FERMI ACCELERATION —
FROM COSMIC RAYS TO DISCHARGE HEATING

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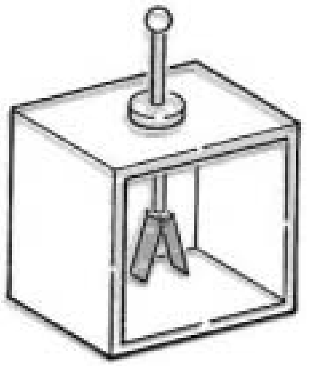
OUTLINE

- Cosmic rays and Fermi acceleration
- Capacitive discharge heating
- Inductive discharge heating

Enrico Fermi (1901–1954)
Nobel Prize 1938

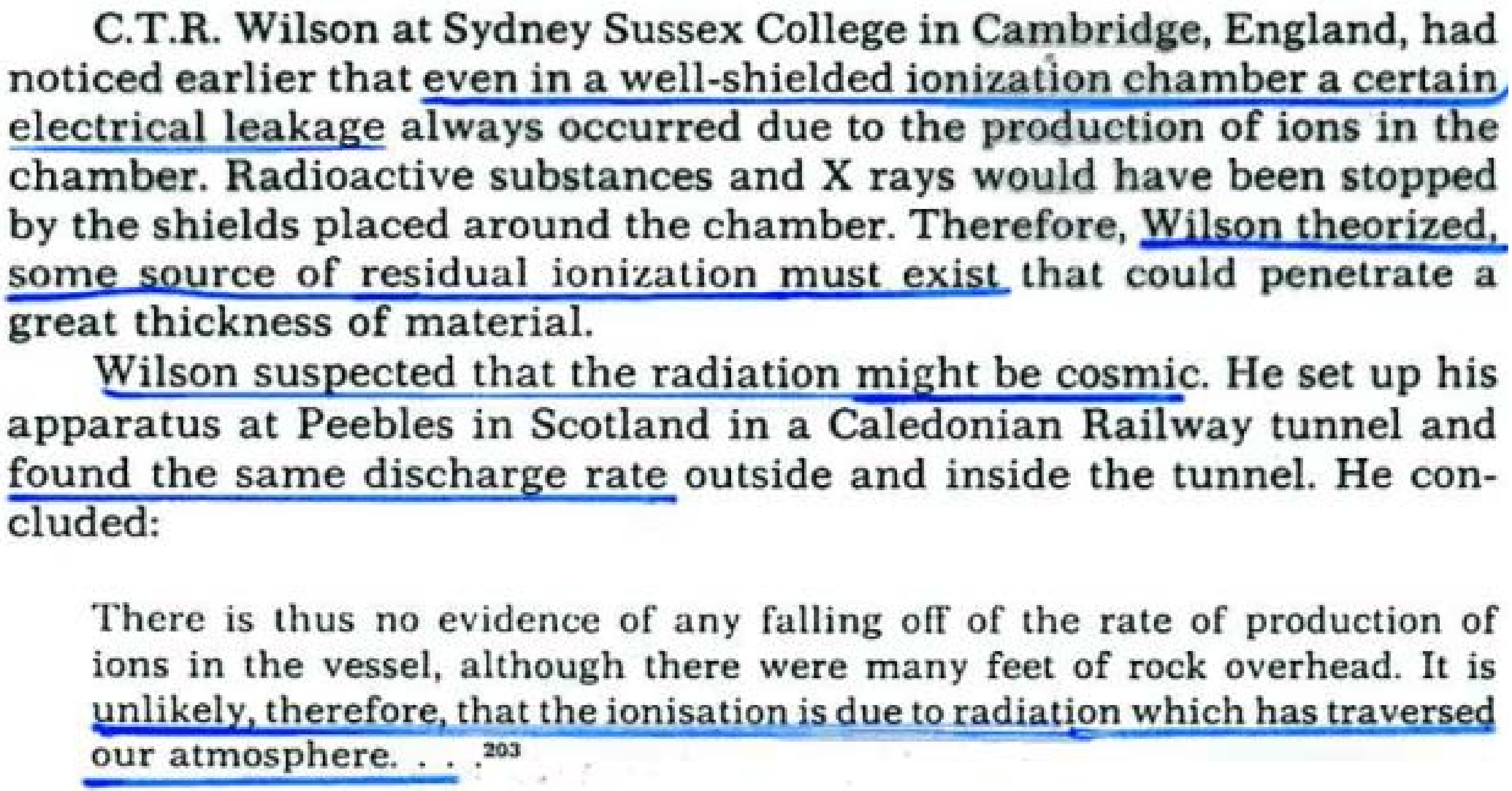
COSMIC RAYS

DISCOVERY MISSED



1895: W.C. Roentgen, x-rays (1927)

1902: C.T.R. Wilson, cloud chamber (1927)



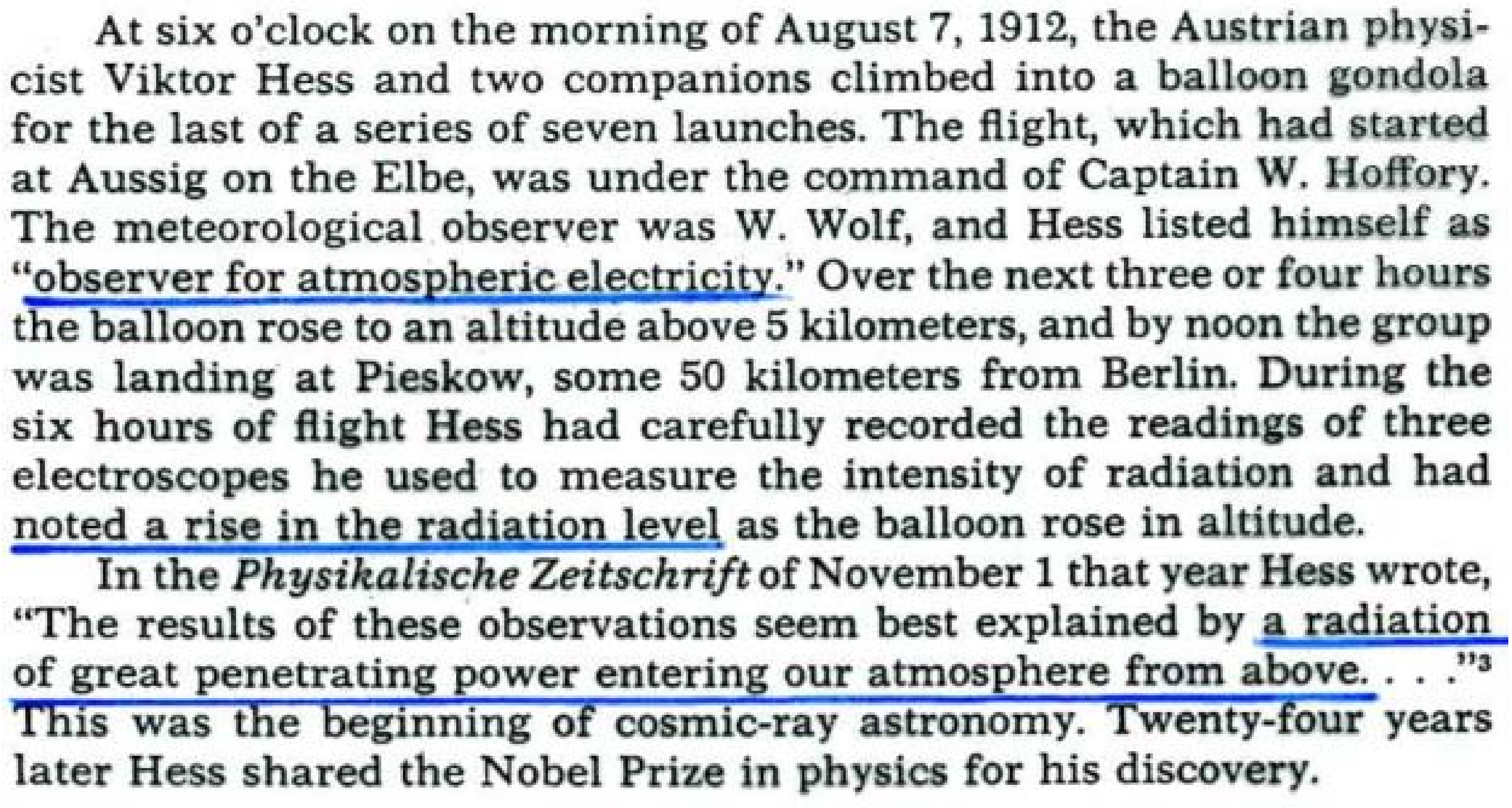
C.T.R. Wilson at Sydney Sussex College in Cambridge, England, had noticed earlier that even in a well-shielded ionization chamber a certain electrical leakage always occurred due to the production of ions in the chamber. Radioactive substances and X rays would have been stopped by the shields placed around the chamber. Therefore, Wilson theorized, some source of residual ionization must exist that could penetrate a great thickness of material.

