Some Economics Models

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Supplementary section for Understanding Networked Applications: A First Course, Morgan Kaufmann, 1999.

A tried-and-true technique in economics is to build a simple (or often sophisticated) mathematical model, and use it to gain insights into the behavior of economic actors. This appendix derives simple mathematical models illustrating some concepts in Chapter 8, using nothing beyond simple algebra.

The Economics of Self-Selection

Versioning is one form of price discrimination in which the customers are induced by pricing to self-select different versions of a product. This can be illustrated by the simple scenario shown in Figure 1., where there are two consumers (with lower and higher willingness to pay) and two versions of a product (with lower and higher quality). The value of the product (willingness to pay) is labeled as V; for example, V_{LH} is the value of the high-value product to the low willingness-to-pay consumer.

The supplier versioning strategy is to offer the lower- and higher-quality goods at price P_L and P_H respectively, setting these prices to maximize total revenue. (This will also be the highest-profit strategy if both versions have the same production cost, or virtually zero production or replication cost, as may be true for software or information products.) The versioning strategy is to induce the consumers to voluntarily self-select—the low-value consumer the low-quality and the high-value consumer the high-quality. For this purpose, the lower-quality product price can be set

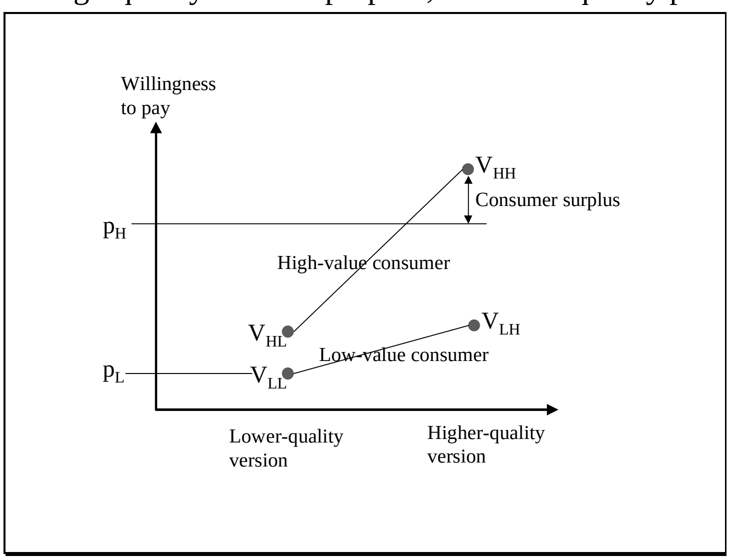

Figure 1. For two consumers and two versions of a product, the values (willingness to pay) for low- and high-value consumers and the prices of the two versions.

at $P_L = V_{LL}$, but setting any higher price and the low-value consumer will not buy. The higher-quality price P_H must not be set so high that the high-value consumer will choose to purchase the low-quality product. The point of indifference between the two choices for the high-value consumer is where the consumer surpluses are equal for the two choices,

$$V_{HH} - P_H = V_{HL} - P_L \text{ or } P_H = V_{HH} - V_{HL} + V_{LL} \quad \text{EQ 1}$$

The total revenue is then

$$P_L + P_H = V_{HH} - V_{HL} + 2 \cdot V_{LL} \quad \text{EQ 2}$$

There are two other strategies the supplier might pursue, both revolving around producing only the high-quality product. It can charge $P_H = V_{LH}$ and sell to both consumers with revenue $2 \cdot V_{LH}$, or it can charge $P_H = V_{HH}$ and sell only to the high-value consumer with revenue V_{HH}. From among these three, the highest-revenue strategy depends on the particulars of the values (see exercises).

The Value of a Locked-In Customer

A simple model can establish the value of a customer that is locked in to the supplier. Suppose a customer buys one upgrade to a product per unit time, but there are two suppliers who offer that upgrade at the same price P, and both suppliers have the same cost C. The customer has a one-time switching cost S to move from one supplier to the other.

Example… A customer might be considering two office-suite applications. Each offers an upgrade once per year at price P, but the customer has a cost S switching associated with training costs.

The customer will be just indifferent to switching if the supplier subsidizes the first sale by offering a one-time introductory price of $P - S$, so the customer's net cost after switching is P, thereafter charging P.

