OPTIMAL CONTROL OF HVAC SYSTEMS IN THE PRESENCE OF IMPERFECT PREDICTIONS

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Motivation

Conventional HVAC Control System

Thermal Modeling
  - Parameter ID
  - unmodeled Dynamics Estimation

Control Design
  - MPC Scheme
  - Min-Max strategy for RMPC
  - OL-RMPC
  - CL-RMPC
  - Proposed Parameterizations: LTS, Toeplitz, TLDS

Simulation Results
Buildings Consume Significant Energy

- Total US annual energy cost $370 Billion
- Increase in US electricity cons. since 1990: 200%
- Total US energy consumption for buildings: 40%
- Total US electricity consumption for buildings: 72%
- Total US natural gas consumption for buildings: 55%.

Source: Buildings Energy Data Book 2007

Related to HVAC
Lack of coordination at a system level
Control logic governing today’s buildings uses simple control schemes dealing with one subsystem at a time...

Local actions are determined without taking into account the interrelations among:

- Outdoor weather conditions
- Internal heat gains
- Indoor air quality
- Cooling demands
- HVAC process components
• **Energy balance for a wall node:**

\[
\frac{dT_{wi}}{dt} = \frac{1}{C_{wi}} \left[ \sum_{j \in \mathcal{N}_{wi}} \frac{T_j - T_{wi}}{R'_{ij}} + r_i \alpha_i A_i q''_{rad_i} \right]
\]

\[
r_i = \begin{cases} 0 & \text{internal wall} \\ 1 & \text{peripheral wall} \end{cases}
\]

• **Energy balance for a room node:**

\[
\frac{dT_{ri}}{dt} = \frac{1}{C_{ri}} \left[ \sum_{j \in \mathcal{N}_{ri}} \frac{T_j - T_{ri}}{R'_{ij}} + \dot{m}_r c_p (T_{si} - T_{ri}) + w_i \tau_{win} A_{win} q''_{rad_i} + q_{int} \right]
\]

_Thermal and circuit model of a wall with window_

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**Nonlinearity**  **Unmodeled dynamics**
Building Thermal Model

Linearize

\[
\dot{x}(t) = Ax(t) + Bu(t) + d(t)
\]

\[
y(t) = Cx(t)
\]
• External heat gain:

\[ q''_{rad,i}(t) = \lambda T_{out}(t) + \gamma \]

**Note:**
Other quantities such as **global horizontal irradiance** (GHI) data can be used here as well.

• Internal heat gain:

\[ \dot{q}_{int}(t) = \mu \Psi(t) + \nu \]

\( \Psi(t) \) is the \( CO_2 \) concentration in the room in (ppm).
For each room:

\[
\begin{align*}
T_1(t) & = \frac{1}{C_r} \left[ \sum_{i \in K} \frac{T_i - T_i^*}{R_i} + m_r(t) (T_{m} - T_i^*) + m_{w_i}(T_{w_i} - T_i^*) + \dot{q}_i \right] \\
T_2(t) & = \frac{1}{C_r} \left[ \sum_{i \in K} \frac{T_i - T_i^*}{R_i} + m_r(t) (T_{m} - T_i^*) + m_{w_i}(T_{w_i} - T_i^*) + \dot{q}_i \right] \\
T_3(t) & = \frac{1}{C_r} \left[ \sum_{i \in K} \frac{T_i - T_i^*}{R_i} + m_r(t) (T_{m} - T_i^*) + m_{w_i}(T_{w_i} - T_i^*) + \dot{q}_i \right] \\
T_4(t) & = \frac{1}{C_r} \left[ \sum_{i \in K} \frac{T_i - T_i^*}{R_i} + m_r(t) (T_{m} - T_i^*) + m_{w_i}(T_{w_i} - T_i^*) + \dot{q}_i \right]
\end{align*}
\]

\[
T(t) = f(C_r, C_{w1}, C_{w2}, C_{w3}, C_{w4}, R_1, R_2, R_3, R_4)
\]

\[
[C_r, C_{w1}, C_{w2}, C_{w3}, C_{w4}, R_1, R_2, R_3, R_4]^* = \arg\min_{C_r, C_{w1}, C_{w2}, C_{w3}, C_{w4}, R_1, R_2, R_3, R_4} \sum_t [e(t)]^2
\]
unmodeled Dynamics Estimation