Wilson suspected that the radiation might be cosmic. He set up his apparatus at Peebles in Scotland in a Caledonian Railway tunnel and found the same discharge rate outside and inside the tunnel. He concluded:

> There is thus no evidence of any falling off of the rate of production of ions in the vessel, although there were many feet of rock overhead. It is unlikely, therefore, that the ionisation is due to radiation which has traversed our atmosphere. . . .[203]

DISCOVERY

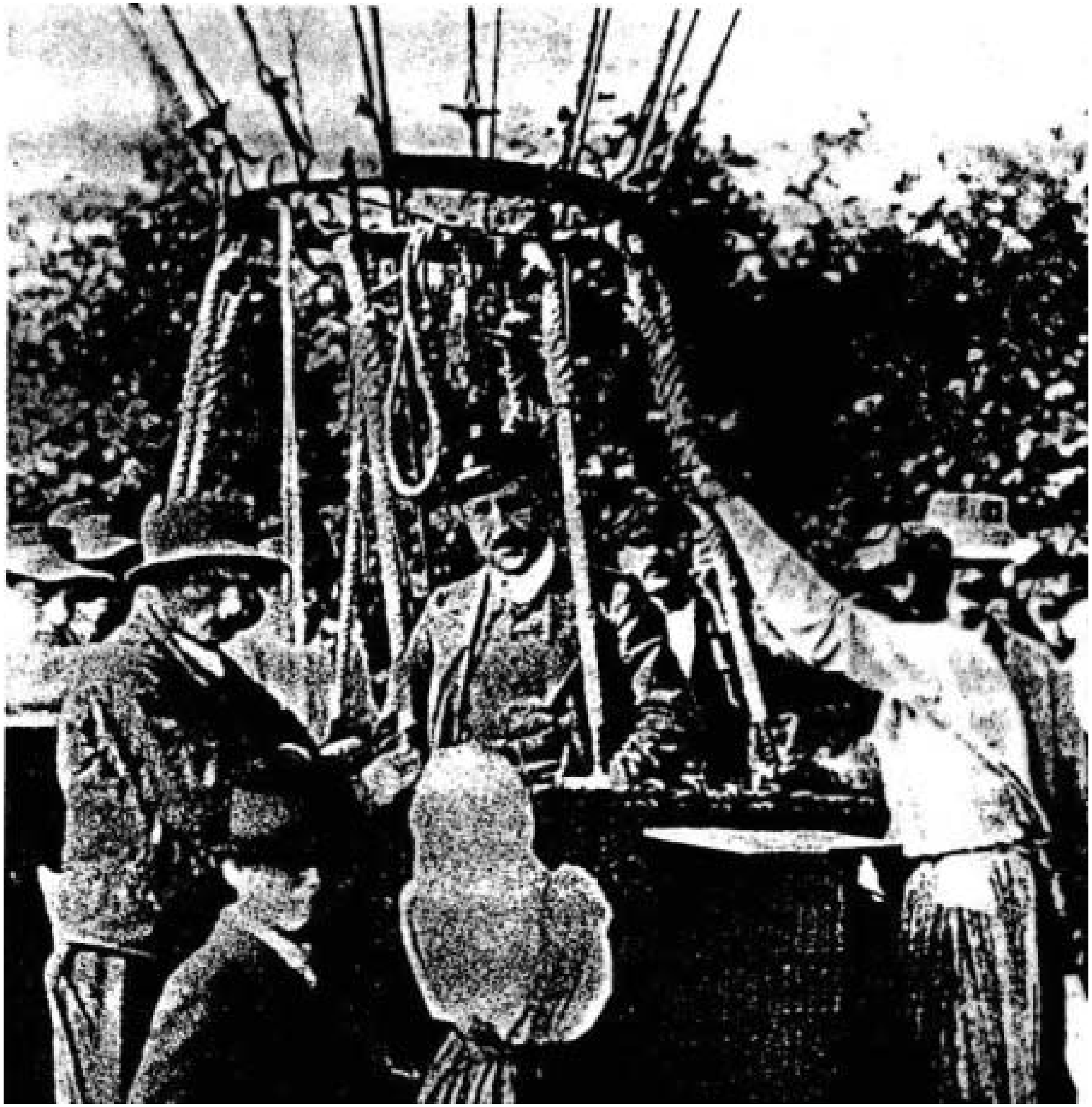
1912: V. Hess (1936)



At six o'clock on the morning of August 7, 1912, the Austrian physicist Viktor Hess and two companions climbed into a balloon gondola for the last of a series of seven launches. The flight, which had started at Aussig on the Elbe, was under the command of Captain W. Hoffory. The meteorological observer was W. Wolf, and Hess listed himself as "observer for atmospheric electricity." Over the next three or four hours the balloon rose to an altitude above 5 kilometers, and by noon the group was landing at Pieskow, some 50 kilometers from Berlin. During the six hours of flight Hess had carefully recorded the readings of three electroscopes he used to measure the intensity of radiation and had noted a rise in the radiation level as the balloon rose in altitude.

In the *Physikalische Zeitschrift* of November 1 that year Hess wrote, "The results of these observations seem best explained by a radiation of great penetrating power entering our atmosphere from above. . . ."[3] This was the beginning of cosmic-ray astronomy. Twenty-four years later Hess shared the Nobel Prize in physics for his discovery.

