Ke Li Jitendra Malik



### Introduction

- Machine learning operates on a data-driven philosophy that favours automatic pattern discovery over manual design.
- Yet, the algorithms that power machine learning are still manually designed.
- Can we learn these algorithms instead?



- Inspired by metacognition (Aristotle, 350 BC), which refers to the ability of humans to reason about their own process of reasoning.
- Goal: learn some general knowledge about the learning outcome or process that is useful across many tasks.
  - Unlike ordinary learning, generalization is not across instances, but across tasks.

#### • Terms:

- Base-learning: instance-level learning
- Meta-learning: task-level learning



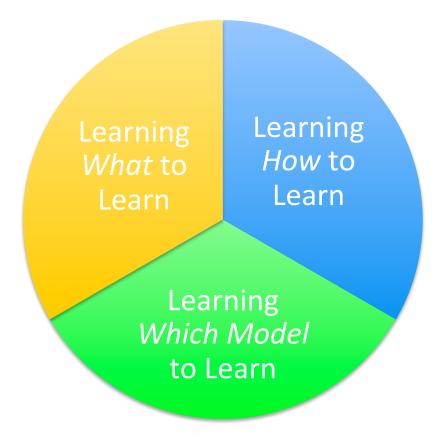
## Fundamental Challenge

- Key Problem: how do we parameterize the space of all possible learning methods such that it is both:
  - 1) expressive, and
  - 2) efficiently searchable?
- Two Extremes:
  - Enumerate a small set of methods: not expressive.
  - Search over all general-purpose programs: takes exponential time.



Different methods differ in the type of meta-knowledge

they learn.

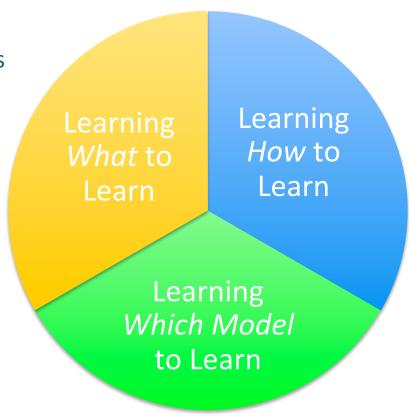




Different methods differ in the type of meta-knowledge

they learn.

Learn parameter values of the base-model that are useful across tasks.



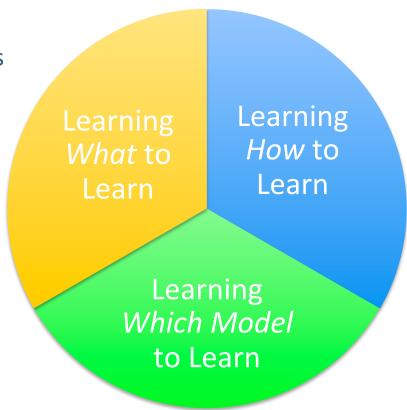


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- Transfer Learning
- Multi-Task Learning
- Few-Shot Learning





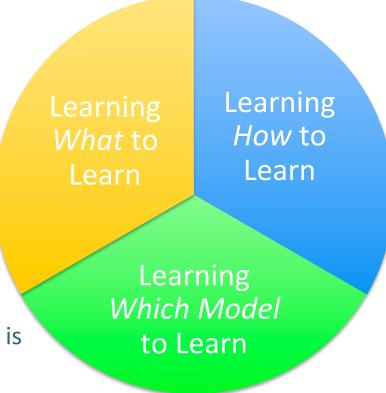
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Learn which base-model is best suited for a task.





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- Multi-Task Learning
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What to Transfer Learning

Learning How to Learn

Learning Which Model to Learn

Learn which base-model is best suited for a task.

Hyperparameter **Optimization** 



Different methods differ in the type of meta-knowledge

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Learn parameter values of the base-model that are useful across tasks.

- Transfer Learning
- Multi-Task Learning
- **Few-Shot Learning**

What to

Learning How to Learn

Learning Which Model to Learn

**Our Contribution:** Learn how to train the base-model.

Learn which base-model is best suited for a task.

Hyperparameter **Optimization** 



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Learn parameter values of the base-model that are useful across tasks.

- Transfer Learning
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Learning What to Learn

Learning
How to
Learn

Our Contribution: Learn how to train the base-model. (Focus of this talk)

Learning Which Model to Learn which base-model is best suited for a task.

