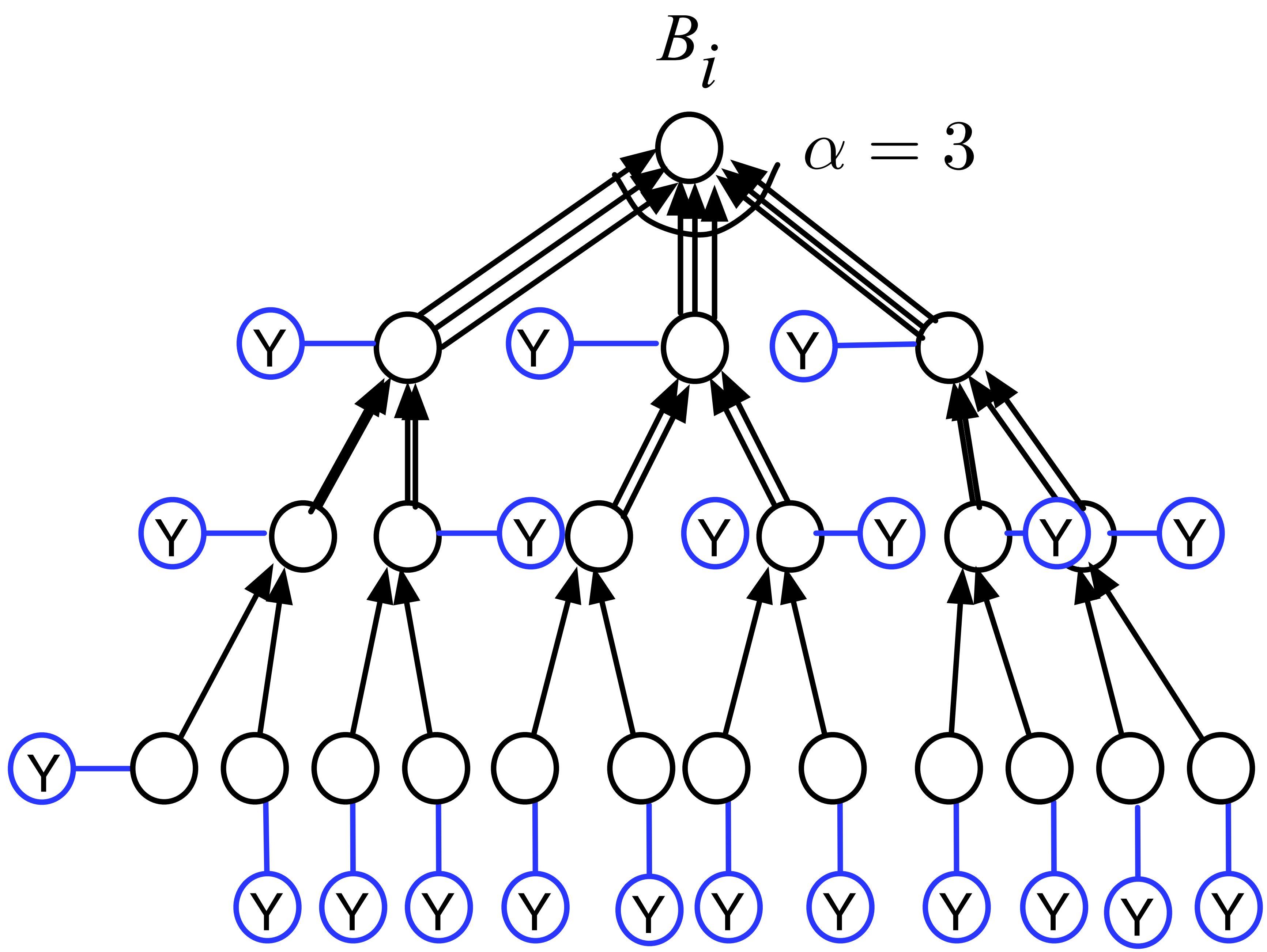
# Decoding neighborhood