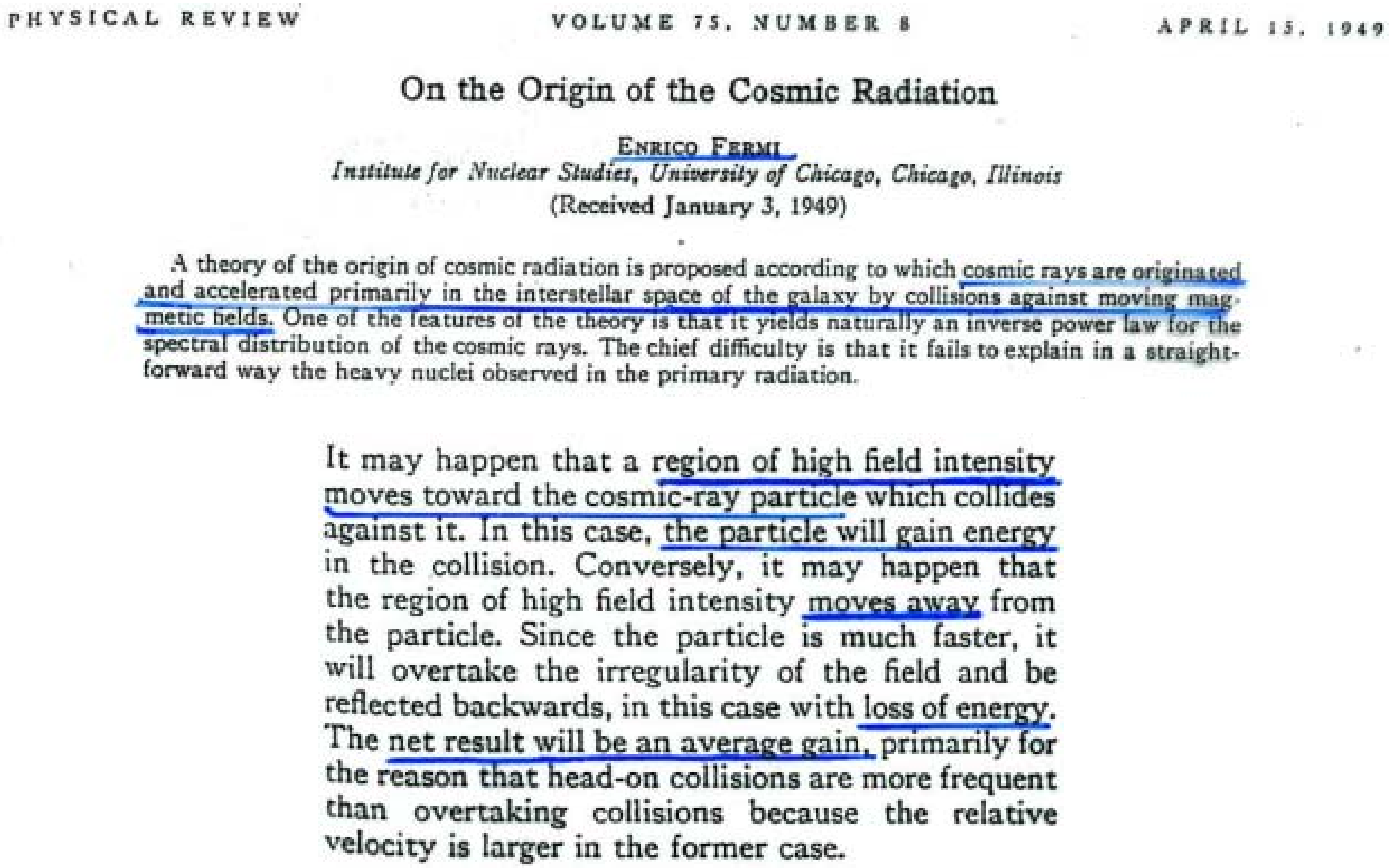
POSING IN HIS BALLOON



Victor Hess, discoverer of cosmic rays, after his 1912 balloon flight reached an altitude of 17,500 feet

FERMI ACCELERATION

- Enrico Fermi proposed an extra-solar origin of cosmic rays in 1949



PHYSICAL REVIEW VOLUME 75, NUMBER 8 APRIL 15, 1949

On the Origin of the Cosmic Radiation

ENRICO FERMI
Institute for Nuclear Studies, University of Chicago, Chicago, Illinois

(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magnetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

It may happen that a region of high field intensity moves toward the cosmic-ray particle which collides against it. In this case, the particle will gain energy in the collision. Conversely, it may happen that the region of high field intensity moves away from the particle. Since the particle is much faster, it will overtake the irregularity of the field and be reflected backwards, in this case with loss of energy. The net result will be an average gain, primarily for the reason that head-on collisions are more frequent than overtaking collisions because the relative velocity is larger in the former case.

FERMI MAPS

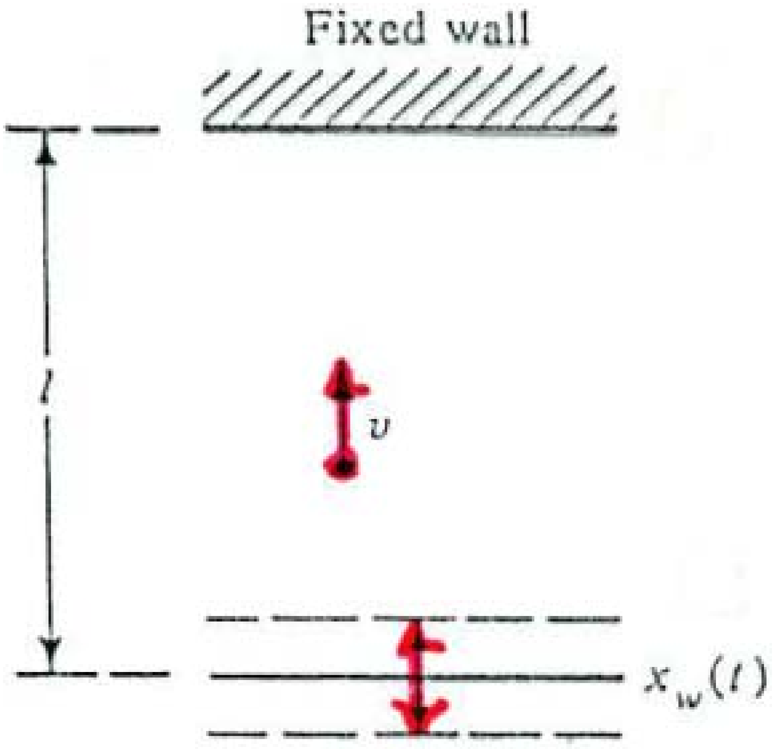
- Repeated collisions lead to heating and to interesting dynamics described by a Fermi mapping

