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EE147/247A Final, Fall 2015 Solutions

NO CALCULATORS, CELL PHONES, or other electronics allowed. Show your work. Use proper units in all answers.

8:07

1. [13] True/False (circle one) 1pt each
   a. True / [✓ False] Vapor HF release causes stiction between polysilicon devices.
   b. True / [✓ False] Self-assembled monolayers like OTS and DDMS cause stiction between polysilicon devices.
   c. True / [✓ False] XeF2 causes stiction between polysilicon devices.
   d. [✓ True] / ~~False~~ Water causes stiction between polysilicon devices.
   e. True / [✓ False] CO2 critical point drying is used as a sacrificial etchant in surface micromachining.
   f. True / [✓ False] Evaporated thin films give good step coverage.
   g. True / [✓ False] All LPCVD thin films are conformal.
   h. [✓ True] / False The DRIE etching rate depends on the size of the features being etched.
   i. [✓ True] / False The resistivity of silicon varies by many orders of magnitude as the fraction of phosphorous dopant atoms increases from 0 to 1%.
   j. True / [✓ False] The stiffness of silicon varies by many orders of magnitude as the fraction of phosphorous dopant atoms increases from 0 to 1%.
   k. True / [✓ False] In a silicon crystal, silicon atoms are just over 1nm apart.
   l. [✓ True] / False The stiffness of polysilicon is roughly the same in all directions.
   m. True / [✓ False] The stiffness of single crystal silicon is roughly the same in all directions.

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2. [22] Short answer. Give a brief description in support of your answer (a sentence fragment is fine).

8:08

a. How does the average thermal noise voltage on a capacitor change if the capacitance increases by a factor of 100? $\tilde{v}_n = \sqrt{\frac{kT}{C}}$ decreases by 10x

b. How does the average thermal noise energy in a capacitor change if the capacitor increases by a factor of 100? $\frac{1}{2}k_BT$ no change

c. How does the spectral density (Watts/Hz) of the noise power in a resistor change if the resistance increases by a factor of 100?

$4k_BT$ no change

d. How does the spectral density (V/sqrt(Hz)) of the noise voltage on a resistor change if the resistance increases by a factor of 100?

$\tilde{v}_n = \sqrt{4k_BTR\Delta f}$ increases by 10x

e. What is the silicon etchant in the Bosch DRIE process?

SF_6 plasma

f. Is the silicon etchant in the Bosch DRIE process isotropic or anisotropic?

mostly isotropic

g. Why is the Bosch DRIE process anisotropic?

alternating etch/dep dep inhibits sidewall etching

h. In a gap-closing electrostatic actuator with a linear spring support, you can set the position of the actuator to one half of the gap by careful choice of applied voltage.

False. Electrostatic pull-in after you've gone $\frac{1}{3}$ of the way in.

i. In a sputtering system, there's a capacitor between the AC supply and one electrode. Why?

creates a negative DC bias on one side to attract positive ions from the plasma

j. In an RIE system, the substrate has a negative bias. Why?

to attract positive reactive ions from the plasma (extract)

k. In most electrostatic gap closing actuators, the voltage needed to close the gap is much larger than the voltage at which the gap will re-open. Why?

Force ~ $\frac{V^2}{gap^2}$ when closed, gap is much smaller, so voltage also needs to be much smaller