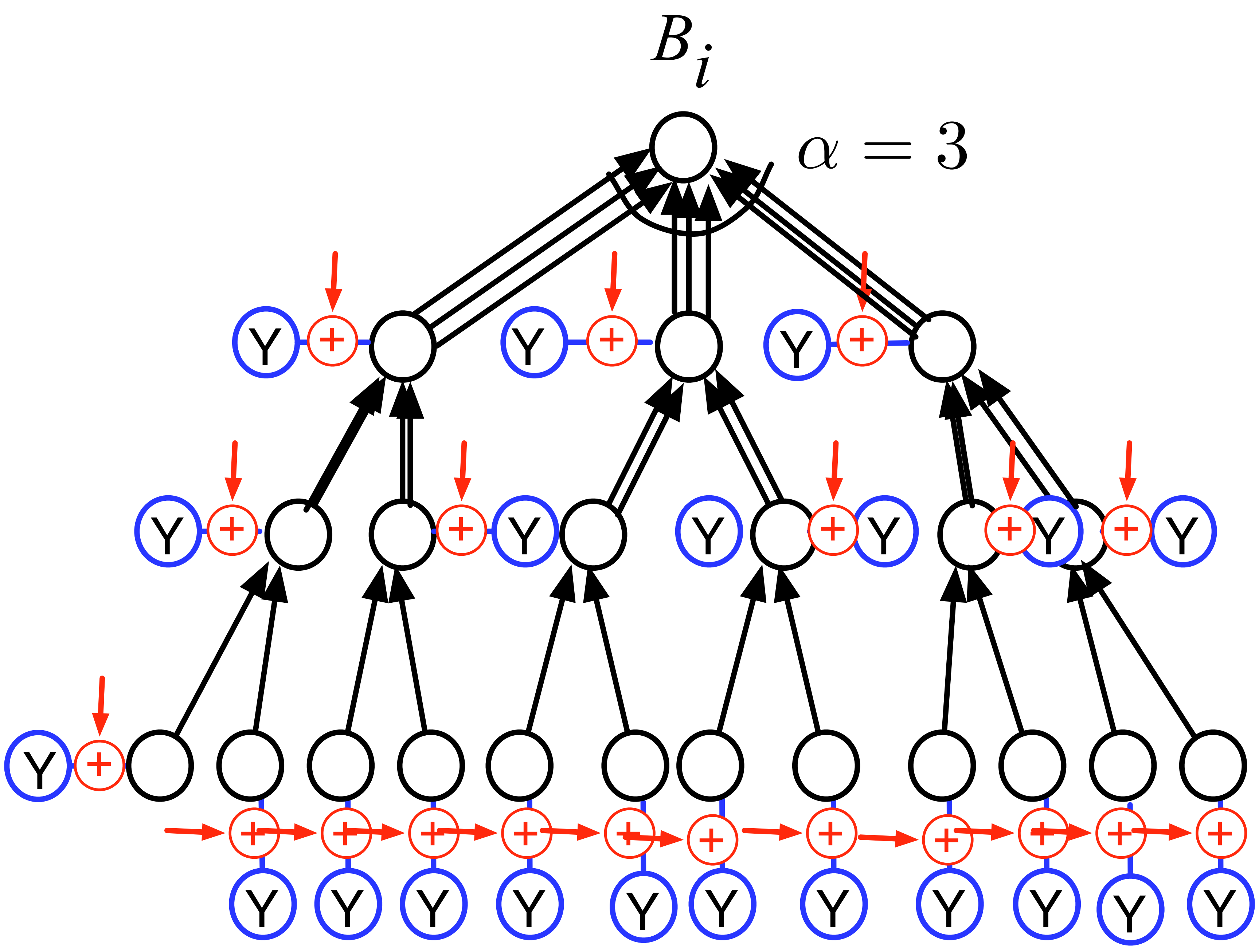
# Decoding neighborhood



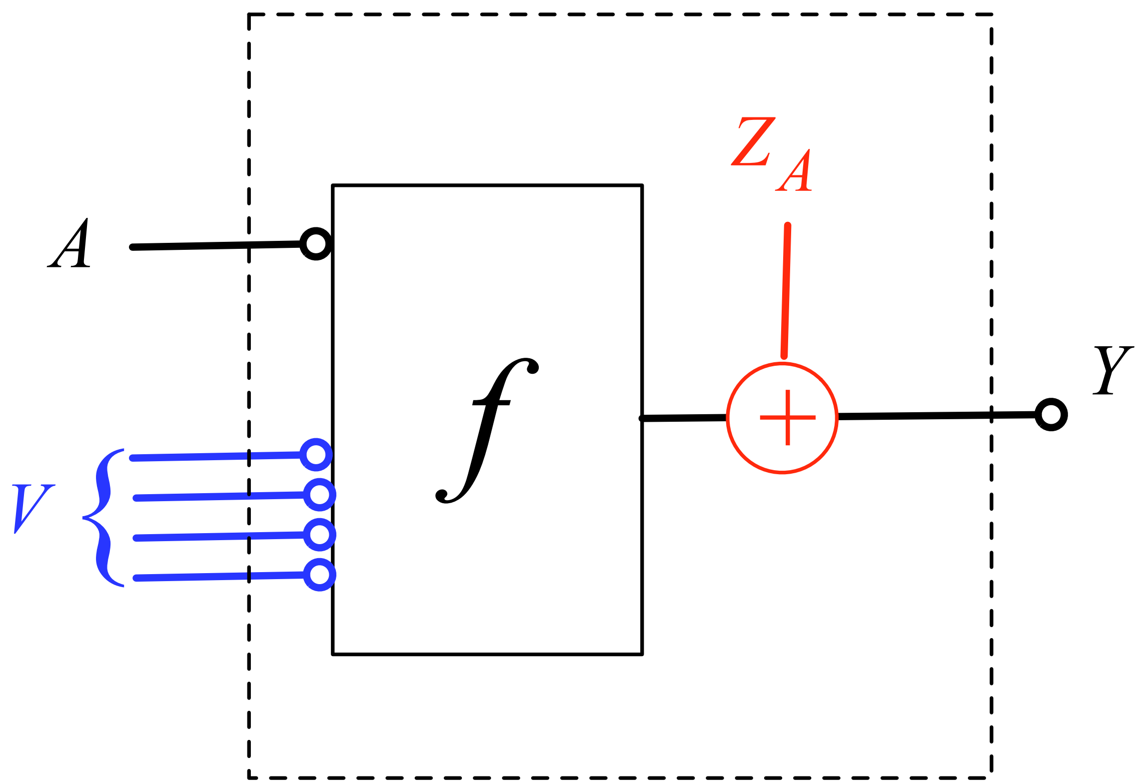