- Simplified Fermi mapping

$$v_{n+1} = v_n + 2\,\omega x_0 \,\sin \omega t_n$$

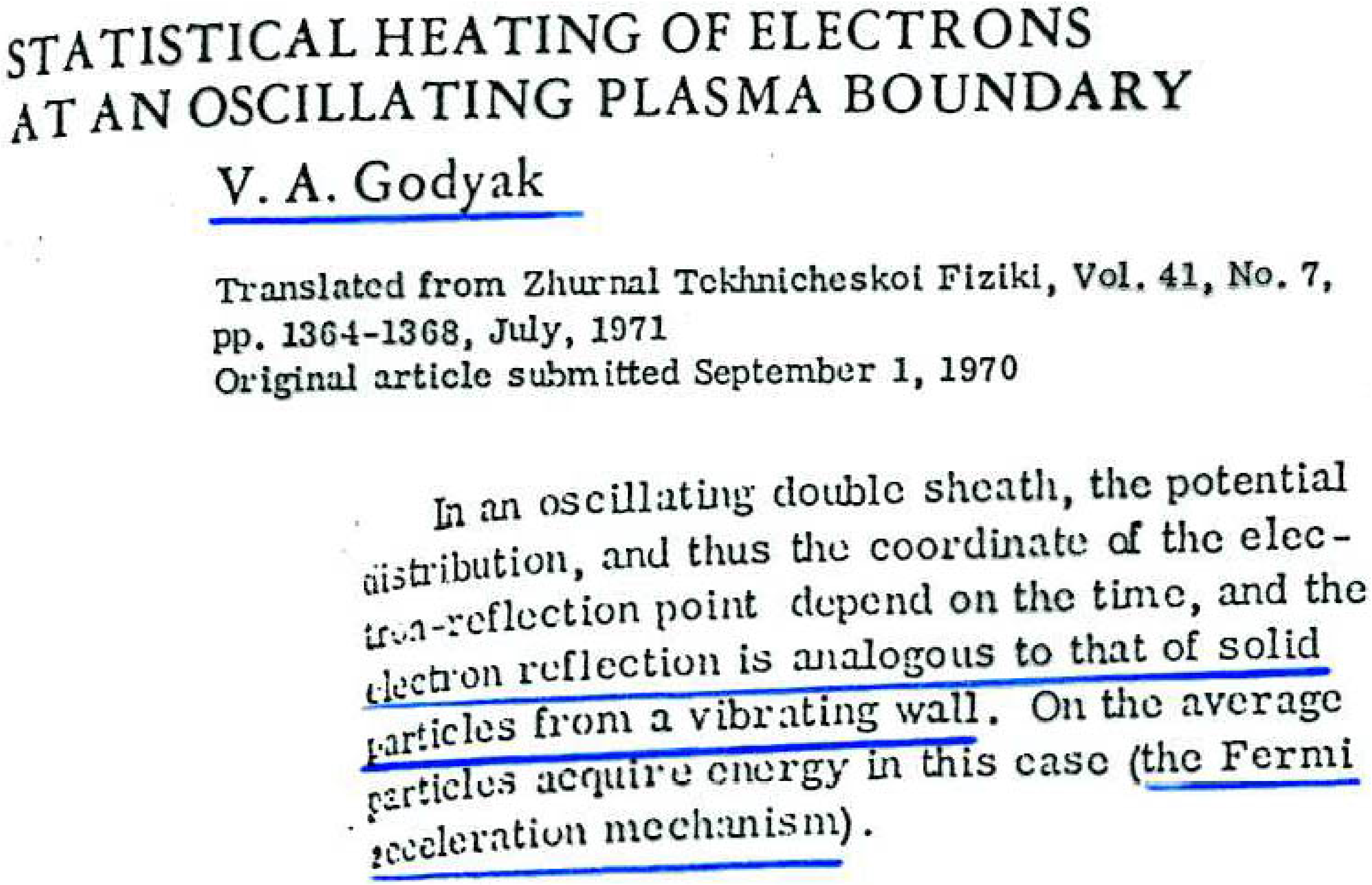
$$t_{n+1} = t_n + \frac{2\,l}{v_{n+1}}$$

- Kick Δv at nth collision is a periodic function of time
- Time between collisions Δt is inversely proportional to the velocity



CAPACITIVE DISCHARGES

CAPACITIVE DISCHARGE HEATING



STATISTICAL HEATING OF ELECTRONS AT AN OSCILLATING PLASMA BOUNDARY

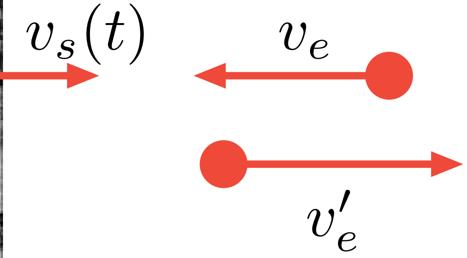
V. A. Godyak

Translated from Zhurnal Tekhnicheskoi Fiziki, Vol. 41, No. 7,
pp. 1364-1368, July, 1971
Original article submitted September 1, 1970

In an oscillating double sheath, the potential distribution, and thus the coordinate of the electron-reflection point depend on the time, and the electron reflection is analogous to that of solid particles from a vibrating wall. On the average particles acquire energy in this case (the Fermi acceleration mechanism).

(V.A. Godyak, *Sov. Phys. Tech. Phys.* **16**, No. 7, Jan. 1972)

STOCHASTIC HEATING

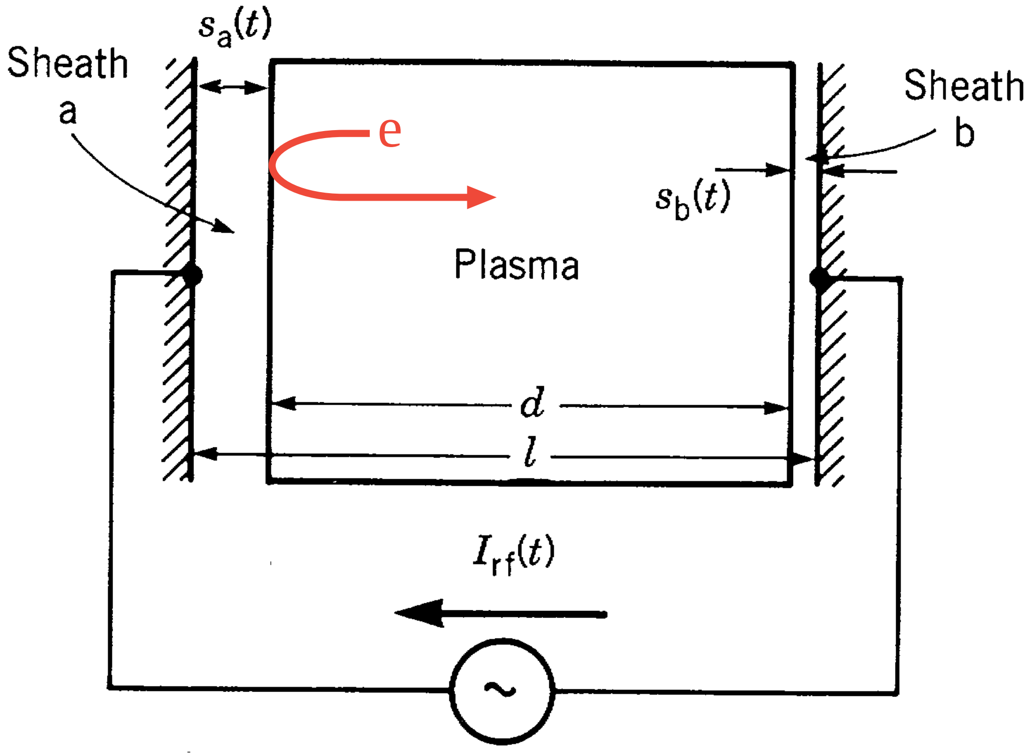


$$v_e' = v_e + 2v_s(t)$$

- $v_s > 0$, energy gain
  $v_s < 0$, energy loss

- Let $v_s = v_{s0} \cos \omega t$
  $\langle v_e'^2 \rangle = v_e^2 + 2v_{s0}^2$
  $\boxed{\text{A net energy gain}}$

