Model-based Optimal Control Design for HVAC Systems of Buildings

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Motivation

Buildings Consume Significant Energy

- Total US annual energy cost $370 Billion
- Increase in US electricity cons. since 1990: 200%
- Total US energy consumption for buildings: 40%
- Total US electricity consumption for buildings: 72%
- Total US natural gas consumption for buildings: 55%

Source: Buildings Energy Data Book 2007

Related to HVAC
Motivation

- Statistically, $\frac{1}{3}$ of buildings are **constantly unoccupied**, but fresh air supplies are provided almost permanently to most buildings.
- ... and air conditioning systems do not take this into account...
Current HVAC control systems

Lack of coordination at a system level
Observations

- Local actions are determined without taking into account the interrelations among:
  - Outdoor weather conditions
  - Indoor air quality
  - Cooling demands
  - HVAC process components
Our Approach

- Derive a model for the building...
  - Simple
  - Descriptive enough

- Design optimal model-based control
  - Provide the desired level of comfort
  - Using minimum possible amount of energy
Outline

- Modeling
- Parameter identification
- Model validation
- Controller design
- Simulation Results
- Conclusion
- Future work
Modeling

- Lumped capacitance model...

- Thermal properties of building elements:
  - Heat **storage** in capacitors (nodes):
    \[ C = mc_p \]
  - Heat **transmission** through resistors (links):
    \[ R'_{\text{cond}} = \frac{T_{s,1} - T_{s,2}}{Q} = \frac{L}{kA} \]
    \[ R'_{\text{conv}} = \frac{T_s - T_\infty}{Q} = \frac{1}{hA} \]
Modeling

- Energy balance for a **wall** node:

\[
\frac{dT_{w_i}}{dt} = \frac{1}{C_{w_i}} \left[ \sum_{j \in N_{w_i}} \frac{T_j - T_{w_i}}{R'_{ij}} + r_i \alpha_i A_i q''_{rad_i} \right]
\]

\[r_i = \begin{cases} 
0 & \text{internal wall} \\
1 & \text{peripheral wall}
\end{cases}\]

- Energy balance for a **room** node:

\[
\frac{dT_{r_i}}{dt} = \frac{1}{C_{r_i}} \left[ \sum_{j \in N_{r_i}} \frac{T_j - T_{r_i}}{R'_{ij}} + \dot{m}_{r_i} c_p (T_{s_i} - T_{r_i}) + w_i \tau_{win_i} A_{win_i} q''_{rad_i} + \dot{q}_{int} \right]
\]

\[w_i = \begin{cases} 
0 & \text{wall } i \text{ doesn’t have window} \\
1 & \text{wall } i \text{ has window}
\end{cases}\]
Building model

\[ \dot{x}(t) = Ax(t) + Bu(t) + d(t) \]

\[ y(t) = Cx(t) \]
Building model

- Where the state vector \( x(k) \in \mathbb{R}^n \) is:

\[
x(k) = [T_{w_1}(k) \cdots T_{w_p}(k) T_{r_1}(k) \cdots T_{r_m}(k)]'
\]

- Input vector \( u(k) \in \mathbb{R}^m \) is:

\[
u(k) = [\dot{m}_1(k) \cdots \dot{m}_m(k)]'
\]

- \( p \): number of walls
- \( m \): number of rooms
- \( n \): number of states
- \( p+m=n \)
Parameter Identification

For each room:

\[ T(t) = f(C_r, C_{w1}, C_{w2}, C_{w3}, C_{w4}, R_1, R_2, R_3, R_4) \]

\[ [C_r, C_{w1}, C_{w2}, C_{w3}, C_{w4}, R_1, R_2, R_3, R_4]^* = \arg\min_{C_r, C_{w1}, R_i} \sum_t [e(t)]^2 \]
Parameter Identification

- **Initial guess** (ASHRAE Handbook)
- Used `fmincon`

- **Training Data:** Conference room in Bancroft library
- UC Berkeley campus

- “Weekend” data: to minimize the disturbance effect of occupants, etc.

Simulated and measured temperature
Model Validation

- Temperature data of the same room
- The following weekend
- Same parameters
Controller design

We are interested in solving:

\[
\min_{U_0} \frac{1}{2} \left[ y_d(N) - y(N) \right]^T S \left[ y_d(N) - y(N) \right] \\
+ \frac{1}{2} \sum_{k=0}^{N-1} \left( [y_d(k) - y(k)]^T Q [y_d(k) - y(k)] + u(k)^T R u(k) \right)
\]

Subject to system dynamics:

\[
x(k + 1) = Ax(k) + Bu(k) \\
y(k) =Cx(k) \\
x(0) = x_0
\]
Controller design

Use dynamic programming to find the optimal control at each time step:

\[ u^o(k) = F(k)b(k + 1) - K(k)x(k) \]

\[ K(k) = \left[ R + B^TP(k + 1)B \right]^{-1}B^TP(k + 1)A \]

\[ F(k) = -\left[ R + B^TP(k + 1)B \right]^{-1}B^T \]

Where \( P(k) \) and \( b(k) \) are calculated backwards in time, which yields:

\[ P(k - 1) = C^TQC + A^TP(k)A - A^TP(k)B\left[ R + B^TP(k)B \right]^{-1}B^TP(k)A \]

\[ b(k - 1) = A^Tb(k) - C^TQy_d(k - 1) - A^TP(k)B\left[ R + B^TP(k)B \right]^{-1}B^Tb(k) \]

With final values:

\[ P(N) = C^TSC \quad \quad b(N) = -C^T Sy^d(N) \]
Hierarchical scheme

- **Higher Level Controller (LQR)**
  - Minimizing cost function
  - Determines set-point for PIDs
  - Coordination between rooms in LQR

- **Lower Level Controller (PID)**
  - Sensing and actuation
  - Tracking the set point
  - No coordination between rooms

\[
J = x^T(N)Sx(N) + \sum_{k=0}^{N-1} \left\{ x^T(k)Qx(k) + u^T(k)Ru(k) \right\}
\]
Simulation:

- Desired output trajectory $y_d(k)$:
Simulation results

- Case I: \( R = \text{eye}(3) \) \( Q = \text{eye}(3) \)
Simulation results

- Case II: \[ R=1e^{-2} \times \text{eye}(3) \quad Q=1e3 \times \text{eye}(3) \]
Conclusions

- **Advantages:**
  - Shown to consume less energy...
  - $P(k)$, $b(k)$ and $K(k)$ matrices can be calculated offline and implemented online.
  - Reasonable amount of computation
  - Pretty good performance (tracking)
  - Easy tuning by changing diagonal entries of $Q$ and $R$

- **Disadvantage:**
  - Cost function not exactly representing energy consumption
Future work

- Developing a model to estimate/predict the **disturbance** using:
  - Outside weather condition forecast
  - Occupancy prediction
  - Internal heat gains

- Considering the model of **chillers, cooling towers, fans, reheat system**, etc.

- Other cost functions more descriptive of the real energy consumption.
Thank You!