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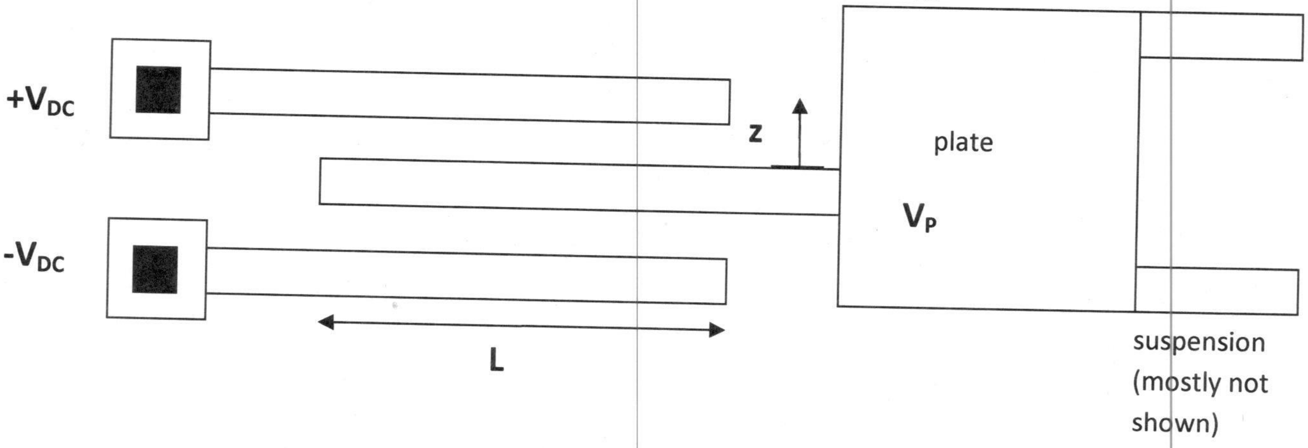
3. [16] You have a force-balanced accelerometer consisting of a suspended plate (mass), and comb fingers for both sensing and force feedback. A simplified diagram is shown below (the full suspension is not shown). There are two fixed fingers biased at plus and minus $\mathbf{V_{DC}}$ (equal magnitude, opposite sign). The finger overlap is **L**, the film thickness is **t**. The plate bias is $\mathbf{V_P}$. Ignoring the sensing component, solve for the force on the plate in the z direction. At rest with **z=0**, the two gaps are both $\mathbf{g_0}$.



a. Write an expression for the force on the plate in the z direction as a function of the plate voltage $\mathbf{V_P}$, and the plate position **z**.

$$F = \frac{1}{2}\epsilon_0\left[\frac{(V_{DC}-V_P)^2}{(g_0-z)^2} - \frac{(-V_{DC}-V_P)^2}{(-g_0-z)^2}\right] Lt$$

b. When **z=0**, calculate the force on the plate (hint: it is linear in $\mathbf{V_P}$!).

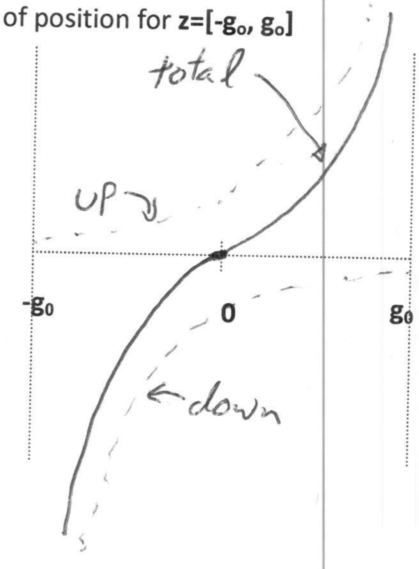
$$F = \frac{1}{2}\epsilon_0 \frac{Lt}{g_0^2}\left[\left(V_{DC}^2 - 2V_{DC}V_P + V_P^2\right) - \left(V_{DC}^2 + 2V_{DC}V_P + V_P^2\right)\right]$$

$$= \frac{1}{2}\epsilon_0 \frac{Lt}{g_0^2}\left(-4V_{DC}V_P\right) = -2\epsilon_0 \frac{Lt}{g_0^2} V_{DC}V_P$$

c. When $\mathbf{V_P}$=0, sketch the electrostatic force on the plate as a function of position for $\mathbf{z=[-g_o, g_o]}$

$$F_{V_P=0} = \frac{1}{2}\epsilon_0 V_{DC}^2 Lt\left(\frac{1}{(g_0-z)^2} - \frac{1}{(-g_0-z)^2}\right)$$

$$K = \left.\frac{df}{dz}\right|_{z=0} = \frac{1}{2}\epsilon_0 V_{DC}^2 Lt\left[\frac{+2}{(g_0-z)^3} - \frac{-2}{(g_0+z)^3}\right]$$



d. What is the electrostatic spring constant when **z=0** and $\mathbf{V_P}$ =0?