# Decoding neighborhood



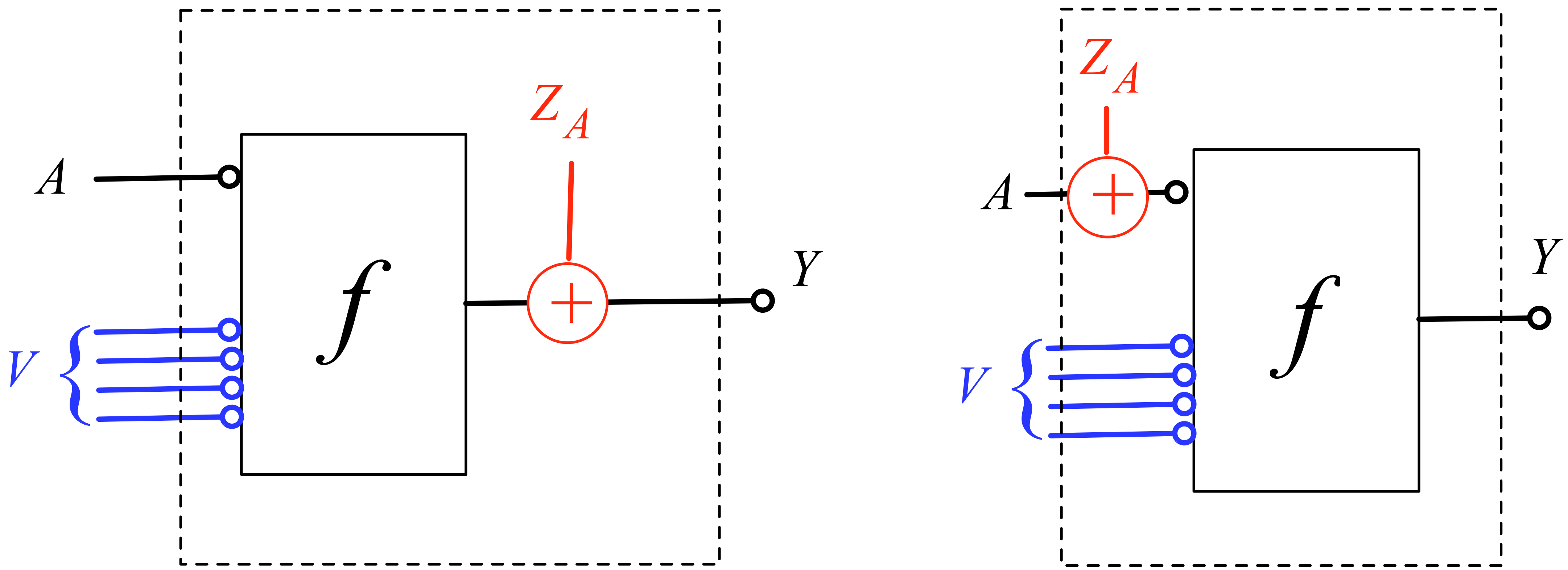
# Decoding