HyperparameterOptimization

Berkeley

### **Learning How to Learn**



# Setting

- Most learning algorithms optimize some objective function.
  - Learning how to learn reduces to learning an optimization algorithm.
- We train an optimization algorithm on a set of objective functions.
- The learner searches the space of possible optimization algorithms and outputs an optimization algorithm that performs well on the set of objective functions.



### Algorithm 1 General structure of optimization algorithms

```
Require: Objective function f x^{(0)} \leftarrow \text{random point in the domain of } f for i=1,2,\ldots do \Delta x \leftarrow \phi(\{x^{(j)},f(x^{(j)}),\nabla f(x^{(j)})\}_{j=0}^{i-1}) if stopping condition is met then return x^{(i-1)} end if x^{(i)} \leftarrow x^{(i-1)} + \Delta x end for
```



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```



### Algorithm 1 General structure of optimization algorithms

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x^{(0)} \leftarrow \text{random point in the domain of } f
\text{for } i = 1, 2, \dots \text{do}
\Delta x \leftarrow \phi(\{x^{(j)}, f(x^{(j)}), \nabla f(x^{(j)})\}_{j=0}^{i-1})
\text{if stopping condition is met then}
\text{return } x^{(i-1)}
```

 $\begin{array}{l} \text{end if} \\ x^{(i)} \leftarrow x^{(i-1)} + \Delta x \end{array}$ 

end for

Gradient Descent 
$$\phi(\cdot) = -\gamma \nabla f(x^{(i-1)})$$

Momentum  $\phi(\cdot) = -\gamma \left(\sum_{j=0}^{i-1} \alpha^{i-1-j} \nabla f(x^{(j)})\right)$ 

Learned Algorithm  $\phi(\cdot) = \text{Neural Net}$ 