- Initial guess (ASHRAE Handbook)
- Used fmincon

- Data of UC Berkeley Bancroft library,
- Conference room

Controller Design

1. Find an operating point of the system
2. Find the closest equilibrium point
3. Linearize about the equilibrium point
Model Predictive Control Scheme

\[
\begin{align*}
\min_{U_t, \bar{e}_t, \underline{e}_t} & \quad \{ |U_t|_1 + \kappa |U_t|_\infty + \rho (|\bar{e}_t|_1 + |\underline{e}_t|_1) \} \\
\min_{U_t, \bar{e}_t, \underline{e}_t} & \quad \left\{ \sum_{k=0}^{N-1} |u_{t+k}|_t + \kappa \max(|u_{t}|_t, \ldots, |u_{t+N-1}|_t) \right\} + \rho \sum_{k=1}^{N} (|\bar{e}_{t+k}|_t + |\underline{e}_{t+k}|_t) \\
\text{s.t.} & \quad x_{t+k+1|t} = Ax_{t+k|t} + Bu_{t+k|t} + Ed_{t+k|t}, \quad k = 0, \ldots, N - 1 \\
& \quad y_{t+k|t} = Cx_{t+k|t}, \quad k = 1, \ldots, N \\
& \quad 0 \leq u_{t+k|t} \leq \bar{U}, \quad k = 0, \ldots, N - 1 \\
& \quad T_{t+k|t} - \underline{e}_{t+k|t} \leq y_{t+k|t} \leq T_{t+k|t} + \bar{e}_{t+k|t}, \quad k = 1, \ldots, N \\
& \quad \underline{e}_{t+k|t}, \bar{e}_{t+k|t} \geq 0, \quad k = 1, \ldots, N
\end{align*}
\]
Schematic of Robust MPC Implementation

\[ x^+ = Ax + Bu + Ed + Fw \]

State update equation

\[ W_\lambda = \{ w : \|w\|_\infty \leq \lambda \} \]

Additive uncertainty

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Min-Max Strategy (Open-Loop) for RMPC

\[ J_0(x(t), U_t) \triangleq \]
\[ \max_{w[\cdot]} \left\{ \sum_{k=0}^{N-1} |u_{t+k}|t| + \kappa \max(|u_t|t|, \ldots, |u_{t+N-1}|t)| + \rho \sum_{k=1}^{N} (|\bar{e}_{t+k}|t| + |\tilde{e}_{t+k}|t|) \right\} \]

s.t.
\[ x_{t+k+1|t} = Ax_{t+k|t} + Bu_{t+k|t} + Ed_{t+k|t} + Fw_{t+k|t} \]
\[ w_{t+k|t} \in \mathbb{W} \]
\[ k = 0, \ldots, N - 1 \]

Robust counterpart of an uncertain optimization problem

TOO CONSERVATIVE!!!

\[ J^*_0(x(t)) \triangleq \min_{U_t} J_0(x(t), U_t) \]
subject to
\[ x_{t+k+1|t} = Ax_{t+k|t} + Bu_{t+k|t} + Ed_{t+k|t} + Fw_{t+k|t} \]
\[ y_{t+k|t} = Cx_{t+k|t} \]
\[ T_{t+k|t} - \tilde{e}_{t+k|t} \leq y_{t+k|t} \leq \bar{T}_{t+k|t} + \bar{e}_{t+k|t} \]
\[ \tilde{e}_{t+k|t}, \bar{e}_{t+k|t} \geq 0 \]
\[ \forall \ w_{t+k|t} \in \mathbb{W} \quad \forall \ k = 0, \ldots, N - 1 \]
Closed-loop min-max problem:

\[
\begin{align*}
\min & \max \cdots \min \\
& u_{k|k} \quad w_{k|k}
\end{align*}
\]

\[
\sum_{j=0}^{N-1} p(x_{k+j|k}, u_{k+j|k})
\]

- State feedback prediction: \( U = MX + v \)
- New decision variables: \( v = [v_{k|k}, v_{k+1|k}, \ldots, v_{k+N-1|k}] \)
- Parameter matrix \( M \) is causal:
  in the sense that \( u_{k+j|k} \) only depends on \( x_{k+i|k}, i \leq j \).

Sometimes \( M \) is incorporated as a decision variable...