neighborhood



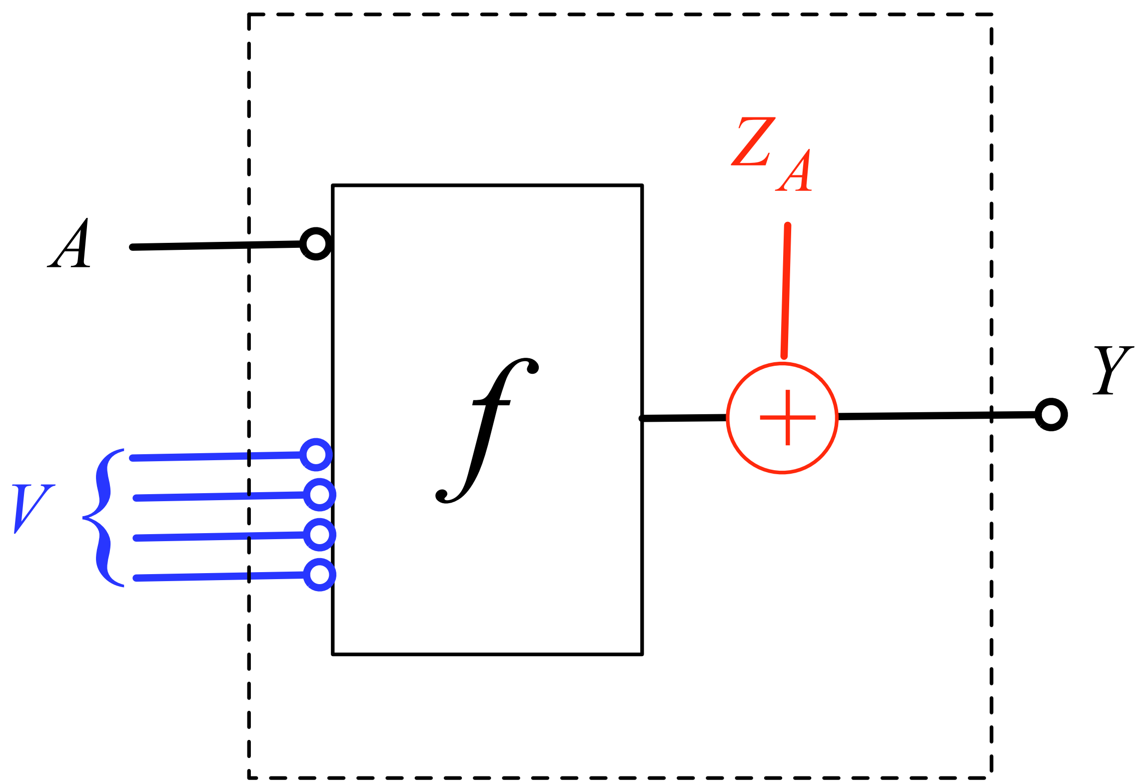
# A noisy computation lemma