HOMOGENEOUS MODEL OF STOCHASTIC HEATING



- Time-average stochastic heating power/area for one sheath

$$S_{\text{stoc}} = \left\langle n \int_{v_s}^{\infty} (v - v_s) \frac{1}{2} m (v'^2 - v^2) f_e(v)\, dv \right\rangle = \frac{1}{4} n \bar{v}_e \cdot 2 m v_{s0}^2$$

with $\bar{v}_e = (8eT_e/\pi m)^{1/2}$ the mean thermal speed

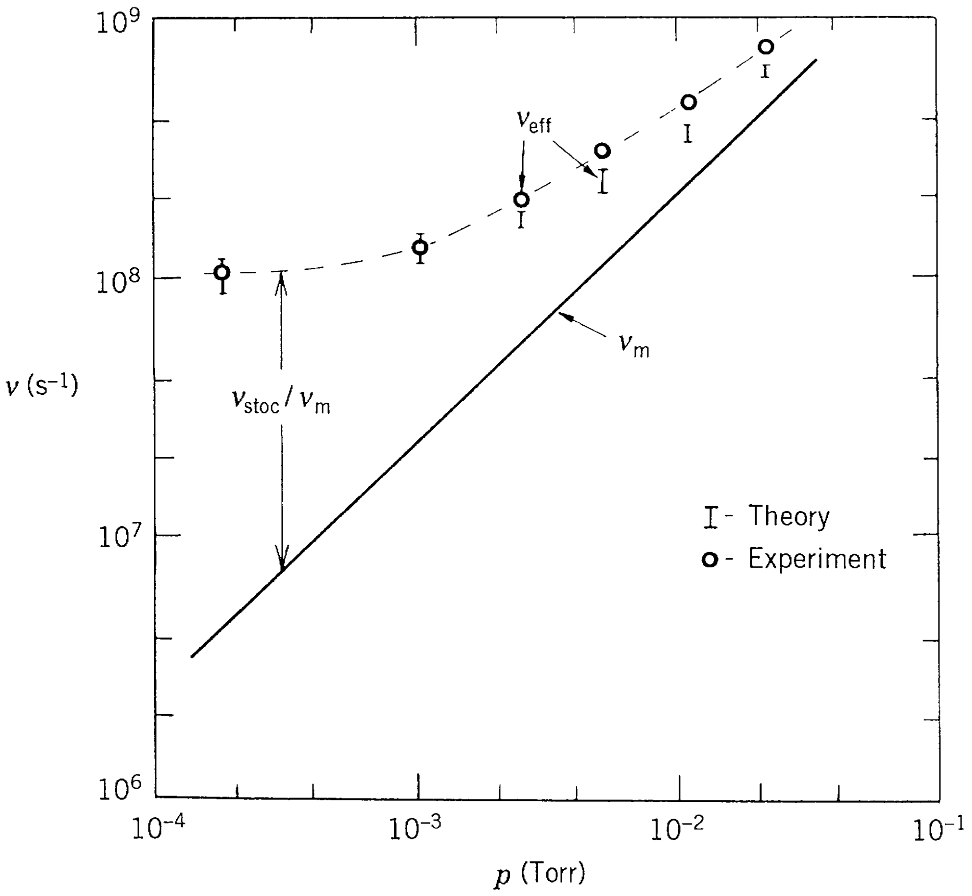
EXPERIMENTAL EVIDENCE

- Including the ohmic heating in the bulk plasma and the stochastic heating for the two sheaths

$$\boxed{\begin{aligned} S_{\rm tot} &= \frac{1}{2} m n v_{s0}^2 \nu_{\rm eff}\, d \\ \nu_{\rm eff} &= \underbrace{\nu_m}_{} + \underbrace{2\bar{v}_e/d}_{} \end{aligned}}$$

Ohmic Stochastic

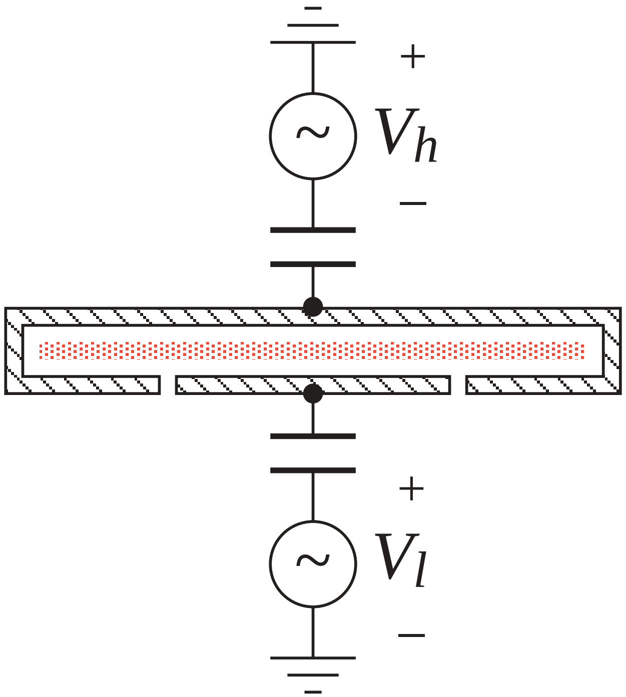
ν_m ($\propto$ gas pressure)=electron-neutral collision frequency

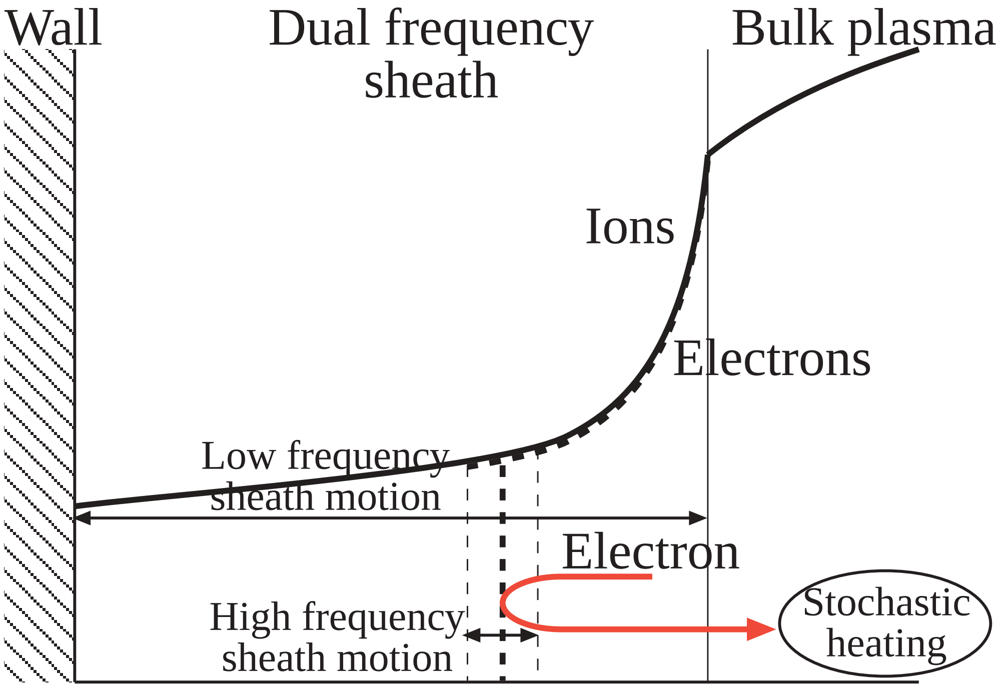


(V.A. Godyak, R.B. Piejak, and B.M. Alexandrovich, *Plasma Sources: Sci. Technol.* **1**, 36, 1992)