$$K = \frac{1}{2}\epsilon_0 V_{DC}^2 \frac{4Lt}{g_0^3} = 2\epsilon_0 V_{DC}^2 \frac{Lt}{g_0^3}$$

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4. [16] A silicon dioxide cantilever beam of length **L**, width **w**, and thickness **t** has a small polysilicon resistor **R** patterned on its top surface at the base. You can assume that the polysilicon is thin enough that it does not affect the stress distribution in the cross-section when the beam deflects. The resistor is the only varying element in a Wheatstone bridge with an excitation voltage of $\mathbf{V_X}$. The beam has a spring constant **K**, damping coefficient **b**, and a mass **m**.

   a. What is the axial strain **ε** in the polysilicon resistor as a function of a force **F** applied at the tip of the beam?

$$\mathcal{E} = \frac{3}{2}\frac{aF}{L^2 k} = \frac{6FL}{Ea^2 b}$$

   b. What is the change in resistance in the resistor as a function of the strain **ε**?

$$\frac{\Delta R}{R} = G\epsilon$$

   c. What is the change in the bridge output voltage as a function of resistance change?

$$V_o = \frac{\Delta R}{R}\frac{V_x}{4}$$

   d. What is the noise force on the beam due to thermal noise?

$$F_n = \sqrt{4kbT\Delta f}$$

   e. What is the noise voltage in the resistor due to thermal noise?

$$V_n = \sqrt{4k_B T R\Delta f}$$

   f. What is the total noise displacement of the spring?

$$\frac{1}{2}K x_n^2 = \frac{1}{2}k_B T \rightarrow x_n = \sqrt{\frac{k_B T}{K}}$$

   g. Assuming that the resistor noise dominates the total output noise (typically a good assumption) and R=1kΩ, Vx=1V, what is the output noise in a 10 kHz bandwidth?

$$V_{n,R} = \frac{4\,nV}{\sqrt{Hz}}(10^4\,Hz)^{1/2} = 400\,nV$$

   h. You apply a 1mN force to the end of the beam and see a 1V signal. What is the minimum detectable signal (noise equivalent force) in a 10 kHz bandwidth with R=1kΩ, Vx=1V as above?

$$V_{out} = \left(\frac{1\,V}{mN}\right)F_{signal} + \overline{V_{n,R}}$$

$$F_{N.Eq} = \frac{\overline{V_{nR}}}{1\,V/mN} = \frac{400\,nV}{1\,V/mN} = 400\,pN$$

$$= 0.4\,nN$$

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5. [4] Write an equation that would let you calculate the angle between the $\overset{x}{(110)}$ plane and the $\overset{y}{(100)}$ plane in a silicon crystal (you don't need to solve the inverse trig problem, but simplify the easy parts)

$\cos\theta \cdot |x||y| = x \cdot y$

$|x| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$|y| = \sqrt{1} = 1$

~~$x \cdot y = 1 \cdot 1 + 1 \cdot 1 + 0 \cdot 0$~~

$x \cdot y = 1 \cdot 1 + 1 \cdot 0 + 0 \cdot 1 = 1$

$\sqrt{2} \cdot 1 \cos\theta = 1 \rightarrow \cos\theta = \frac{1}{\sqrt{2}}$

~~$\sqrt{3} \sqrt{2} \cos\theta = 2 \rightarrow \cos\theta =$~~

6. [10] Write down a process flow that would let me make the following cross-section.

