Base Station Association Game in Multi-cell Wireless Networks

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***Abstract*—We consider a multi-cell wireless network with a large number of users. Each user selfishly chooses the Base Station (BS) that gives it the best throughput (utility), and each BS allocates its resource by some simple scheduling policy. First we consider two cases: (1) BS allocates the same time to its users; (2) BS allocates the same throughput to its users. It turns out that, combined with users' selfish behavior, case (1) results in a single Nash Equilibrium (NE), which achieves system-wide Proportional Fairness. On the other hand, case (2) results in many possible Nash Equilibria, some of which are very inefficient. Next, we extend (1) to the case where the users have general concave utility functions. It is shown that if each BS performs intra-cell optimization, the total utility of all users is maximized at NE. This suggests that under our model, the task of joining the "correct" BS can be left to individual users, leading to a distributed solution.**

I. INTRODUCTION

It is common to have multiple Base Stations (or Access Points) in wireless networks (e.g., IEEE 802.16 [1], IEEE 802.11, CDMA cellular networks). For a population of users with heterogeneous physical locations, signal strengths and transmission data rates, a suboptimal user-station association may lead to poor system performance. At the same time, it is complicated for the Base Stations to centrally control the associations of a large number of users. In this paper, we study whether distributed selfish choices of base station by users themselves can yield good or optimal system performance.

We consider a network with multiple base stations (BS's) and a large number of users. Different users have different PHY data rates (related to the signal strengths) to different BS's. Assume each BS independently allocates its resource to its users by some simple scheduling policies, such as allocating the same time or the same throughput to each user. It can also perform an intra-cell optimization of the total utility of all its users. We assume no explicit communications among the BS's, and the users are allowed to choose the BS's freely. This is different from [9] where an "association control" protocol is developed.

It is not surprising that the users should not simply join the base station with the best PHY data rate; they should also consider the current load in different BS's. This is the spirit of the protocol designed in [6]. There, before association, the user estimates the attainable throughput if he would join each available BS, and picks the BS that would give the best throughput. Simulation in [6] shows good throughput performance of the protocol in some cases. Reference [7] made similar observations. However, the underlying game-theoretic principle is not well understood in these references.

[5] uses a maximum-utility based formulation for optimal association in IEEE 802.11 wireless LANs. The paper assumes that the utility is the logarithm of the throughput, and provides the centralized optimal association results for some simple cases: 1) All users can associate with all APs at the same rate. 2) All users can associate with each AP at the same rate, but with different APs at different rates. Compared to [5], our model here is much more general without making the limiting assumptions. Also, we allow distributed association and different utility functions for all users.

Usually, a user can associate only with one base station at a time. This makes the optimal BS-association a hard combinatorial problem [5][9]. To avoid this, we use a continuous population model [2][8], which assumes that the number of users is large and can be modeled as a continuous variable. Another result of the assumption is that a single user is relatively "small". That is, the association decision of a single user does not affect the throughput of other users by much, which we think is a reasonable assumption in practice. This is a major difference from the models in [5] and [9].

II. BASE STATION ASSOCIATION GAME

Consider a multi-cell wireless network with J base stations (BS's). Assume there is a large number of users, which makes a single user relatively small, then the number of users can be modeled as a continuous variable [2]. Let R_j be the physical (PHY) data rate between a user and BS j. R_j depends on the signal strength from BS j to this user and on the interference from other BS's. Usually, R_j is chosen from a discrete, finite set of possible data rates, depending on the modulation and coding scheme used (e.g., in the IEEE 802.16 standard [1]). If the signal strength from a BS j' is so low that communication is impossible, then denote $R_{j'} = 0$. For a given user, $R_j, j = 1, 2, \ldots, J$ (the PHY rates from different BS's) forms a rate vector. If some users are geographically close, then they are likely to share the same rate vector. Similar to [8], we define a "class" as a group of users with the same rate vector. Assume

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that there are K such classes, and the rate vector for class k are denoted as $R_{kj}, j = 1, 2, \ldots, J$. (Even if the data rates are continuous, most conclusions in the paper should still apply. But we assume discrete data rates to simplify the analysis.)