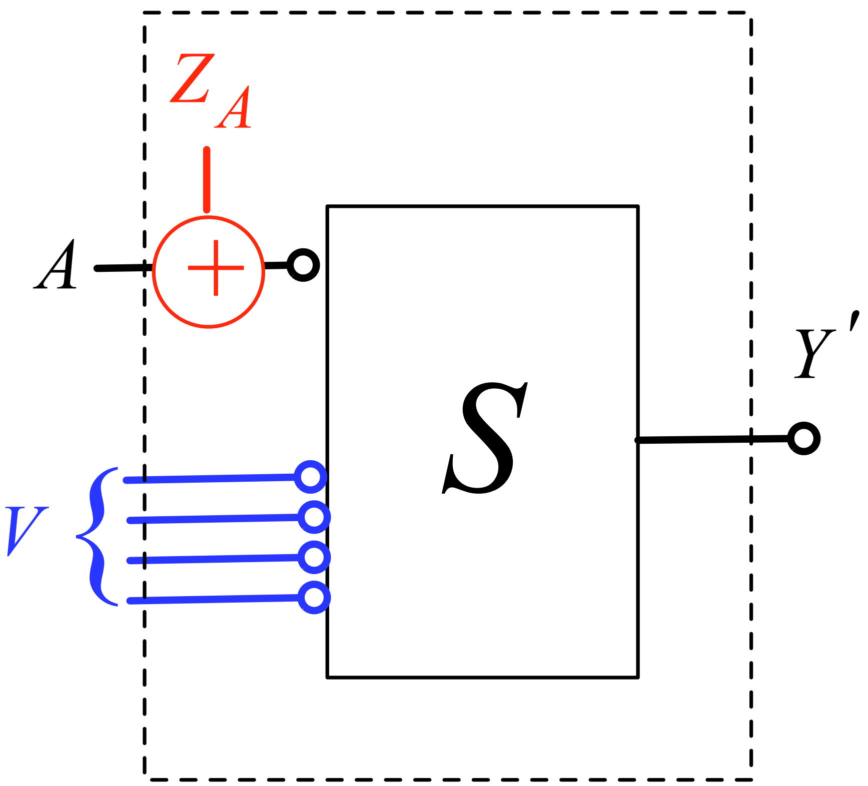


# A noisy computation lemma

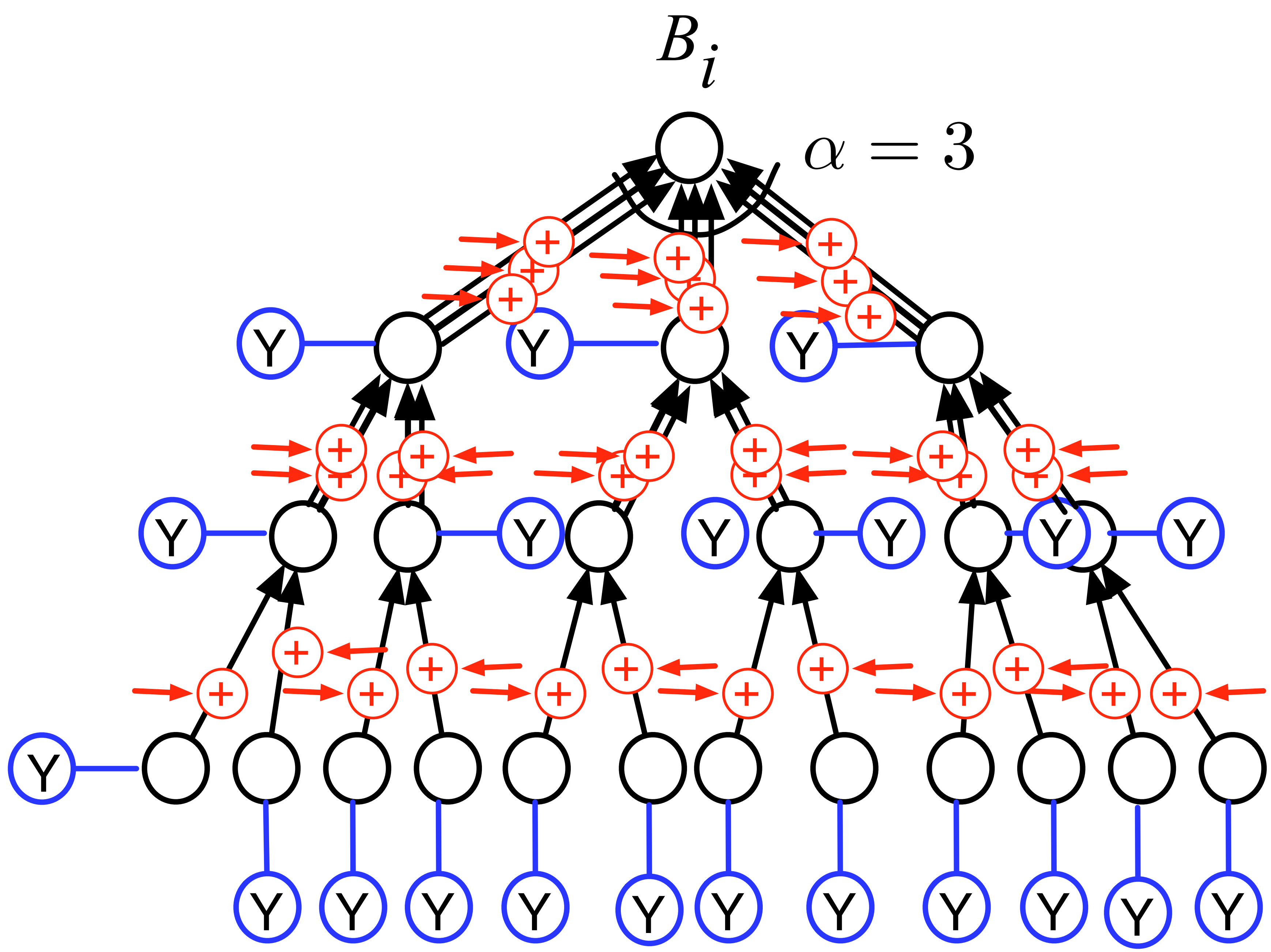