DUAL FREQUENCY HEATING

- Dual frequency capacitive discharges used extensively to fabricate integrated circuits





High frequency voltage controls ion flux
Low frequency voltage controls ion energy

STOCHASTIC HEATING POWER

- Fermi acceleration theory in dual frequency regime:

$$\boxed{S_{\text{stoc}} = \underbrace{\frac{1}{2} m \bar{v}_e \frac{J_h^2}{e^2 n_s}}_{\text{High freq part}} \times \underbrace{\left(1 + \frac{\pi}{4} H_l\right)\left(\frac{H_l}{H_l + 2.2}\right)}_{\text{Low freq part } F(H_l)}}$$

S_{stoc} = stochastic heating power per unit electrode area

m = electron mass

$\bar{v}_e = (8e\mathrm{T}_e/\pi m)^{1/2}$ = mean thermal electron speed

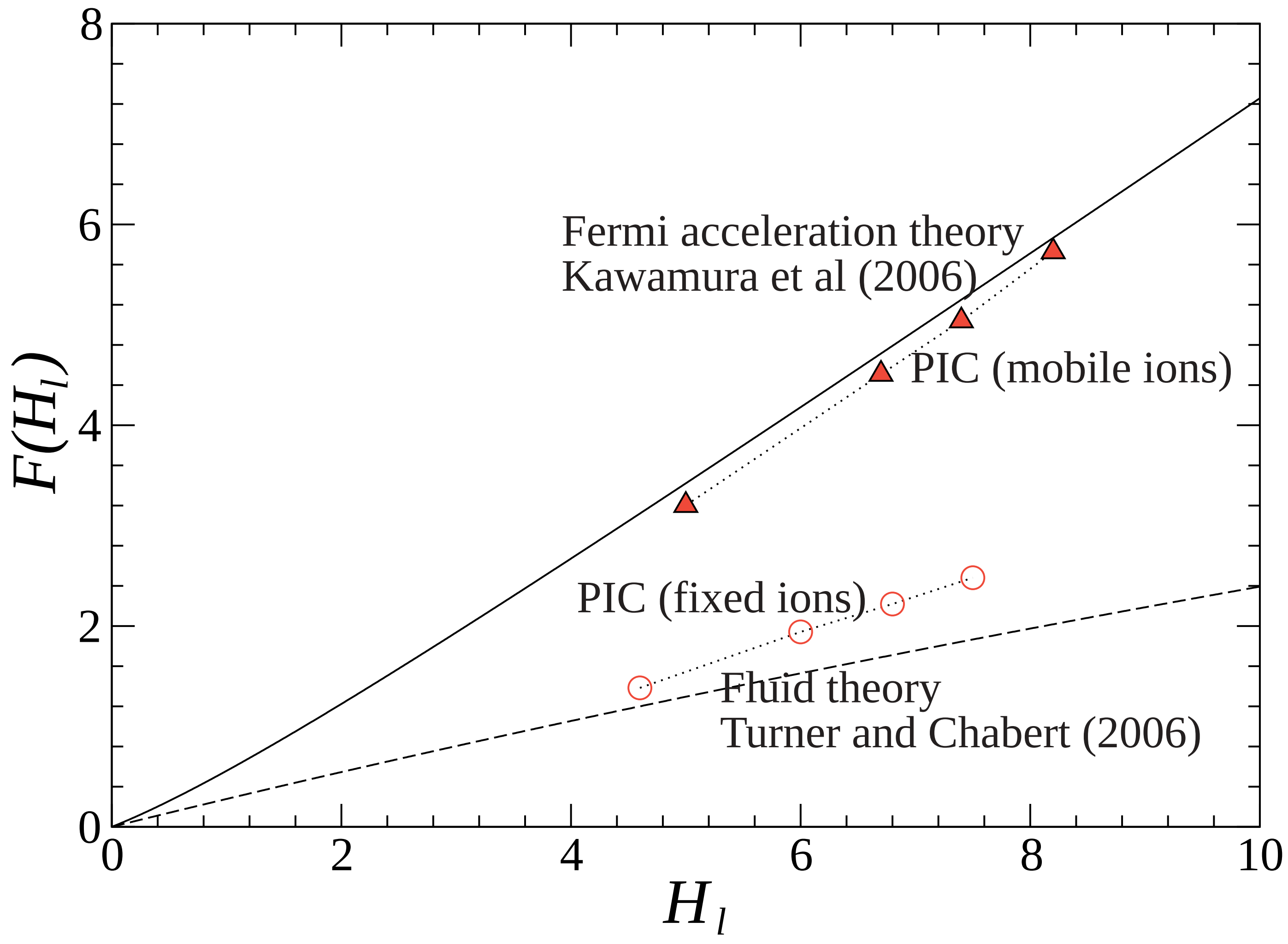
J_h = high frequency current density

n_s = plasma density at bulk plasma–sheath edge

$H_l = 0.55(V_l/\mathrm{T}_e)^{1/2}$ =low frequency enhancement factor

(E. Kawamura, M.A. Lieberman, and A.J. Lichtenberg, *Phys. Plasmas* **13,** 053506/1–14, 2006)