Let the discount rate be q; that is, due to the time value of money the value of revenue n time periods in the future is only q^n as large as the same revenue in the current period. Then the asset value (present worth) of an existing customer is the present value of the revenue stream,

$$V = (P - C) \cdot \sum_{i=0}^{\infty} q^i = \frac{P - C}{1 - q} \quad \text{EQ 3}$$

In the absence of lock-in, and for a market with perfect competition, the price will equal the cost and $V = 0$.

When will the supplier be just indifferent to taking on a new customer by offering a lower introductory price? The future asset value of the new customer is qV, and the one-time cost is $C - (P - S)$, since the one-time price is $P - S$, presumably below cost. Thus, the supplier is just indifferent to taking on a new customer if the one-time cost equals the future asset value, or $C + S - P = qV$. The price the supplier can get away with charging is thus

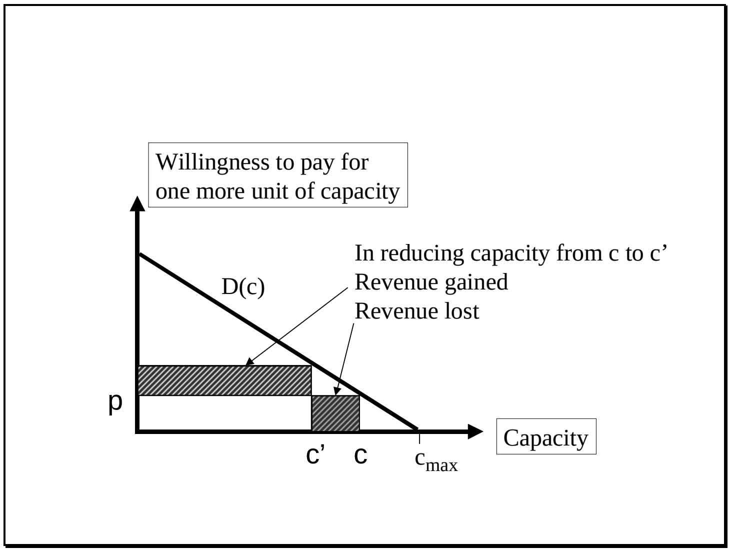

Figure 2. When the service provider charges a fixed price per unit of capacity, revenue is maximized at a much smaller capacity than the users are willing to pay for.

$$P = C + S - qV \text{ where } V = \frac{P - C}{1 - q} . \qquad \text{EQ 4}$$

Solving these simultaneous equations, $P = C + S \cdot (1 - q)$ and $V = S$. Thus, the asset value of a current customer is just equal to that customer's switching cost, with a profit per unit time equal to $S \cdot (1 - q)$. The introductory price is $P - S = C - qS$, which is indeed below cost.

Example of Network Services Pricing

It is easy to illustrate, using a simple economics model, the perverse incentives to a service provider that result from overly simplistic pricing schemes. An example is "fixed price per unit of bitrate capacity" pricing—a user would pay twice as much for twice the capacity—which would cause a revenue maximizing service provider to provision less network capacity than the user is willing to pay for, as illustrated in Figure 2.. Shown is a user demand curve $D(c)$, interpreted as the willingness to pay for one more unit of bitrate capacity as a function of current capacity c. As capacity increases, a user will be willing to pay less for the *next* unit of capacity, and thus $D(c)$ decreases with c. The maximum capacity the user is willing to buy is c_{max}. The provider can sell capacity c by charging a price $P = D(c)$, receiving revenue $c \cdot P$. As shown, a revenue-maximizing provider will reduce capacity far below c_{max} in order to charge a higher price. (For the linear demand curve shown, the maximum revenue is achieved when the capacity is $c_{max}/2$.) Reducing capacity also reduces capital expenditures as well as increases revenue, giving the provider a double benefit.

A tiered pricing approach makes more sense for both provider and user, with additional increments in capacity charged at a lower price. Then a provider could increase capacity and derive additional revenue without reducing the price of capacity already sold.

Exercises

E1. Describe a "variable price per unit of capacity" model that would get around the perverse incentive illustrated in Figure 2..

E2. For the consumers demands represented in the example in Figure 1., find conditions on the willingness to pay for each of the following, and interpret those conditions intuitively:

a. A single high-quality product with a fixed low price is more advantageous than versioning

b. A single high-quality product with a fixed high price is more advantageous than versioning

E3. For the consumers demands represented in the example of Figure 1., find conditions on the willingness to pay for each of the following, and interpret those conditions intuitively:

a. A single high-quality product with a fixed low price is more advantageous than versioning

b. A single high-quality product with a fixed high price is more advantageous than versioning