# A noisy computation lemma

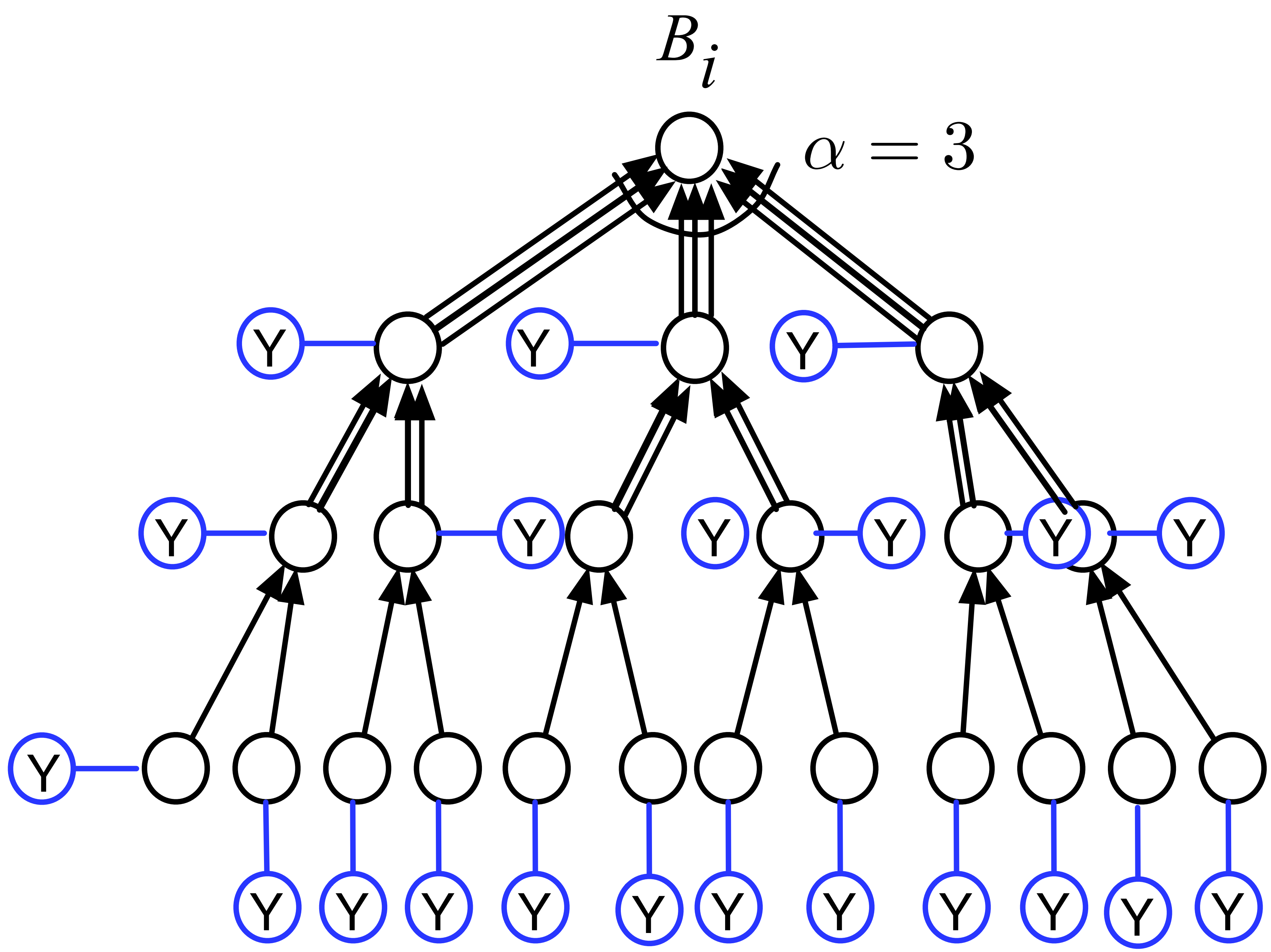