-2 for PR KOH mask
-1 for no PIT

LPCVD SiO_2 /PIT

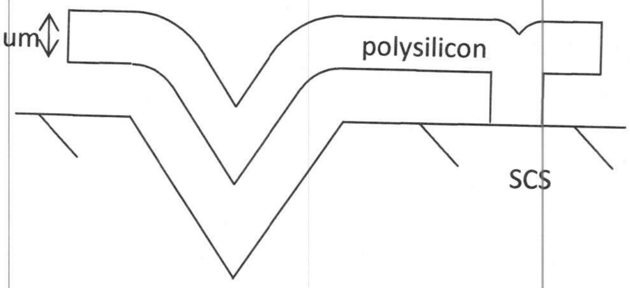
KOH etch

HF oxide etch

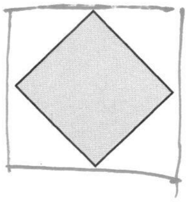
2um LPCVD SiO2 / CONTACT

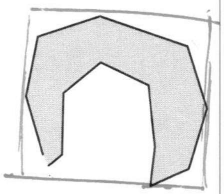
2um LPCVD poly / BEAM

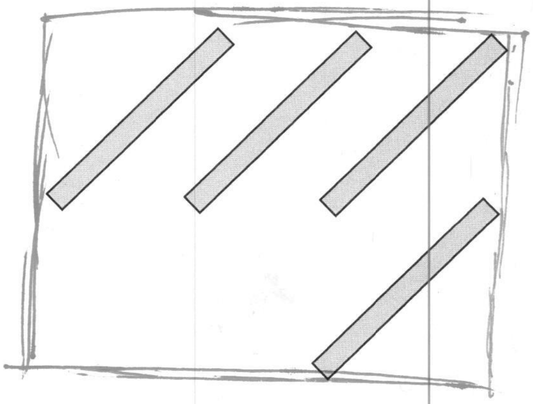
HF release



7. [6] A (100) wafer coated with silicon dioxide has the following regions opened to the silicon surface. The wafer is dropped in a KOH etch and the etch runs until only {111} planes are exposed. What is the outline of the etched regions under the silicon dioxide (i.e. where is the region where the SiO2 will not be supported by silicon)? Assume that this page is oriented with the wafer flat.







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8. [12 total] In a Tang, Nguyen, Howe comb drive resonator you apply 15V DC to the resonator, and 1.5 V sin(ωt) to one of the combs. There are 10 comb gaps on each side. The suspension stiffness is 10 N/m. The Q in vacuum is 10,000.
   a. [6] What is the total force applied to the resonator (magnitude in N, and phase for each term)?

$$\frac{F}{t/g} = (10)\left(\tfrac{1}{2}\epsilon_0 V_{DC}^2\right) + 10\left(\tfrac{1}{2}\epsilon_0 2V_{DC}V_{AC}\right)\sin(\omega t) + 10\left(\tfrac{1}{2}\epsilon_0 \frac{V_{AC}^2}{2}\right)\cos(2\omega t)$$

$$F = \frac{t}{g}\left[10\,nN + 2\,nN\sin(\omega t) + 0.05\,nN\cos(2\omega t)\right]$$

-1 if you leave ϵ_0 in the equation! $\tfrac{1}{2}\epsilon_0(15V)^2 = 1nN$!

   b. [6] What is the total displacement of the structure if the frequency of the 1.5 V sin wave is equal to the resonant frequency ω_n ? (magnitude in meters and phase for each term)

$$\frac{x(t)}{t/g} = \left\{\frac{10\,nN}{10\,N/m} + \frac{Q}{10\,N/m}\,2\,nN\sin\left(\omega t - \frac{\pi}{2}\right) - \frac{1/4}{10\,N/m}\,0.05\,nN\cos(2\omega t - \pi)\right.$$

$$\frac{x(t)}{\cancel{t/g}} = \left[1\,nM + 2\,\mu m\sin\left(\omega_n t - \frac{\pi}{2}\right) + 1.25\,pm\cos(2\omega_n t)\right]\frac{t}{g}$$

9. [6] In a comb-drive resonator how does the suspension stiffness, resonant frequency, and quality factor, Q, change
   a. if Young's modulus, E, increased by a factor of 2?

   b. if the length of the support beams increased by a factor of 2?

10. [4] In a comb-drive resonator, how does the damping and quality factor change if
    a. the gap under the mass increases by a factor of 2? (you can ignore damping between comb fingers and above the mass)

    b. the air pressure increases by a factor of 2? (assume we start at atmospheric pressure)

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8. [12 total] In a Tang, Nguyen, Howe comb drive resonator you apply 15V DC to the resonator, and 1.5 V sin(ωt) to one of the combs. There are 10 comb gaps on each side. The suspension stiffness is 10 N/m. The Q in vacuum is 10,000.
   a. [6] What is the total force applied to the resonator (magnitude in N, and phase for each term)?