Algorithm 1 General structure of optimization algorithms

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for i=1,2,\ldots do
\Delta x \leftarrow \phi(\{x^{(j)},f(x^{(j)}),\nabla f(x^{(j)})\}_{j=0}^{i-1})
if stopping condition is met then
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How do we learn  $\phi(\cdot)$ ?



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```

How do we learn  $\phi(\cdot)$ ? We use reinforcement learning.



### Background on Reinforcement Learning

- Set of states:  $\mathcal{S} \subseteq \mathbb{R}^D$
- Set of actions:  $\mathcal{A} \subseteq \mathbb{R}^d$
- Probability density over initial states:  $p_i\left(s_0
  ight)$
- State transition probability density:  $p\left(s_{t+1} \mid s_t, a_t\right)$
- Cost function:  $c:\mathcal{S} 
  ightarrow \mathbb{R}$
- Time horizon: T
- Typically, the reinforcement learning algorithm does not know what  $p\left(s_{t+1} \mid s_t, a_t\right)$  is.



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This is crucial.



### Background on Reinforcement Learning

- Policy:  $\pi\left(\left.a_{t}\right|s_{t},t\right)$ 
  - When it is independent of t, it is known as stationary.
- The goal is to find:

$$\pi^* = \arg\min_{\pi} \mathbb{E}_{s_0, a_0, s_1, \dots, s_T} \left[ \sum_{t=0}^{T} c(s_t) \right]$$

where the expectation is taken w.r.t.

$$q(s_0, a_0, s_1, \dots, s_T) = p_i(s_0) \prod_{t=0}^{T-1} \pi(a_t | s_t, t) p(s_{t+1} | s_t, a_t)$$



### Reduction to Reinforcement Learning

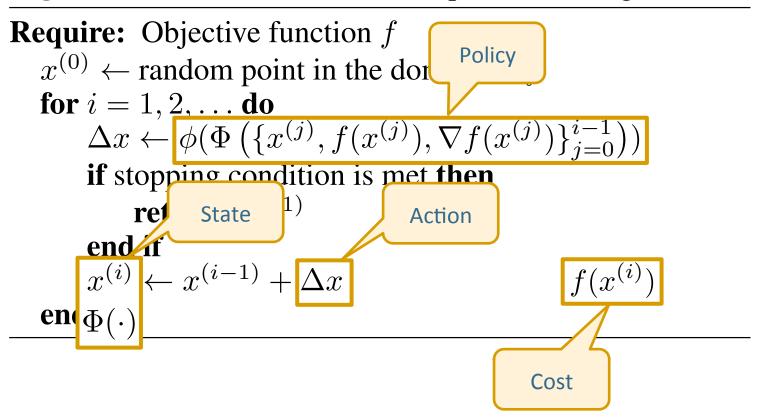
### Algorithm 1 General structure of optimization algorithms

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```