Some other variables to be used in this paper include

- x_{kj}: Number of class-k users associated with BS j
- d_k: Total number of class-k users. $d_k = \sum_j x_{kj}$
- S_{kj}: Throughput of a class-k user associated with BS j

We assume that the (average) PHY data rates, R_{kj}, is constant for the purpose of the analysis. This is suitable for modeling the downlink traffic in multi-cell 802.16 (WiMAX) or CDMA networks. For the downlink traffic, the interference suffered by the users comes from other BS's. Assume the BS's keep transmitting data with constant powers, then the average interference level perceived by a user is fixed, no matter how other users choose their BS's. This in turn, gives a fixed PHY data rate. (An analysis of uplink traffic is more complex and is left for future research. We will explain the reason later.)

Note that the overhead of specific protocols (such as IEEE 802.16) can be included in this formulation by using an "effective" data rate $\bar{R}_{kj} = (1-\rho)R_{kj}$ instead of R_{kj}, where ρ is the fraction of bandwidth consumed by the protocol overhead.

As mentioned before, assume users are selfish, and free to choose the BS that give it the best throughput. The throughput a user can receive, on the other hand, is assumed to be controlled by the BS (e.g., in an 802.16 network). Depending on the scheduling policy used by the BS's, the game may have different outcomes. In the following two subsections, we first study two specific scheduling policies used by the BS's: equal-time allocation and equal-throughput allocation.

The handshake needed to enable selfish association are easy to implement. For example, before association, the user sends a Request packet to all available BS's, possibly reporting its application type (i.e., utility function). Each BS computes and reports the would-be throughput of this user, derived from the user's PHY data rate to the BS, and other users currently in the cell. Then, the user can join the BS that would give it the best throughput (utility). If this protocol is not enabled in some users or BS's, then a user can simply choose its BS by "trial and error".

A. Equal-time allocation by the BS

Assume that each BS j assign equal time to each user that is associated to it. Then, the fraction of time in BS j for each user is $1/\sum_k x_{kj}$. Thus the throughput of a class-k user in BS j is

$$S_{kj} = \frac{R_{kj}}{\sum_k x_{kj}} \tag{1}$$

which is the PHY rate R_{kj} multiplied by the fraction of time.

Since the number of users is modeled as continuous, S_{kj} is also continuous. Then, at Nash Equilibrium (NE), for each class k, there exists a positive constant c_k, such that

$$\begin{cases} S_{kj} = c_k & \forall x_{kj} > 0 \\ S_{kj} \le c_k & \forall x_{kj} = 0 \end{cases} \tag{2}$$

That is, any BS used by class-k gives equal throughputs to a class-k user, and any BS not used by class-k would give a lower throughput to a class-k user. As a result, no user has the incentive to change its choice of BS unilaterally from this equilibrium, because he could not receive a higher throughput by doing so (definition of "Nash Equilibrium"). This NE is also known as "Wardrop Equilibrium" [11].

Proposition 1: There is a unique NE, in the sense that each user will receive a unique throughput at NE (but the throughputs may be different for different users). Also, this NE achieves system-wide proportional fairness [3].

Interestingly, this is achieved by the BS's simple scheduling policy (independent of physical data rates R_{kj}'s) and the users' selfish behavior. No explicit communication is needed between the BS's.

Note that although the individual throughput is unique at NE, the associations of individual users may not be unique. For example, assume that in a NE, user A in class 1 is in BS 1, and user B in class 1 is in BS 2. The two users receive the same throughput. Then, exchanging their associations also results in a NE.

Proof: At NE, (2) is satisfied. Also, each user in a BS j gets the same amount of time.

On the other hand, consider proportional fairness. If the individual throughputs solve the following utility maximization problem, then the system-wide proportional fairness is achieved.

$$\begin{aligned} \max_{\mathbf{z},\mathbf{x}} \quad & U = \sum_{k,j} x_{kj} \log\left(\frac{z_{kj} R_{kj}}{x_{kj}}\right) \\ \text{st} \quad & \sum_k z_{kj} = 1, \forall j; \sum_j x_{kj} = d_k, \forall k \end{aligned} \tag{3}$$

Here, z_{kj} is the fraction of time of BS j allocated to class k users. Then $\frac{z_{kj}R_{kj}}{x_{kj}} = S_{kj}$ is the throughput of a class-k user connected to BS j (throughput should be equal for all class-k users in BS j since they share the same PHY data rate and have the same concave utility function). Also, $\log(\cdot)$ is the utility function used for proportional fairness. So the objective function above, denoted by U, is the total utility of all the users. It is easy to see that U is a concave function of $\mathbf{z}, \mathbf{x}$.