~~$F = \frac{1}{2}\varepsilon_0 \frac{t}{g}\left(15V^2 + \frac{15V^2}{20} + 2(15V)^2 \frac{1}{10}\sin\omega t - \left(\frac{15V}{10}\right)^2 \frac{1}{2}\cos(2\omega t)\right)$~~

   b. [6] What is the total displacement of the structure if the frequency of the 1.5 V sin wave is equal to the resonant frequency ω_n ? (magnitude in meters and phase for each term)

9. [6] In a comb-drive resonator how does the suspension stiffness, resonant frequency, and quality factor, Q, change
   a. if Young's modulus, E, increased by a factor of 2?

$k \uparrow \times 2$

$\omega_n \uparrow \times\sqrt{2}$

$Q = \frac{k}{b\omega_n} \propto \frac{2}{\sqrt{2}} \uparrow \times\sqrt{2}$

   b. if the length of the support beams increased by a factor of 2?

$k \downarrow \times 8$ $Q \downarrow \times\sqrt{8}$

$\omega_n \downarrow \times\sqrt{8}$

10. [4] In a comb-drive resonator, how does the damping and quality factor change if
    a. the gap under the mass increases by a factor of 2? (you can ignore damping between comb fingers and above the mass)

$b \propto \frac{1}{g} \downarrow \times 2$

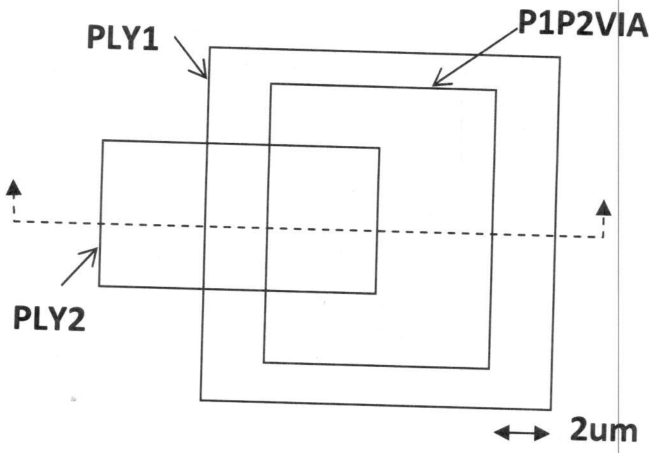
$Q \propto \frac{1}{b} \uparrow \times 2$

    b. the air pressure increases by a factor of 2? (assume we start at atmospheric pressure)

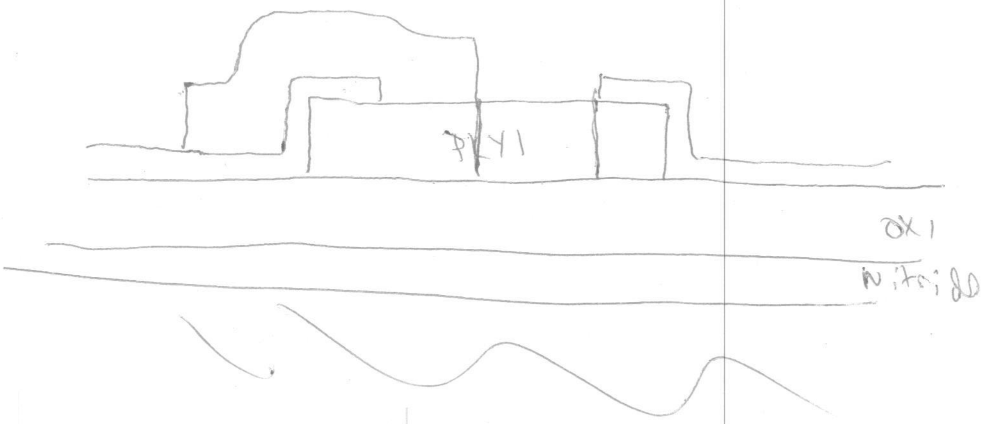
$b \propto \mu \uparrow \times 2$

$Q \propto \frac{1}{b} \downarrow \times 2$

11. [10 total] In the polyMUMPS process you draw the following layout.



