

EE147/247A Midterm, Fall 2014

NO CALCULATORS, CELL PHONES, or other electronics allowed. Show your work, and put final answers in the boxes provided. Use proper units in all answers.

*wrong formula -5      bad math -1  
wrong or missing term -2*

1. [35pts, 5 each] You are characterizing a new process and a new material, a thin film. You have made a variety of test structures, and want to find the material properties of the thin film. The film is  $1\mu\text{m}$  thick. You have made a several beams that are  $10\mu\text{m}$  wide and  $1000\mu\text{m}=1\text{mm}$  long.

a. You apply 1V across the ends of the beam, and measure a current of 1mA. What is the resistivity of the material?

$$R = 10^3 = \rho \frac{L}{A} = \rho \frac{10^{-3}\text{m}}{(10^{-5}\text{m})(10^{-6}\text{m})} = \rho 10^{8\frac{1}{2}} \quad \rho = \frac{10^3 \Omega}{10^{8\frac{1}{2}} \frac{1}{\text{m}}}$$

$\rho = 10^{-5} \Omega\text{m}$

b. You flow  $1\mu\text{W}$  of power along the length of the beam, and measure a temperature difference of  $\Delta T = 10$  Kelvin. What is the thermal conductivity of the material?

$$10^{-6}\text{W} = \dot{Q} = \kappa \frac{A}{L} \Delta T = \kappa \frac{10^{-11}\text{m}^2}{10^{-3}\text{m}} 10\text{K} = \kappa 10^{-7} [\text{mK}] \quad \kappa = \frac{10^{-6}\text{W}}{10^{-7}\text{mK}}$$

$\kappa = 10 \frac{\text{W}}{\text{mK}}$

c. When you heat the beam uniformly by 10 Kelvin, the resistance changes by 5 percent. What is the temperature coefficient of resistance?

$$5 \times 10^{-2} = \frac{\Delta R}{R} = \alpha \Delta T = \alpha 10\text{K} \quad \alpha = 5 \times 10^{-3} \frac{1}{\text{K}}$$

$\text{TCR} = 5 \times 10^{-3} \frac{1}{\text{K}}$

d. When you pull on the beam with a force of 1mN, you measure a change in length of  $1\mu\text{m}$ . What is the modulus of elasticity?

$$\frac{10^{-3}\text{N}}{10^{-6}\text{m}} = \frac{F}{x} = k = E \frac{A}{L} = E \frac{10^{-8}\text{m}^2}{10^{-3}\text{m}} = 10^3 \frac{\text{N}}{\text{m}} \quad E = 10^{11} \frac{\text{N}}{\text{m}^2}$$

$E = 10^{11} \frac{\text{N}}{\text{m}^2}$

e. When you pull on the beam with a force of 1mN, you measure a resistance change of 1 percent. What is the piezoresistive gauge factor?

$$E = \frac{10^{-6}}{10^3} = 10^{-9} \quad \frac{\Delta R}{R} = 10^{-2} = G \epsilon = G \cdot 10^{-3} \quad G = \frac{10^{-2}}{10^{-3}} = 10$$

$G = 10$

f. You pull on the end of the beam and slowly stretch it. At the point when it is  $20\mu\text{m}$  longer, it breaks. What is the strain limit?

$$\epsilon = \frac{\Delta L}{L} = \frac{2 \times 10^{-5}\text{m}}{10^{-3}\text{m}} = 2 \times 10^{-2}$$

$\epsilon_{\text{max}} = 2\%$

g. If one end of the beam is attached to the substrate, you notice that the rest of the beam curls slightly out of plane, with the tip displacement equal to  $10\mu\text{m}$ . What is the unreleased surface strain gradient parameter  $\sigma_1$  divided by the elastic modulus?

$$y = \frac{\sigma_1}{E} \frac{L^2}{a} \quad \frac{\sigma_1}{E} = \frac{ya}{L^2} = \frac{(10^{-5}\text{m})(10^{-6}\text{m})}{(10^{-3}\text{m})^2} = 10^{-5}$$

$\sigma_1/E = 10^{-5}$

2. [10] In the polyMUMPS process, if you draw three large concentric circles in your layout on layers ANCHOR2, POLY2, and METAL what is the list of materials in the center of the circle just before the HF release etch? Start at the silicon substrate and work up, in order.

*oxide, nitride, poly 2, gold*

*extra layer: -1 each  
missing layer: -3 each*

35

45



3. [20] You have made a Wheatstone bridge with four identical silicon resistors. The nominal resistance of each is  $1k\Omega$ . They have a gauge factor of  $G=100$ , and a temperature coefficient of resistance of  $\alpha=0.1\%/K$ . The bridge excitation voltage is  $V_x=10V$ .

a. If all of the resistors heat up by  $\Delta T=10K$ , what is the output voltage of the bridge.

$$V_{out} = 0V$$

b. If three of the resistors heat up by  $10K$ , and one heats up by  $20K$ , what is the output voltage of the bridge?

$$\frac{\Delta R}{R} = \left(10^{-3} \frac{1}{K}\right) (10K) = 10^{-2}$$

$$V_{out} = \frac{1}{4} V_x \cdot 10^{-2}$$

$$V_{out} = 25mV$$

c. Assume that all resistors are the same temperature, and one of them experiences a strain of  $\epsilon=10^{-6}=1ppm$ . What is the output voltage of the bridge?

$$\frac{\Delta R}{R} = G\epsilon = 10^{-4} \quad V_{out} = \frac{V_x}{4} \cdot 10^{-4}$$

$$V_{out} = 250\mu V$$

d. If your sense electronics has a noise floor of  $V_{noise}=100nV$ , what is the noise-equivalent (minimum detectable) strain?

$$V_{out} = \frac{V_x}{4} G\epsilon = 250V\epsilon = V_{noise} = 100nV$$

$$\epsilon = \frac{100nV}{250V} = 0.4 \times 10^{-9}$$

$$\epsilon_{neq} = 4 \times 10^{-10}$$

4. [24] You have made a classic "Tang, Nguyen, Howe" comb drive resonator in an SOI process. You apply pure force  $f(t)=F_0\sin(\omega t)$  and measure a resonant frequency of  $\omega_n=1,000rad/s$  and a  $Q$  of  $1,000$  in vacuum. The DC response to  $F_0$  is a  $10nm$  displacement. You sweep the force over a range of frequencies.

a. [6] What is the displacement at the resonant frequency (should be a sinusoid, show magnitude in meters, and phase)

$$x_n(t) = 10\mu m \sin(1000t - 90^\circ)$$

b. [6] What is the displacement at  $\omega=1 rad/s$ ? (again, w/ magnitude and phase)

$$x_1(t) = 10nm \sin(t)$$

c. [6] What is the displacement at  $\omega=10,000 rad/s$ ? (again, w/magnitude and phase)

$$\omega = 10\omega_n \Rightarrow |x| = \frac{10nm}{10^2} = 0.1nm$$

phase is  $-180$

$$x_{10k}(t) = 0.1nm \sin(10^4 t - 180)$$

d. [6] You apply a force  $F(t) = F_0\sin(\omega_n t) + F_0\sin(t) + F_0\sin(10,000t)$ . What is the resulting displacement in terms of  $x_n(t)$ ,  $x_1(t)$ , and  $x_{10k}(t)$  and possibly other things? (box your answer)

$$x(t) = x_n(t) + x_1(t) + x_{10k}(t)$$