### Reduction to Reinforcement Learning

### Algorithm 1 General structure of optimization algorithms





### Reduction to Reinforcement Learning

- Under this formulation, the state transition probability density  $p(s_{t+1}|s_t,a_t)$  captures how the gradient and objective value are likely to change for any given step vector.
  - In other words, it encodes the distribution of local geometries of the objective functions of interest.
- The geometry of an unseen objective function is unknown.
  - This is OK, since reinforcement learning does not assume knowledge of  $p(s_{t+1}|s_t, a_t)$ .



### Why Reinforcement Learning?



# Simultaneous Discovery

 A similar idea was also proposed independently by Andrychowicz et al. soon after our paper appeared:



#### **Learning to Optimize**

Ke Li Jitendra Malik

Department of Electrical Engineering and Computer Sciences
University of California, Berkeley



### Learning to learn by gradient descent by gradient descent

Marcin Andrychowicz Matthew W. Hoffman Misha Denil David Pfau Nando de Freitas Sergio Gomez Tom Schaul



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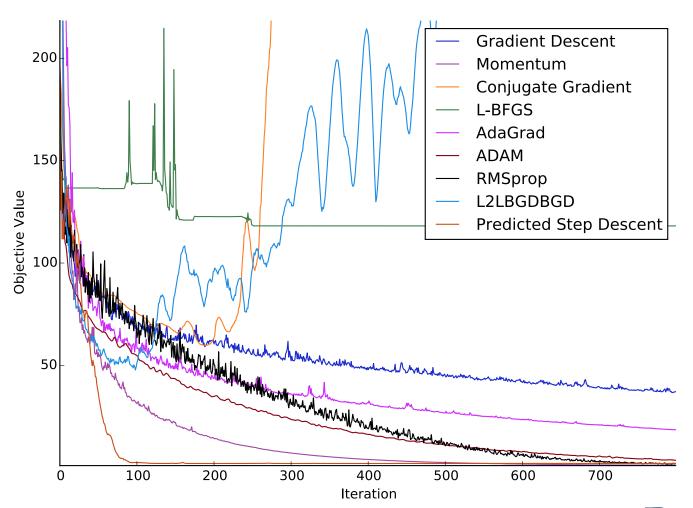
**Learning to Optimize** 

Uses Reinforcement Learning

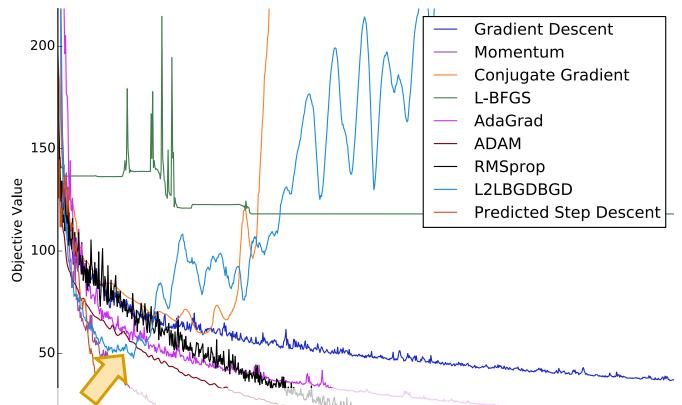


Learning to learn by gradient descent by gradient descent

**Uses Supervised Learning** 

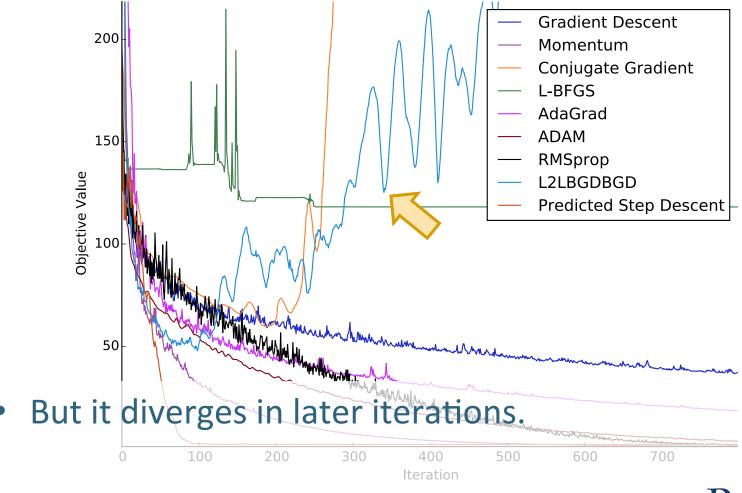




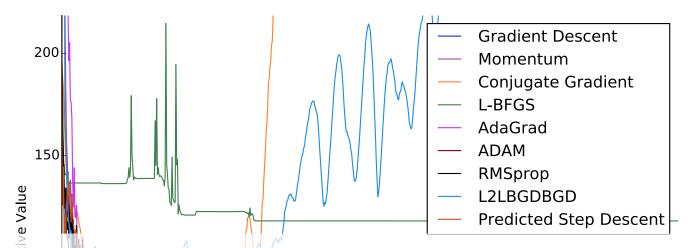


 Optimization algorithm trained using supervised learning does reasonably well initially.