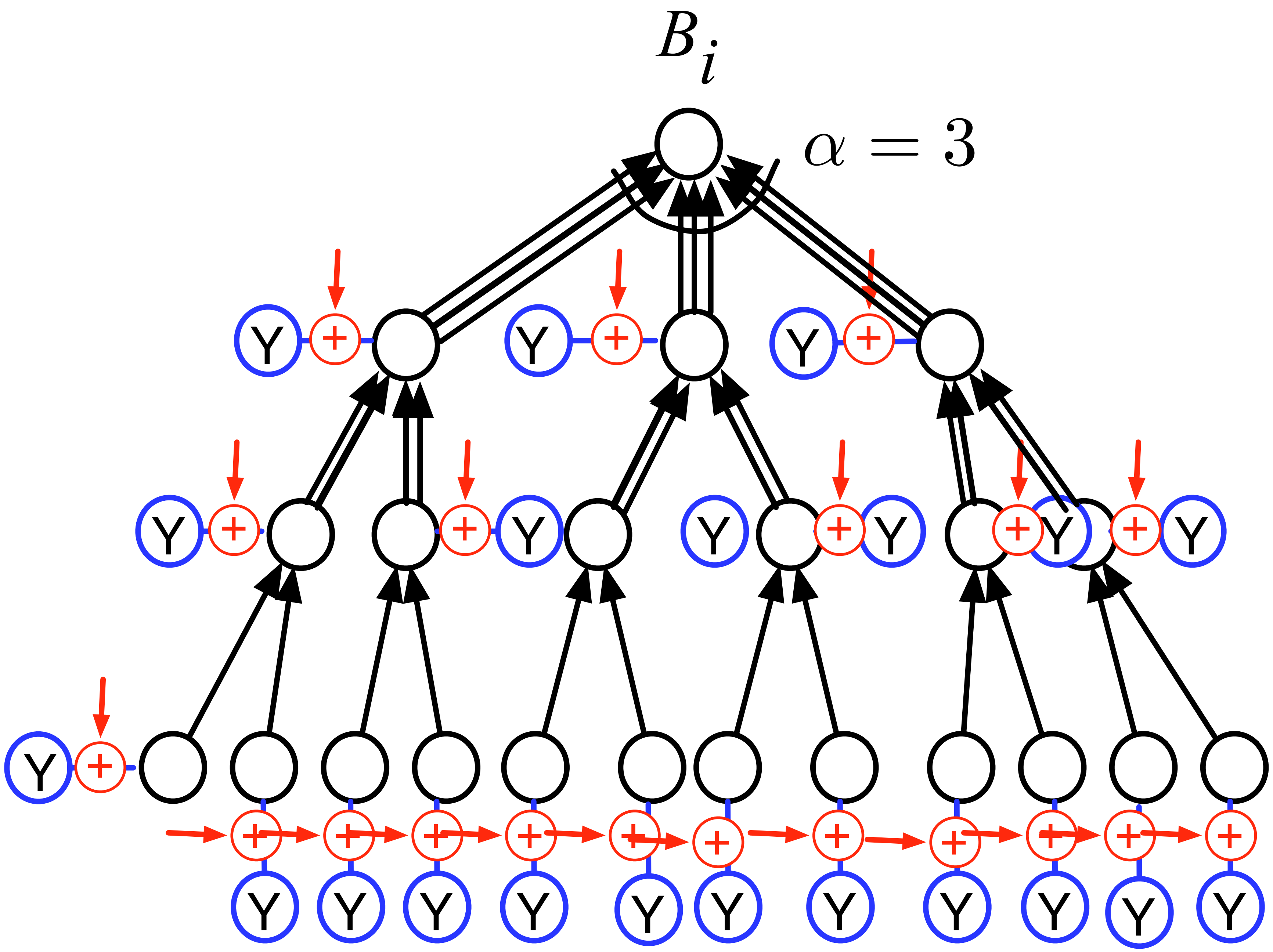
# From noisy computation to a lower bound