The KKT condition [10] for Problem (3) is

$$\begin{cases} \frac{\partial U}{\partial x_{kj}} = \lambda_k & \forall x_{kj} > 0 \\ \frac{\partial U}{\partial x_{kj}} \le \lambda_k & \forall x_{kj} = 0 \\ \frac{\partial U}{\partial z_{kj}} = \mu_j & \forall z_{kj} > 0 \\ \frac{\partial U}{\partial z_{kj}} \le \mu_j & \forall z_{kj} = 0 \end{cases} \tag{4}$$

Since $\frac{\partial U}{\partial x_{kj}} = \log(\frac{z_{kj}R_{kj}}{x_{kj}}) - 1 = \log(S_{kj}) - 1$, and $\frac{\partial U}{\partial z_{kj}} = \frac{x_{kj}}{z_{kj}}$, then from (4), we have

$$\begin{cases} S_{kj} = \exp(\lambda_k + 1) & \forall x_{kj} > 0 \\ S_{kj} \le \exp(\lambda_k + 1) & \forall x_{kj} = 0 \\ \frac{z_{kj}}{x_{kj}} = \mu_j^{-1} & \forall z_{kj} > 0 \end{cases}$$

Here, the first two inequalities are the same as (2); while the last equation means that the times allocated to the users in BS j are equal. (Note that when $z_{kj} = 0$, then $x_{kj} = 0$. This is the case when no class-k users are in BS j.) Therefore, the NE solves the utility maximization problem (3) and thus achieves proportional fairness. Also, since (3) is strictly convex and has a unique optimal solution $(\mathbf{z}, \mathbf{x})$, the throughput of each user at NE is also unique. ■

B. Equal-throughput allocation by the BS

Now assume that each BS ensures that all associated users receive the same throughput (i.e., it schedules the same amount of traffic, in bits, to all its users). Thus the BS may need to allocate different times to different users, depending on the PHY data rates. Let S_j be the (equal) throughput of each user associated with BS j. Then the fraction of time in BS j used by a class k user is $\frac{S_j}{R_{kj}}$. Since all the time fractions sum up to 1, we have $\sum_k \frac{S_j}{R_{kj}} x_{kj} = 1, \forall j$. Therefore,

$$S_j = (\sum_k \frac{x_{kj}}{R_{kj}})^{-1}, \forall j. \tag{5}$$

The conditions for a NE are

$$\begin{cases} S_{kj_1} = S_{kj_2} & \forall x_{kj_1} > 0, x_{kj_2} > 0 \\ S_{kj_1} \geq S_{kj_2} & \forall x_{kj_1} > 0, x_{kj_2} = 0 \\ S_{kj} = S_j & \forall k, j \end{cases} \tag{6}$$

where the third condition reflects the equal-throughput allocation by each BS.

Proposition 2: If $R_{kj} > 0, \forall k, j$, then

$$S_{j_1} = S_{j_2}, \text{ for all non-empty BS's } j_1, j_2. \tag{7}$$

As a result, the individual throughputs of all users (of all classes) are the same.

Proof: Suppose there exist two non-empty BS's j_1, j_2 such that $S_{j_1} > S_{j_2}$. Since BS j_2 is not empty, then $x_{kj_2} > 0$ for some class k. If $x_{kj_1} > 0$, then according to (6), $S_{j_1} = S_{j_2}$; if $x_{kj_1} = 0$, then $S_{j_1} \leq S_{j_2}$. Both cases contradict the assumption $S_{j_1} > S_{j_2}$. Similarly, $S_{j_1} < S_{j_2}$ cannot be true. ■

Proposition 3: There can be infinite number of NE's in this game. The NE's may not be efficient.

Proof: For simplicity, assume there are only 2 BS's and 2 classes. Then, for any $\{x_{kj}\}, k = 1, 2, j = 1, 2$ such that $S_1 = S_2$, a NE is reached: if a user switches from BS 1 to BS 2, he increases S_1 but decreases S_2 (from equation (5)), which decreases its own throughput and vice versa.

