EECS 225A Spring 2005

Homework 1

---

1. Choose any two of the identities involving *finite* summations in Table 2.3 of Hayes.

   a. Verify those identities *numerically* for $0 \le N \le 1000$ using Matlab.
   b. Verify those identities for all N using (and trusting) the *symbolic* manipulation capabilities of Matlab.

Solution

See M-file hmwk01.m

---

2. Repeat 1. for Table 2.4.

Solution

See hmwk01.m (I did not do the numerical case since this didn't seem very worthwhile)

The fifth formula in Table 2.4 requires a little cleverness, since Matlab deals with one-sided sequences. We can write

$$\sum_{n=-\infty}^{\infty} a^{|n|} \cdot z^{-n} = \sum_{n=0}^{\infty} a^n \cdot z^{-n} + \sum_{n=0}^{\infty} a^n \cdot z^n - 1 = H(z) + H(z^{-1}) - 1$$

where $H(z) = \sum_{n=0}^{\infty} a^n \cdot z^{-n}$ is easily calculated by Matlab.

---

3. Do Problem 2.6 of Hayes. Use Matlab!

Solution

See hmwk01.m

---

4. Do Problem 2.19 of Hayes.

Solution

This is an analytic function of two complex variables, so the objective function is analytic:

$$Q(z_1, z_2) = 3z_1^2 + 3z_2^2 + 4z_1 z_2 + 8 + \lambda(z_1 + z_2)$$

The stationary point is at

$$6z_1 + 4z_2 + \lambda = 0 \text{ and } 6z_2 + 4z_1 + \lambda = 0 .$$

Solving for the two complex variables, we get $z_1 = z_2 = -\frac{\lambda}{10}$ and substituting in the constraint we find that $\lambda = -5$ and thus $z_1 = z_2 = \frac{1}{2}$ where the value of the function is 10.5.

---

5. Consider the complex-valued function of a complex variable $H(z) = \frac{1}{z - z_p}$ where z_p is a constant.

a. For what values of z is $H(z)$ an analytic function? Justify your answer.
b. List all possible double- and single-sided sequences for which $H(z)$ is the Z-transform. For each such sequence, give the ROC.

Solution

a. Examining the differentiation limit:

$$\frac{H(z+\Delta z) - H(z)}{\Delta z} = \frac{-1}{(z + \Delta z - z_p)\cdot(z - z_p)} \xrightarrow{\Delta z \to 0} \frac{-1}{(z - z_p)^2}$$

As long as $z \neq z_p$, the limit exists and is independent of the angle of Δz. Thus, the function is analytic for all $z \neq z_p$.

b. Since there is a single pole, the two possibilities on the ROC are $|z| > |z_p|$ and $|z| < |z_p|$.

$$\text{ROC} = |z| > |z_p|,\ H(z) = \frac{z^{-1}}{1 - z_p z^{-1}} = \sum_{k=1}^{\infty} z_p^{k-1} z^{-k},\ h_k = \begin{cases} 0, & k \le 0 \\ z_p^{k-1}, & k \ge 1 \end{cases}$$

Note in this case that the signal is causal and a decaying exponential if $|z_p| < 1$, a growing exponential if $|z_p| > 1$, and oscillatory if $|z_p| = 1$. By “exponential”, we mean the modulus of the signal; in addition, there is a linearly increasing phase. The Z-transform converges even when the signal is a growing exponential.

$$\text{ROC} = |z| < |z_p|,\ H(z) = \frac{-z_p^{-1}}{1 - z_p^{-1} z} = -\sum_{k=-\infty}^{0} z_p^{k-1} z^{-k},\ h_k = \begin{cases} -z_p^{k-1}, & k \le 0 \\ 0, & k \ge 1 \end{cases}$$

Note in this case that the signal is anti-causal and a decaying exponential if $|z_p| > 1$, a growing exponential if $|z_p| < 1$, and oscillatory if $|z_p| = 1$.

These answers are a little messy because $H(z)$ is not normalized. A natural normalization would force $H(z)$ to be *monic*, meaning $h_0 = 1$. Looking at the one-sided series representation, this is equivalent to $H(\infty) = 1$ in the causal case, and $H(0) = 1$ in the anti-causal case. Thus, the monic first-order rational transfer functions would be:

Causal case: $H(z) = \frac{z}{z - z_p}$, Anti-causal case: $H(z) = \frac{-z_p}{z - z_p}$.