EECS 225A Spring 2005

Homework 8 solutions

1. In class we showed that if the transfer function for the $n-1$ order lattice filter $A_{n-1}(z)$ is minimum phase, and $|\Gamma_n| < 1$ then $A_n(z)$ is also minimum phase.
   a. Loosen the assumption and assume that only m zeros of $A_{n-1}(z)$ fall inside the unit circle, and none are on the unit circle (this may happen if one or more of the previous reflection coefficients were chosen to be larger than unity in magnitude, which of course cannot happen if they are chosen for the optimum linear predictor). For the two cases $|\Gamma_n| < 1$ and $|\Gamma_n| > 1$, how many zeros of $A_n(z)$ fall inside the unit circle (ignore the case $|\Gamma_n| = 1$)?
   b. Assume that the 3rd, 6th, and 9th reflection coefficients are greater than unity in magnitude, but all the remaining are less than unity in magnitude. How many zeros of $A_{10}(z)$ fall interior to the unit circle?

Solution

a. Recall the basic lattice update equation, written in terms of only positive powers of z:

$$\frac{z^n A_n(z)}{z \cdot z^{n-1} A_{n-1}(z)} = 1 + \Gamma_n \cdot \frac{A_{n-1}^*\left(\frac{1}{z^*}\right)}{z \cdot z^{n-1} A_{n-1}(z)}.$$

Rewriting this equation replacing numerator and denominator with a count of the number of roots interior to the unit circle for the corresponding polynomial,

$$\frac{k}{m+1} = 1 + \Gamma_n \cdot \frac{(n-1)-m}{m+1}.$$

- When $|\Gamma_n| < 1$ there are no encirclements of $z = 0$, and we get $k = m+1$ (the number of zeros interior to the unit circle increases by one). It is as if all the zeros previously interior the unit circle stayed there (but of course changed location), and the new zero is also interior to the unit circle.
- When $|\Gamma_n| > 1$ the number of encirclements of both rational polynomials must be equal, and thus the number of zeros in the numerators must be equal, or $k = (n-1) - m = n - (m+1)$. The number of zeros outside the unit circle is now $n - k = m+1$. That is, it is as if all the zeros previously exterior to the unit circle moved inside, and all the zeros previous interior (plus the added zero) moved outside.

b. We can fill in the following table:

Order	Reflection coefficient outside unit circle?	Interior to unit circle	Exterior to unit circle
1		1	0
2		2	0
3	yes	0	3
4		1	3
5		2	3
6	yes	3	3
7		4	3
8		5	3
9	yes	3	6
10		4	6

So we end up with 4 zeros interior to the unit circle, out of 10 total.

2. Hayes problem 9.3

Solution

(a) Evaluating the gradient vector we have

$$\nabla\xi(n) = 2E\{e(n)\nabla e(n)\} = -2E\{e(n)\mathbf{x}(n)\} = -2\mathbf{r}_{dx} + 2\mathbf{R}_x\mathbf{w}_n$$

Thus,

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \tfrac{1}{2}\mu\mathbf{R}_x^{-1}\left[2\mathbf{R}_x\mathbf{w}_n - 2\mathbf{r}_{dx}\right]$$

and we have

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \mu\mathbf{w}_n + \mu\mathbf{w} = (1-\mu)\mathbf{w}_n + \mu\mathbf{w}$$

Thus, the Newton algorithm is stable for $0 < \mu < 2$.

(b) The convergence is the fastest when $\mu = 1$. Note, in fact, that when $\mu = 1$, the Newton iteration converges in one step to $\mathbf{w}$.

(c) The gradient approximation is

$$\nabla e^2(n) = -2e(n)\mathbf{x}(n)$$

Therefore the LMS-type algorithm is

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu e(n)\mathbf{R}_x^{-1}\mathbf{x}(n)$$

Comparing this to the LMS algorithm we see that the step direction is changed from $\mathbf{x}(n)$ to $\mathbf{R}^{-1}\mathbf{x}(n)$.

(d) From (c) we see that

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu\mathbf{R}_x^{-1}\mathbf{x}(n)d(n) - \mu\mathbf{R}_x^{-1}\mathbf{x}(n)\mathbf{x}^T(n)\mathbf{w}_n$$

Assuming that $\mathbf{x}(n)$ is uncorrelated with the filter tap weight vector, $\mathbf{w}_n$, then

$$E\{\mathbf{w}_{n+1}\} = E\{\mathbf{w}_n\} + \mu\mathbf{R}_x^{-1}\mathbf{r}_{dx} - \mu\mathbf{R}_x^{-1}\mathbf{R}_x E\{\mathbf{w}_n\}$$

which becomes

$$E\{\mathbf{w}_{n+1}\} = (1-\mu)E\{\mathbf{w}_n\} + \mu\mathbf{w}$$