To make $S_1 = S_2$, we should satisfy $\frac{x_{11}}{R_{11}} + \frac{x_{21}}{R_{21}} = \frac{x_{12}}{R_{12}} + \frac{x_{22}}{R_{22}}$, as well as $x_{11} + x_{12} = d_1, x_{21} + x_{22} = d_2$. These are 3 equations but 4 variables, so there can be infinite number of solutions, each of which constitutes a NE. (In practice, there are a finite number of users. So the number of NE's is large, but not infinite.)

Some NE's may be very inefficient. For example, let $R_{11} = R_{22} = 10, R_{12} = R_{21} = 1, d_1 = d_2$. Then, if all class-1 users connect to BS 2, and all class-2 users connect to BS 1, a NE is reached. Clearly, this is inefficient. It only achieves 1/10 of the throughput in the NE when all users connect to the other BS.

Generally, suppose there are K classes and J BS's, and $R_{kj} > 0, \forall k, j$. At each NE, all S_j's in non-empty BS's should be the same. Otherwise, some user has an incentive to switch from one BS to another, regardless of its PHY data rates. Like the 2-BS case discussed above, there are infinitely many NE's. Depending on system dynamics, it may settle on different NE's. The resulting NE, however, may be very inefficient. ■

III. BS-ASSOCIATION GAME WITH GENERAL CONCAVE UTILITY FUNCTIONS

A. General concave utility functions

In section II-A, we notice an interesting property: if each BS allocates its resource to achieve proportional fairness (PF) within the cell (i.e., allocates the same time to each user), and if we allow each user to choose its best BS, then a system-wide PF is achieved. In other words, the BS's do not need to communicate with each other or control the association of the users; BS's intra-cell optimization and the users' selfish behavior automatically achieves a form of system optimality.

In this section, we explore whether this generally holds if each user has its own increasing, strictly-concave utility function, which depends on his specific application and preference. It turns out to be true: BS's intra-cell optimization and the users' selfish choice lead to social optimum.

Lemma 1: Given any z_{kj}'s (the fraction of time allocated by BS j to class k), where $\sum_k z_{kj} = 1, \forall j$, class-$k$ users' selfish choice of BS will lead to the optimal total utility $V_k(z_{k1}, z_{k2}, \ldots, z_{kJ})$ within class k.

Proof: Without loss of generality, we consider a particular class k. Since each BS performs intra-cell optimization, BS j solves

$$\begin{aligned} \max_{\mathbf{t}} \quad & \sum_{i \in j} u_i(R_{kj} \cdot t_i) \\ \text{st} \quad & \sum_{i \in j} t_i = z_{kj} \end{aligned} \tag{8}$$

where i is the index of class-k users, t_i is the fraction of time used by user i, and the summation is over all class-k users in BS j.

Let λ be the Lagrange multiplier (or "price") for the above problem. Then the Lagrangian is $L(\mathbf{t}, \lambda) = \sum_{i \in j} u_i(R_{kj} \cdot t_i) - \lambda(\sum_{i \in j} t_i - z_{kj})$. Let λ_{kj} be the value of λ when the optimal solution is reached. Then,

$$u_i'(R_{kj} \cdot t_i^*) = \lambda_{kj}/R_{kj} \tag{9}$$

where t_i^* is the time allocated to user i at the optimal solution. And the utility of user i is $u_i(R_{kj} \cdot t_i^*)$.

Define $P_i(\cdot)$ as the inverse function of $u_i'(\cdot)$. Since $u_i(\cdot)$ is strictly concave, $P_i(\cdot)$ is strictly decreasing. Then we have

$$P_i(\lambda_{kj}/R_{kj}) = R_{kj} t_i^* = S_i^* \tag{10}$$

Now, when a NE is reached, no user should have incentive to switch across different BS's with $z_{kj} > 0$. So, for all BS j

where $z_{kj} > 0$, S_i^* would be the same had user i join any of them. (Since we have assumed that a single user is small, its decision on BS-association will not affect the "prices" in the BS's.) So,

$$\lambda_{kj}/R_{kj} = \alpha_k, \forall j \text{ such that } z_{kj} > 0, \tag{11}$$

where α_k is some positive constant. It follows that $P_i(\alpha_k) = S_i^*$ for any user i in class k.