The mapping from \( M \) and \( v \) to \( X \) and \( U \) is nonlinear!
Disturbance Feedback Policy:
- parameterize future inputs as affine functions of past disturbances.

\[ U = Mw + v \]

i.e.
\[ u_i := \sum_{j=0}^{i-1} m_{i,j} \omega_j + v_i \quad \forall i = 1, \ldots, N - 1 \]

Where \( M_{i,j} \in \mathbb{R}^{m \times p} \) and \( v_i \in \mathbb{R}^m \).
Drawback:

- Main **problem** with the *min-max formulations* based on these parameterizations is:

  the **excessive** number of decision variables and constraints

To resolve this issue

we study some other parameterizations
Toeplitz Structure

- **Lower Triangular Toeplitz** (diagonal-constant) structure:

\[ U = Mw + v \]

\[ M = \begin{pmatrix}
  k_1 & & \\
  k_2 & k_1 & \\
  k_3 & k_2 & k_1 \\
  \vdots & \ddots & \ddots \\
  k_{N-1} & \cdots & \cdots & k_2 & k_1 \\
  k_N & k_{N-1} & \cdots & \cdots & k_2 & k_1 \\
\end{pmatrix} \]

- was shown to deteriorate the performance of the CL-RMPC in our simulations!
Two Lower Diagonal Structure (TLDS)

- By analyzing the structure of the optimal matrix \( M \), we observed:
  - the parameterization of the input need not consider feedback of more than past two values of \( w \) at each time.

\[
\begin{align*}
  u_i & := m_{i,i-2}w_{i-2} + m_{i,i-1}w_{i-1} + v_i \\
  & = \sum_{j=i-2}^{i-1} m_{i,j}w_j + v_i \quad \forall i = 1, \ldots, N - 1
\end{align*}
\]

we exploit the sparsity of the \( M \) matrix to enhance the computational cost of the optimization problem.
Comfort Zone Definition

- Unoccupied hours: $Q_1, R_1$
- Occupied hours: $Q_2, R_2$
- Unoccupied hours: $Q_1, R_1$
Simulation Results

Comparison of ECS, MPC, OL-RMPC and CL-RMPC

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RMPC: Energy vs. Comfort

\[
P_c(t) = \dot{m}_c(t) c_p [T_{out}(t) - T_c(t)]
\]

\[
P_h(t) = \dot{m}_h(t) c_p [T_h(t) - T_{out}(t)]
\]

\[
P_f(t) = \alpha m^3(t)
\]

\[
I_D = \int_{t=0}^{24} \left[ \min \{|T(t) - \overline{T}(t)|, |T(t) - T(t)|\} \cdot 1_{B(t) c(T(t))} \right] dt
\]

\[
I_E = \int_{t=0}^{24} [P_c(t) + P_h(t) + P_f(t)] dt
\]
Simulation Results

- Comparison of **LTS** and **TLDS** uncertainty feedback parameterizations and Open Loop min-max results for the case of $\delta = 50\%$.

<table>
<thead>
<tr>
<th>Controller</th>
<th>Number of feedback decision variables</th>
<th>Average simulation time for $N = 24$ [s]</th>
<th>$I_e$ [kWh]</th>
<th>$I_d$ [$^\circ$Ch]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LTS</strong></td>
<td>$lmr\left(\frac{N(N+1)}{2}\right)$</td>
<td>200</td>
<td>16467</td>
<td>0</td>
</tr>
<tr>
<td><strong>TLDS</strong></td>
<td>$3lmr(N - 1)$</td>
<td>138</td>
<td>16467</td>
<td>0</td>
</tr>
<tr>
<td><strong>OL</strong></td>
<td>-</td>
<td>159</td>
<td>22592</td>
<td>0.84</td>
</tr>
</tbody>
</table>
Conclusion

- Presented a:
  - MPC strategy that is robust against additive uncertainty.
- Study the performance of two robust optimal control strategies, i.e.
  - Open-loop (OL-RMPC)
  - Closed-loop (CL-RMPC)
- Proposed (TLDS): a new uncertainty feedback parameterization for the CL-RMPC which results in:
  - Same energy and discomfort indices as LTS.
  - Fewer decision variables, (linear in N, as opposed to quadratic for LTS).
  - Average simulation time of 30% less than LTS.
Future work

- Co-design of Controller and embedded platform for HVAC systems
  - Sensing error modeling
    - Relation between the sensing errors and the number and locations of temperature and CO2 sensors using:
      1) computational fluid dynamics (CFD)
      2) real sensor readings from test-beds.
  - Co-design Formulation
    - Optimal design of controller having in mind the computation and communication limitations of the embedded platform.

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Thank you for your attention...


Questions?

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