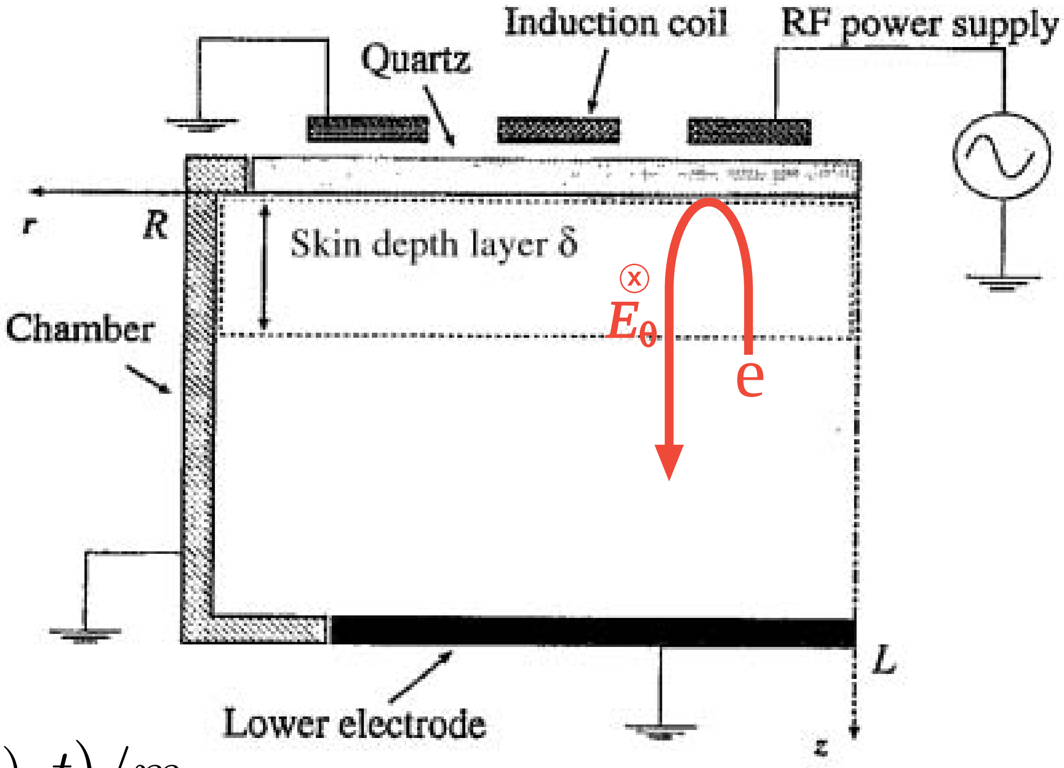
PARTICLE-IN-CELL SIMULATIONS



(M.M. Turner and P. Chabert, *Phys. Rev. Lett.* **96,** 205001/1–4, 2006)

INDUCTIVE DISCHARGES

PLANAR INDUCTIVE DISCHARGE HEATING

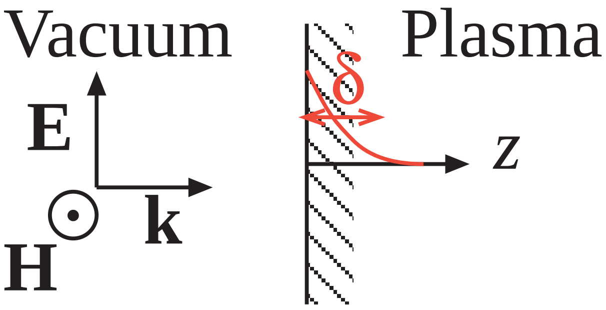


- Let $E_\theta(z) = E_0 e^{-z/\delta}\cos(\omega t + \phi)$
- $z(t) = \begin{cases} -v_z t, & t < 0 \\ v_z t, & t > 0 \end{cases}$
- $\Delta v_\theta(\phi) = -\int_{-\infty}^{\infty} dt\, eE_\theta\big(z(t), t\big)/m$

$$\boxed{S_{\text{stoc}} = n\int_0^\infty dv_z\,\frac{1}{2}m\langle(\Delta v_\theta)^2\rangle_\phi\, v_z f_e(v_z)}$$

(V. Vahedi, M.A. Lieberman, G. DiPeso, T.D. Rognlien, and D. Hewett, *J. Appl. Phys.* **78**, 1446, 1995)

SKIN DEPTHS



$$\kappa_p = 1 - \frac{\omega_p^2}{\omega(\omega - j\nu_m)}$$

with ω_p the plasma frequency and ν_m the electron-neutral collision frequency

- Local theory (E-field decays exponentially)

  (a) Collisionless ($\nu_m \ll \omega$) $\Rightarrow \delta = \dfrac{c}{\omega_p}$

  (b) Collisional ($\nu_m \gg \omega$) $\Rightarrow \delta = \dfrac{c}{\omega_p}\left(\dfrac{2\nu_m}{\omega}\right)^{1/2}$

- Non-local theory (non-exponential decay of E-field)

  (c) Anomalous ($\bar{v}_e/\delta \gg \omega,\ \nu_m$) $\Rightarrow \delta = \dfrac{c}{\omega_p}\left(\dfrac{2\bar{v}_e\omega_p}{\omega c}\right)^{1/2}$

  with $\bar{v}_e$ the mean electron speed

Anomalous regime corresponds to Fermi acceleration

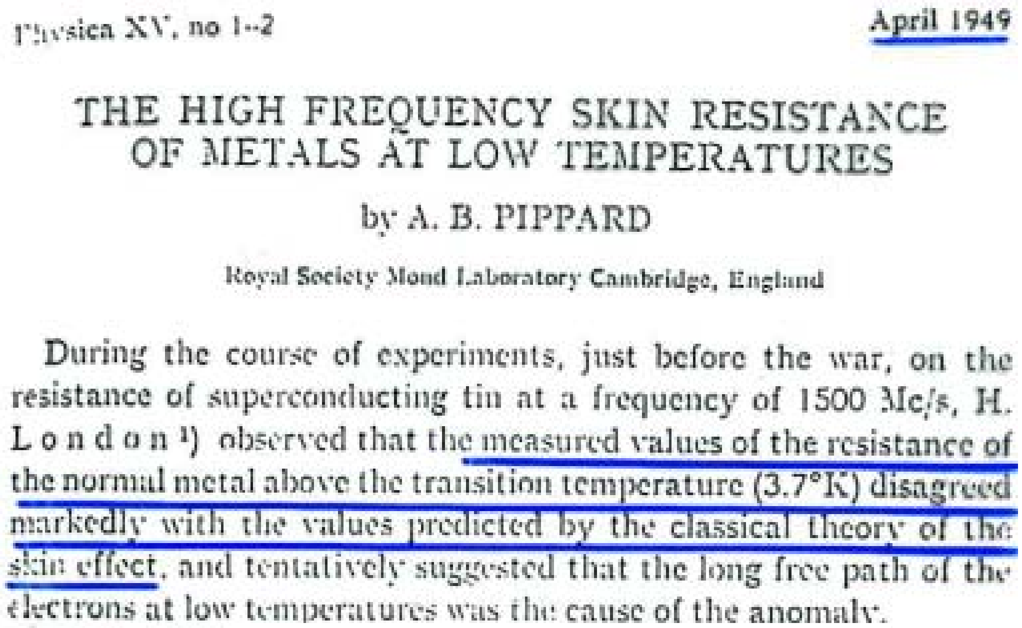
DISCOVERIES OF ANOMALOUS HEATING

- Measurements in inductive discharges (Godyak et al, 1994)
- Collisionless theory and PIC simulations (Turner, 1993)
- Theory of rf magnetic field effects (Cohen and Rognlien, 1994)
- Negative power absorption (Godyak and Kolobov, 1997)
- Measurements of RF magnetic field effects (Godyak et al, 1999)

- First plasma experiment (Demirkhanov et al, 1964)
- First plasma kinetic theory (Weibel, 1967)
- Theory of RF magnetic field effects (Blevin and Thonemann, 1962)
- RF magnetic fields (Rotomaks) (Jones, 1980)

- First experiment on anomalous skin effect in metals (Pippard, 1949)
- First kinetic theory of anomalous skin effect (Reuther and Sondheimer, 1949)