Note that given z_{kj}, the total throughput of class-k users is fixed. That is, $\sum_i S_i = C_k$, where $C_k \triangleq \sum_j R_{kj} z_{kj}$. Then the utility maximization problem (within class k) is

$$\begin{aligned} \max \quad & \sum_i u_i(S_i) \\ \text{st} \quad & \sum_i S_i = C_k \end{aligned} \tag{12}$$

where the summation "$\sum_i$" is over all class-k users.

The optimality condition is that there exists $\beta_k > 0$ such that $u_i'(S_i^{**}) = \beta_k$ and $\sum_i S_i^{**} = \sum_j R_{kj} z_{kj}$. Notice that letting $\beta_k = \alpha_k$ meets the condition, therefore $P_i(\alpha_k) = S_i^{**}$. We have known $P_i(\alpha_k) = S_i^*$. So, the NE maximizes the total utility with class k.

Note that the optimal solution of (12) may require a few "marginal" users to split their traffic into more than one cells. However, since we have assumed that individual users are "small", the total utility will not be affected much if these users concentrate their traffic on only one cell. ■

Recall that $V_k(z_{k1}, z_{k2}, \ldots, z_{kJ})$ is the optimal total utility of class k, as a function of $z_{kj}, j = 1, 2, \ldots, J$. According to convex optimization theory, V_k is concave in $z_{kj}, j = 1, 2, \ldots, J$ [10]. In problem (12), $\beta_k = \alpha_k$ gives the sensitivity of V_k under a perturbation of C_k. Since $C_k = \sum_j R_{kj} z_{kj}$, the sensitivity of V_k under a perturbation of z_{kj} is $\alpha_k \cdot R_{kj} = \lambda_{kj}$.

Theorem 1: The base stations' intra-cell optimization and the users' selfish choices of BS lead to the maximal sum of the utilities of all users. The resulting NE is unique, in the sense that each user gets a unique throughput (utility) at NE.

Proof: Consider the equilibrium when intra-cell utilities have been maximized and users have reached the NE of choosing BS's. At this point, let Z_{kj} be the fraction of time that BS j allocates to class k. According to Lemma 1, users' selfish choices of BS achieves the maximal total utility $V_k(Z_{k1}, Z_{k2}, \ldots, Z_{kJ})$ for class k. And the price for class-k users in BS j, λ_{kj}, gives the sensitivity of V_k under a perturbation of Z_{kj} if $Z_{kj} > 0$. That is,

$$\frac{\partial V_k(Z_{k1}, Z_{k2}, \ldots, Z_{kJ})}{\partial z_{kj}} = \lambda_{kj}, \text{ if } Z_{kj} > 0.$$

Now, since an intra-cell optimization for all users in BS j is performed, then the Lagrange multiplier, or price, is the same for all classes of users in BS j. So, $\lambda_{kj} = \gamma_j, \forall k : Z_{kj} > 0$, where γ_j is a positive constant. Therefore, in BS j,

$$\frac{\partial V_k(Z_{k1}, Z_{k2}, \ldots, Z_{kJ})}{\partial z_{kj}} = \gamma_j, \forall k \text{ such that } Z_{kj} > 0. \tag{13}$$

If there is no class-k user in BS j at the NE, then it must be the case that the price at BS j is too high to class k, i.e., $\gamma_j/R_{kj} \geq \alpha_k$. Since $\frac{\partial V_k(Z_{k1}, Z_{k2}, \ldots, Z_{kJ})}{\partial z_{kj}} = \alpha_k \cdot R_{kj}$, then

$$\frac{\partial V_k(Z_{k1}, Z_{k2}, \ldots, Z_{kJ})}{\partial z_{kj}} \leq \gamma_j, \forall k \text{ such that } Z_{kj} = 0. \tag{14}$$

Now, (13) and (14) are also the optimality conditions for the overall utility maximization problem (for all users)

$$\begin{aligned} \max_{\mathbf{z}} \quad & \sum_k V_k(z_{k1}, z_{k2}, \ldots, z_{kJ}) \\ \text{st} \quad & \sum_k z_{kj} = 1, \forall j. \end{aligned} \tag{15}$$

Therefore the optimal total utility is achieved. Since (15) is a strictly convex optimization problem, it has a unique solution $\mathbf{z}$. Therefore, the NE is unique, in the sense that each user gets a unique throughput (utility) at NE. ■

We have shown that the Nash Equilibrium maximizes the total utility. There is a remaining question whether the system will converge from an initial state to the equilibrium.