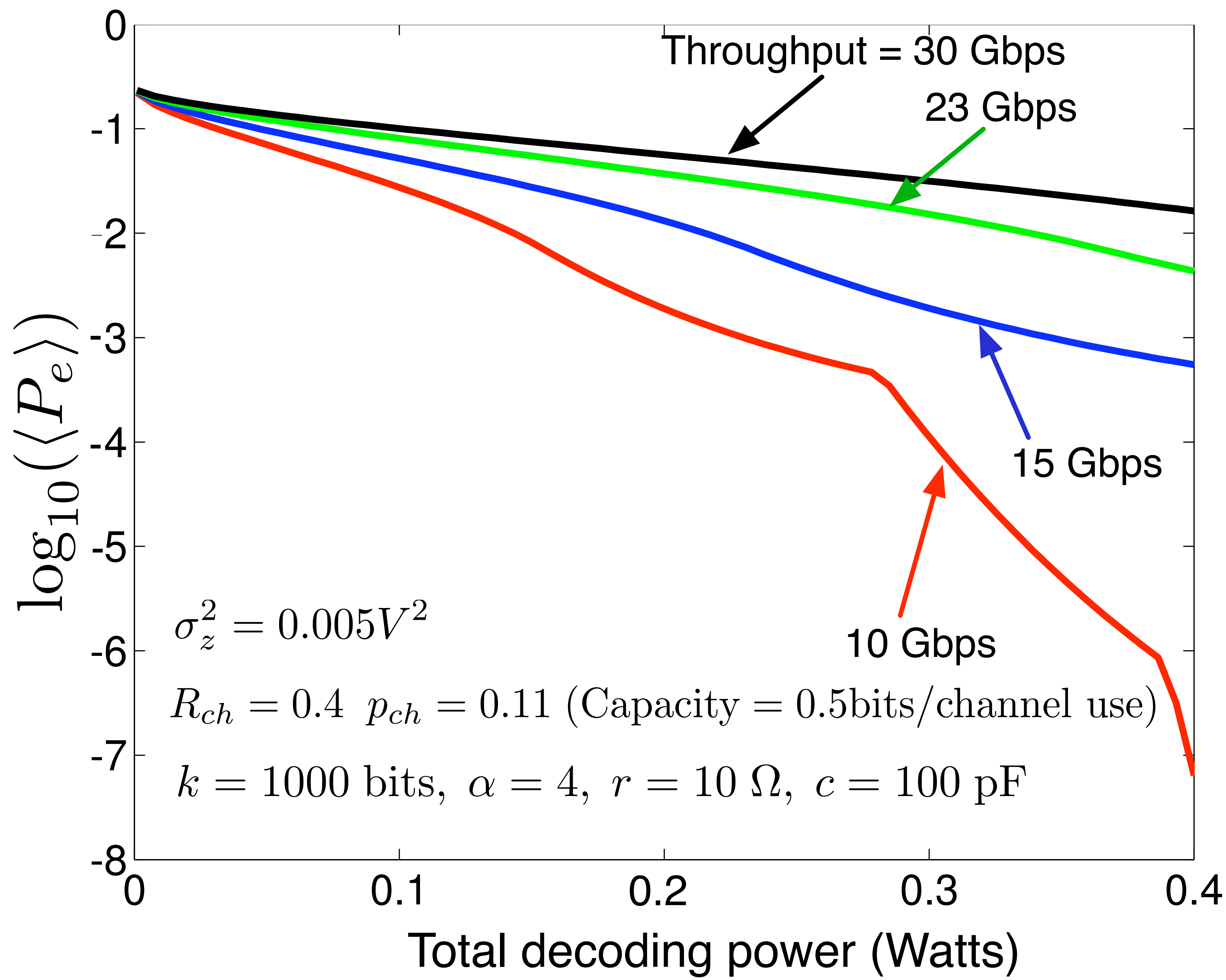
# From noisy computation to a lower bound



# From noisy computation to a lower bound



# lower bound on decoding power



# Where do we go from here?

- fewer assumptions
- tighter bounds using finite-ness of bit-pipes?
- better codes to bridge the gap
- code design for specific decoding architectures