PIPPARD'S WORK



Physica XV, no 1–2 April 1949

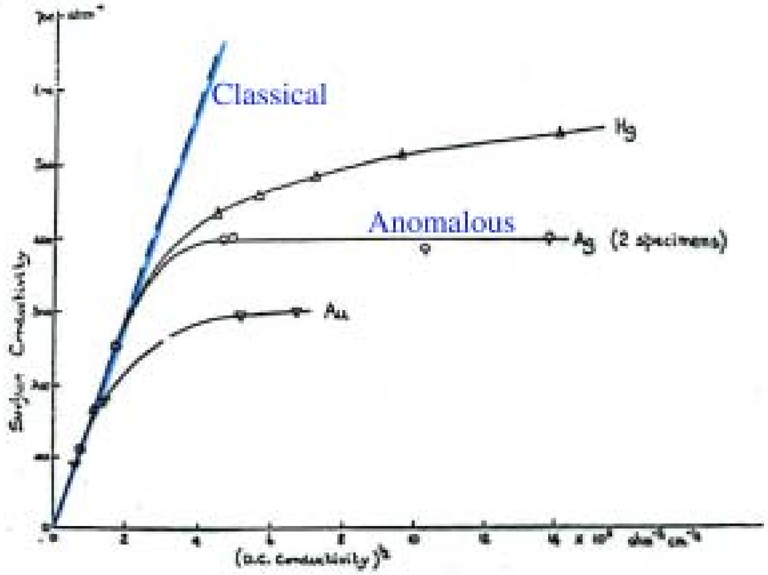
THE HIGH FREQUENCY SKIN RESISTANCE
OF METALS AT LOW TEMPERATURES

by A. B. PIPPARD

Royal Society Mond Laboratory Cambridge, England

During the course of experiments, just before the war, on the resistance of superconducting tin at a frequency of 1500 Mc/s, H. London[1]) observed that the measured values of the resistance of the normal metal above the transition temperature (3.7°K) disagreed markedly with the values predicted by the classical theory of the skin effect, and tentatively suggested that the long free path of the electrons at low temperatures was the cause of the anomaly.

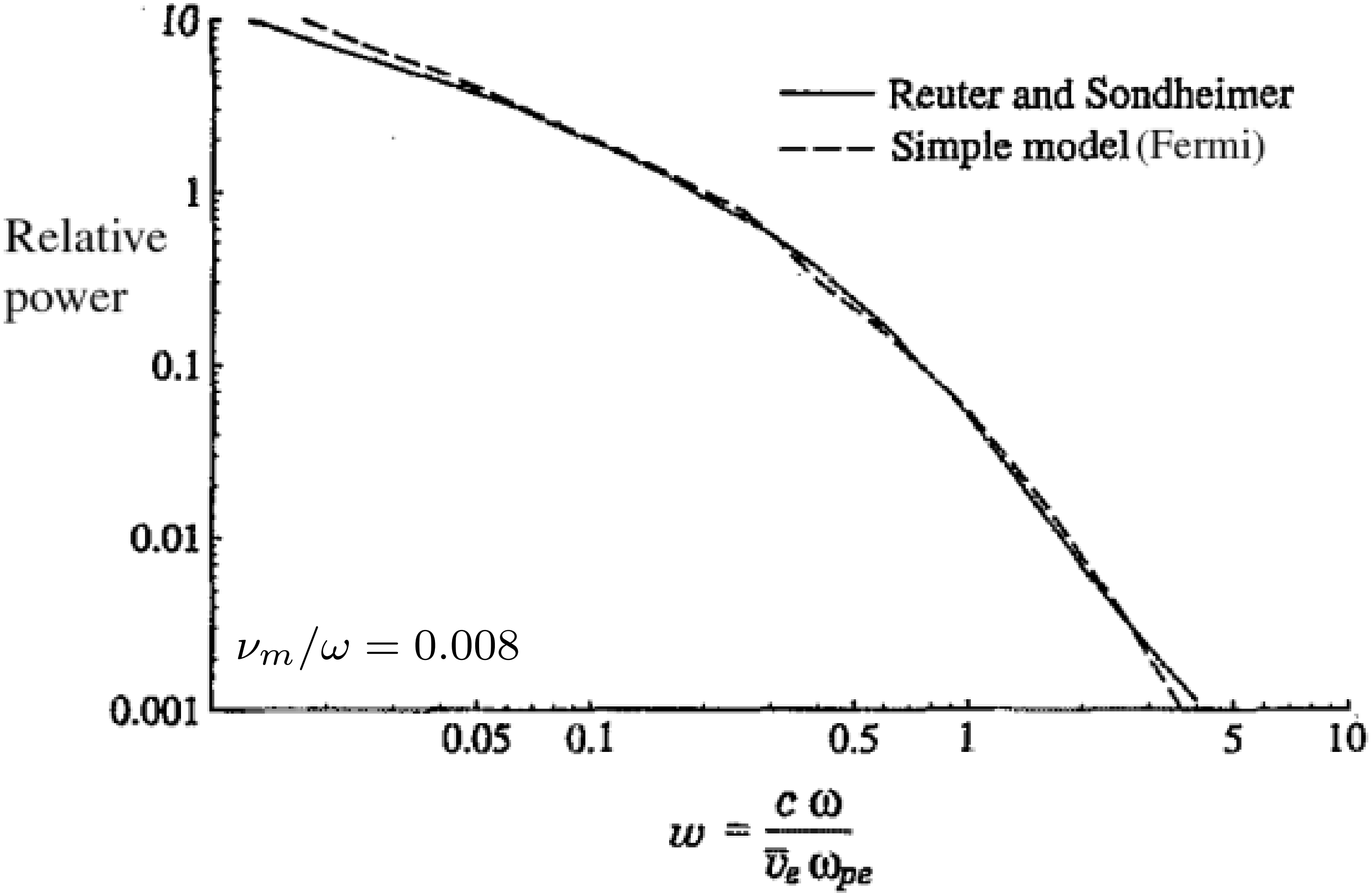
Power
absorbed
↓



← Collisionality

LiebermanDPP06

COMPARISON OF KINETIC AND FERMI MODELS

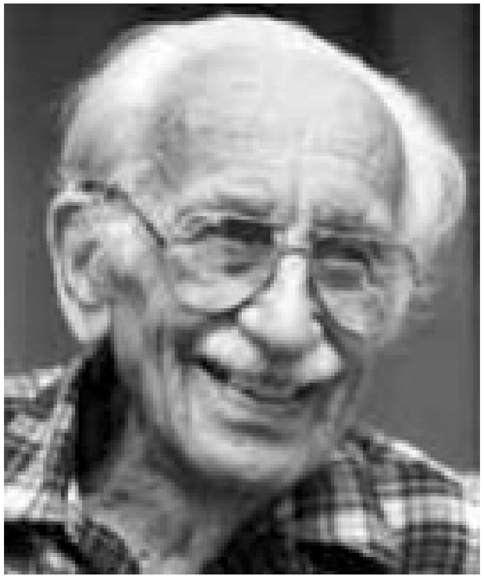


CONCLUDING REMARKS

- Collisionless electron heating, which is usually associated with high temperature space and fusion plasmas, is a fundamental process in the warm, low pressure plasmas used in today's industrial plasma technology
- The Fermi acceleration model of a ball bouncing between a fixed and an oscillating wall is a useful model to describe such heating

Review article: M.A. Lieberman and V.A. Godyak, *IEEE Trans. Plasma Sci.* **26**, 955, 1998

I am greatly indebted to A.J. Lichtenberg, C.K. Birdsall, V.A. Godyak, Emi Kawamura, and V. Vahedi



W.P.Allis (1901–1999)

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