Proposition 4: If a user decides to switch from one BS j_1 to another BS j_2 because BS j_2 would give a higher throughput, then the total utility of all users will also improve if the user makes the switching.

As a result, as long as users' switchings of BS's are slow enough such that the BS's can adjust their allocations in time, the total utility will constantly increases with the switchings until convergence.

Proof: If user i, assumed to be in class k, would like to switch from BS j_1 to BS j_2, then $\lambda_{kj_1}/R_{kj_1} > \lambda_{kj_2}/R_{kj_2}$. Let t_{j_1} be the time used by i in BS j_1. If we keep the throughput of user i unchanged after the switching, then BS j_2 should allocate a time fraction $t_{j_2} = R_{kj_1} \cdot t_{j_1}/R_{kj_2}$ to the user. Since t_{j_1}, t_{j_2} are small, following a sensitivity analysis [10], the total utility in BS j_1, except user i, is increased by $\lambda_{kj_1} \cdot t_{j_1}$; while the total utility in BS j_2, except user i, is decreased by $\lambda_{kj_2} \cdot t_{j_2} = \lambda_{kj_2} \cdot R_{kj_1} \cdot t_{j_1}/R_{kj_2} < \lambda_{kj_1} \cdot t_{j_1}$. Therefore, the total utility in the two BS's increases.

Since we have kept the throughput of user i unchanged, the resulting intra-cell allocations may not be optimal. Now, let each BS performs an intra-cell optimization, which will further increase the total utility.

Since each switching by a user increases the total utility, the system will converge. Because the equilibrium solves a strictly convex optimization problem, the vector $\mathbf{z} = \{z_{kj} : \forall k, j\}$ is unique and the system will converge to it. ■

In the above proof, we have used the fact that each individual user is small. Otherwise, the dual variables $\lambda_{kj_1}, \lambda_{kj_2}$ may change significantly with the switching, making the sensitivity analysis inaccurate. In that case, optimality is hard to obtain since the problem becomes combinatorial (see [5] [9]).

The result has mathematical connections with [3], which shows how to use pricing for distributed congestion control. From Lemma 1, class k can be viewed as a "big user" with utility function $V_k(z_{k1}, z_{k2}, \ldots, z_{kJ})$, and the outcome of congestion control among these K big users coincides with the NE of the game.

We can further extend the result to *multi-channel multi-cell wireless networks*, by regarding each channel as a virtual BS, and allowing the users to freely choose their BS's and channels.

B. Relations with BS's equal-time and equal-throughput allocation

Since the concave utility functions can be arbitrary, Theorem 1 indicates that a more general notion of fairness, (p, α)-fairness [4], can be achieved if it is achieved within each cell. (p, α)-fairness is said to be reached iff the utility maximization problem is solved, with the utility function $u_k(s) = p_k f_\alpha(s)$, where p_k is the weight associated with class-k users, and $f_a(\cdot)$ is defined as

$$f_\alpha(s) = \begin{cases} \frac{s^{1-\alpha}}{1-\alpha} & \alpha \neq 1, \alpha > 0 \\ \log(s) & \alpha = 1 \end{cases} \tag{16}$$

When $\alpha = 1$, weighted proportional fairness (WPF) is achieved. Proportional fairness (PF), as considered in Subsection II-A, is a special case when all the weights are equal.

For (p, α)-fairness, the system has a unique NE (which is also social optimal). When $\alpha \to \infty$, (p, α)-fairness approaches (or approximates) max-min fairness [4]. Interesting, however, there can be many NE's which generally are not social optimal, as shown in section II-B, if the BS allocates the same throughput to its users (i.e., achieving intra-cell max-min fairness). This does not constitute a contradiction since (p, α)-fairness only *approximates* max-min fairness when $\alpha \to \infty$.

IV. SIMULATIONS

A. Equal-time allocation by the BS's

First assume that each BS allocates equal time to each user that connects to it. In our simulation, $K = 2$, $J = 2$, $d_1 = 20, d_2 = 30$, $R_{11} = 10, R_{12} = 20, R_{21} = 15, R_{22} = 15$.

Initially, each user is associated randomly with BS 1 or BS 2. Then, at each time slot, we randomly pick a user, and let the user choose the best BS. (The spacing of time slots may not be even in practice.) This process eventually converges to the Nash equilibrium. This NE is unique and achieves proportional fairness according to Proposition 1. Fig. 1 "Run 1" shows the evolution of the number of users (of both classes) in BS 1. (The remaining users are in BS 2.)