Draw a cross-section of the resulting device assuming infinite selectivity in all etches, 20% overetch in all etches except PLY2, which has 200%.



-1 No nitride

-1 with PLXO, METAL,

-2 No PLX1 etch w/ PLX2

-1 partial PLY1 etch w PLY2

-1 layer thicknesses way off

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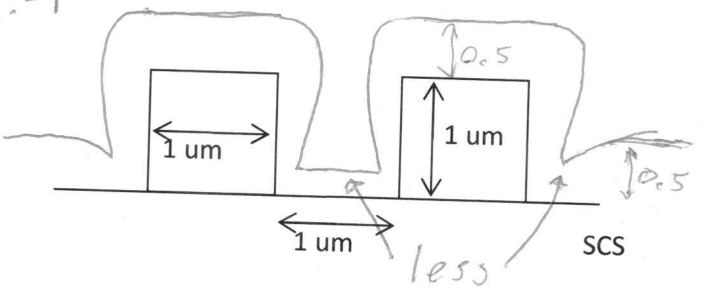
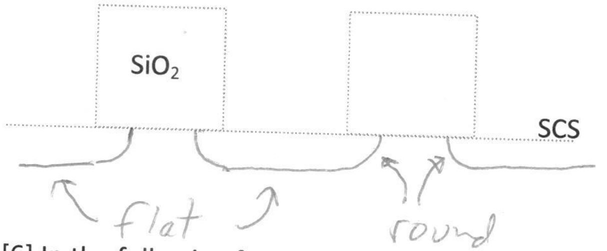
12. [12] In the following figures, assume that an SiO_2 film 0.5um thick has been patterned with a 1um line and space on two different silicon wafers. The cross section of two lines is shown below. Draw the expected cross-section for three different processes: 1) 0.5um sputtered aluminum; 2) 0.3um etch in XeF2; 3) 0.6um LPCVD polysilicon followed by a 1 um silicon RIE. RIE selectivity to oxide is 2:1. The substrate is single-crystal silicon. For process 2, draw the cross-section after each of the two steps.

directional: -2
conformal: -1

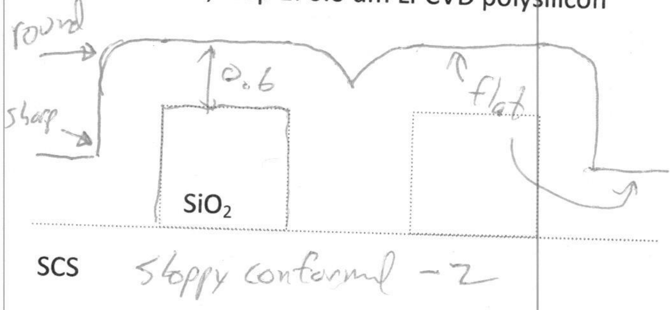
Process 1 : 0.5um sputtered aluminum



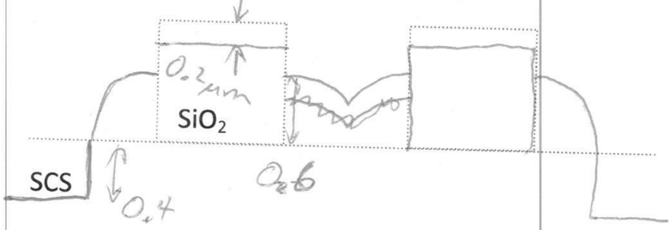
Process 2 : 0.3 um XeF_2 etch



Process 3, step 1: 0.6 um LPCVD polysilicon



Process 3, step 2: 1 um RIE silicon etch



13. [6] In the following figures, a 1 micron thick SiO2 layer has been patterned to expose bare [100] silicon. Both apertures are 10um square, one is oriented in the 110 direction (parallel to the wafer flat), one is oriented in the 100 direction (45 degrees from the wafer flat). Draw the cross section after a 3 um etch in KOH.