To verify there is indeed a unique NE (in the case, a unique vector $\mathbf{x}$), we let all users connect to BS 1 initially, and repeat the above process. Fig. 1 "Run 2" shows the evolution, which converges to the same point as "Run 1".

B. Equal-throughput allocation by the BS's

We use the same parameters as before, except that $R_{11} = 10, R_{12} = 1, R_{21} = 1, R_{22} = 10$.

Fig. 2 shows the evolution of the throughput (for each user) at cell 1 and cell 2, starting with different initial associations. (Recall that the throughput is always kept equal within a cell.) In "Run 1", the initial association is random. In "Run 2", initially all class-1 users are in cell 1 and all class-2 users are in cell 2 (a "good" association). In "Run 3", initially all class-1 users are in cell 2 and all class-2 users are in cell 1 (a "bad" association). As expected, the system converges to different NE's. The initial BS-association seems to have a significant effect on the efficiency of the resulting NE. The NE in "Run 2" is more than 7 times better than the NE in "Run 3". (Remark: even with a given initial association, the final NE may not be unique since the users can move in different orders.) In "Run 2", the throughputs are not exactly the same, since we have assumed a continuous population model, which is not exact when the number of users is not very large.

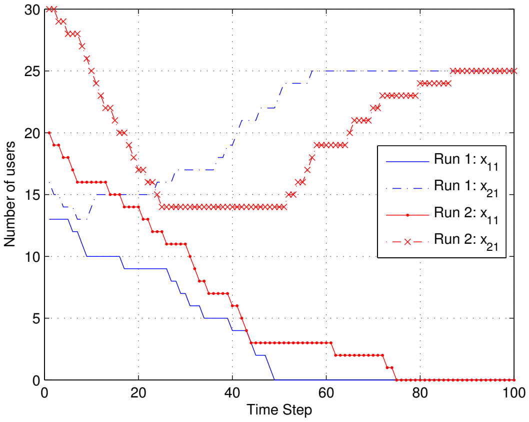


Fig. 1. Convergence to NE, with BS's equal-time allocation. Note that $x_{12} = d_1 - x_{11}$, $x_{22} = d_2 - x_{21}$ are not shown here. "Run 1": Random initial association; "Run 2": Initially all users are associated with BS 1.

For this example, we can compute the maximal and minimal possible throughput $S_1 = S_2 := 1/C$, by minimizing or maximizing $C = \frac{x_{11}}{R_{11}} + \frac{x_{21}}{R_{21}} = \frac{x_{12}}{R_{12}} + \frac{x_{22}}{R_{22}}$ using Linear Programming. The result is $S_{max} = 0.3437, S_{min} = 0.0478$, close to the throughputs shown in Fig. 2 "Run 2" and "Run 3".

C. Intra-cell optimization by BS's, with general concave utility functions

We use the same parameters as in section IV-A, but allowing arbitrary concave utility functions. For convenience, assume there are two kinds of utility functions, possibly depending on application types: $u_A(s) = \log(s)$, and $u_B(s) = \sqrt{s}$, where s is the user's throughput. Each of the 50 users adopt one of the functions. Assume that 11 users in class 1 and 12 users in class 1 have the utility function $u_A(\cdot)$, and other users have the utility function $u_B(\cdot)$.

Initially, each user is associated randomly with BS 1 or BS 2. Then the users can switch to their favorite BS selfishly, as in subsection IV-A. In Fig. 3(a), the two curves marked with "Run 1" shows the evolution of the variables z_{11}, z_{12} with time. They are the fraction of time allocated to class 1 in both BS's. (z_{21}, z_{22} are not shown, since $z_{21} = 1 - z_{11}$, $z_{22} = 1 - z_{12}$.) In Fig. 3(b), the curve marked with "Run

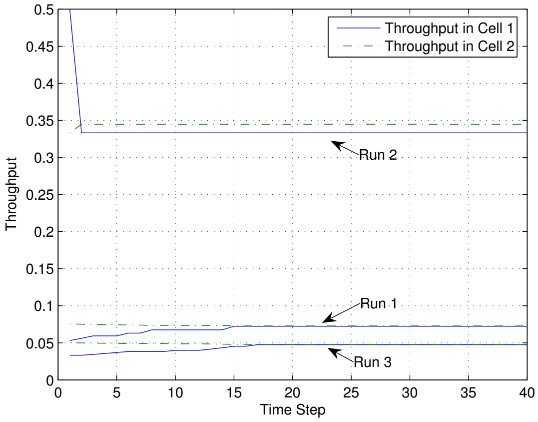


Fig. 2. Evolution of the throughput (of each user), with BS's equal-throughput allocation. "Run 1": Random initial association; "Run 2": Good initial association; "Run 3": bad initial association.

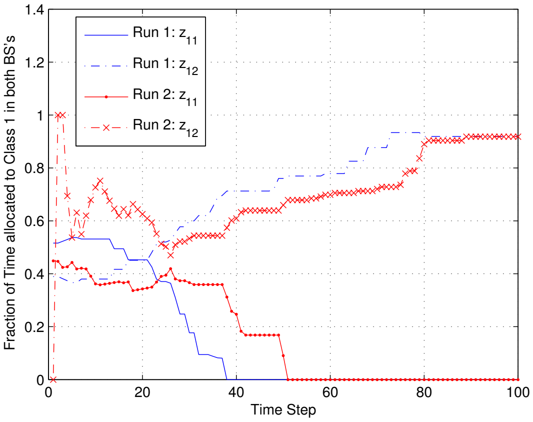
1" shows the evolution of the total utility of all users. As expected, it increases monotonically.

To verify there is indeed a unique NE (in this case, unique $\mathbf{z}$ leads to unique utility for each user), we re-run the simulation starting with the initial state when all users connect to BS 1. The curves marked with "Run 2" in Fig. 3 (a), (b) shows that the system converges to the same point as "Run 1".

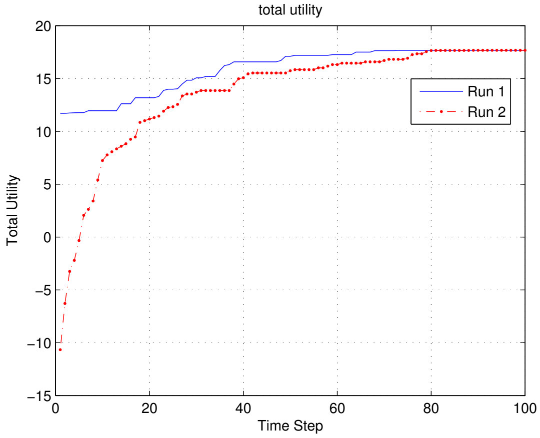
V. CONCLUSION

We have studied the Base-Station-association game, in which users are allowed to selfishly choose their BS's. Depending on how the BS's allocate their resource, the game has different outcomes. We have shown that if each BS allocates the same throughput to its users, then the result may be inefficient, and all users may receive low throughput (utility). Also, the Nash Equilibria are not unique and it's uncertain which NE the system converges to. However, if each BS allocates its resource to maximizes the total utility of the users within the cell (i.e., intra-cell optimization), then the resulting NE achieves optimal total utility of all users. The NE is unique and the system converges to it from any initial state, given that the users' switchings of BS's are not too fast. A special case here which admits particularly simple implementation is "proportional fairness", where the utility function of each user is assumed to be $\log(\cdot)$. In this case, if each BS allocates the same time to all its users, then the system-wide proportional fairness is achieved.

In this paper we have assumed that the PHY data rates are fixed from the BS's to the users. This is suitable for modeling the downlink traffic in multi-cell WiMAX or CDMA networks, as explained before. But for the uplink traffic, the interference is caused by the transmissions of other users. Therefore the interference level perceived by a user depends on the activity of other users, which in turn depends on which BS's other users choose and how much resource is allocated to them. So, a game-theoretic analysis for uplink traffic is more complex and needs further study.



(a) Fraction of time allocated to class 1 in 2 BS's



(b) Total utility

Fig. 3. Evolution of the resource allocation vector $\mathbf{z}$ and total utility. In "Run 1", the initial association is random; in "Run 2", all users are in cell 1 initially.

Another direction of future research is to accommodate non-concave utility functions, such as those of real-time voice or video connections. Usually, this leads to difficulty to achieve overall optimal solutions [13].

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