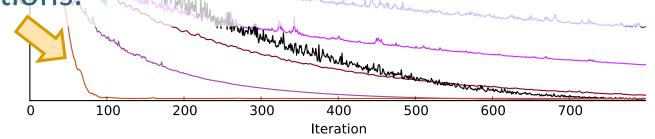








 The optimization algorithm trained using reinforcement learning does not diverge in later iterations.





### What Generalization Means

- Each example is an objective function.
  - In the learning-to-learn setting, it is the loss function for training a base-model on a task.
  - Objective functions can differ in two ways:
    - The base-model
    - The task
- Generalization is across objective functions.
  - In the learning-to-learn setting, it is across base-models and/or tasks.
- We should train the optimization algorithm on some base-models and tasks, and test it on different basemodels and tasks.

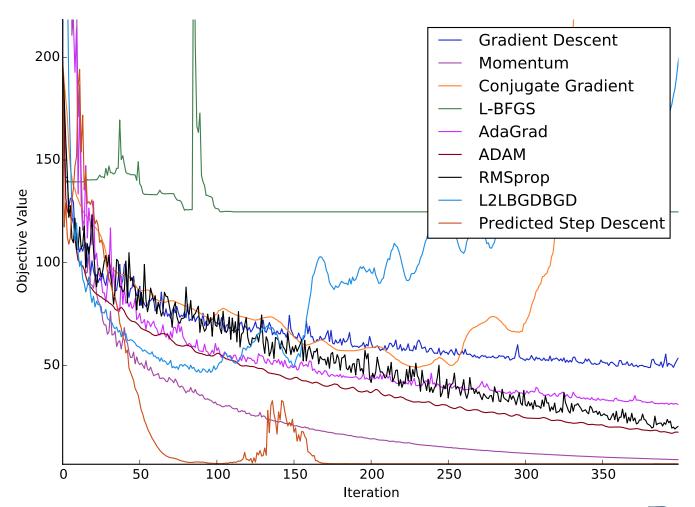


## **Experimental Setting**

- The training set consists of one objective function: the cross-entropy loss function for training a neural net on MNIST.
- The test set consists of the loss functions for training neural nets with different architectures on different datasets, i.e.: the Toronto Faces Dataset (TFD), CIFAR-10 and CIFAR-100.
- In other words, the optimization algorithm is:
  - Meta-trained on the problem of training a neural net on MNIST.
  - Meta-tested on the problems of training neural nets on TFD, CIFAR-10 and CIFAR-100.

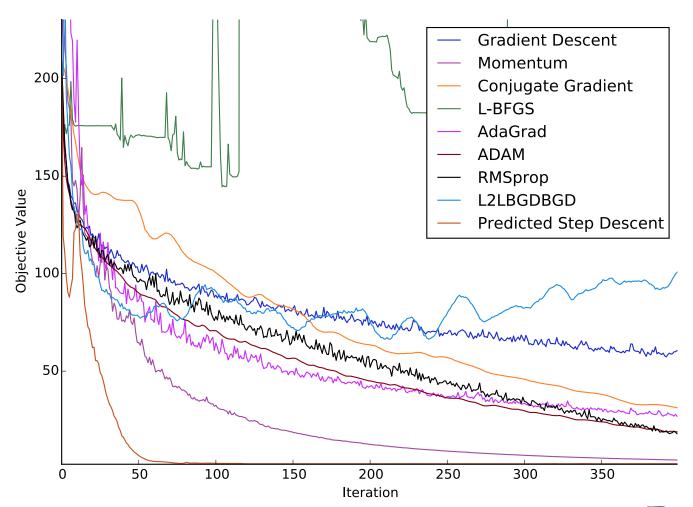


# Larger Architecture (TFD)



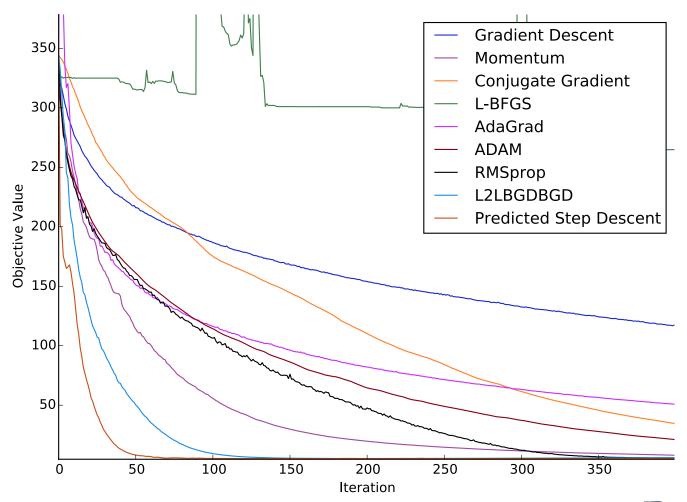


# Larger Architecture (CIFAR-10)



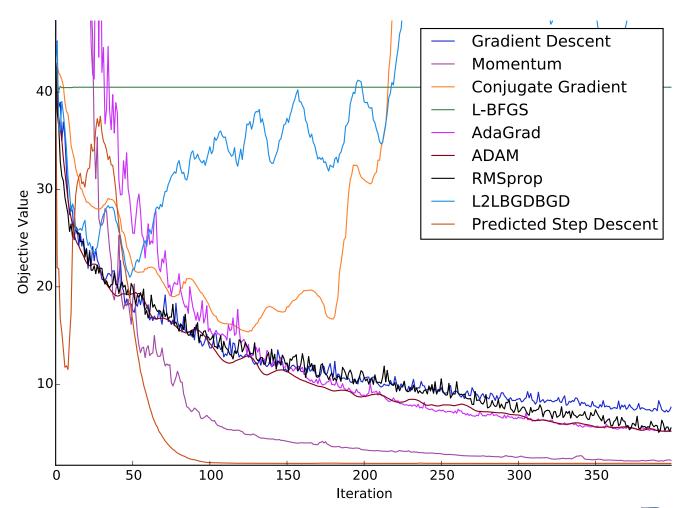


# Larger Architecture (CIFAR-100)



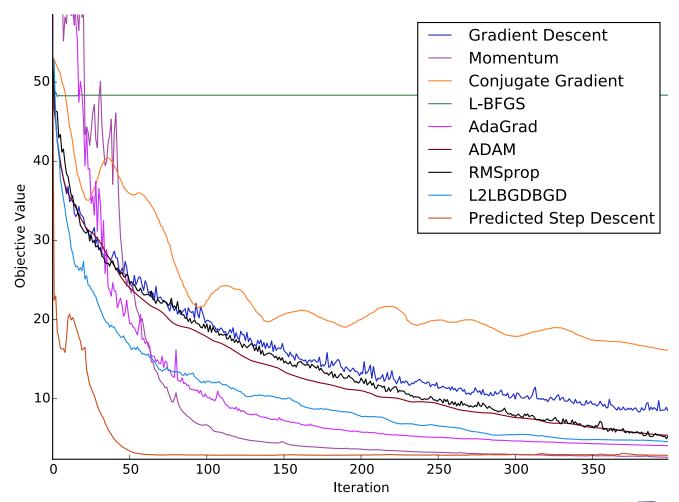


# Noisier Gradients (TFD)



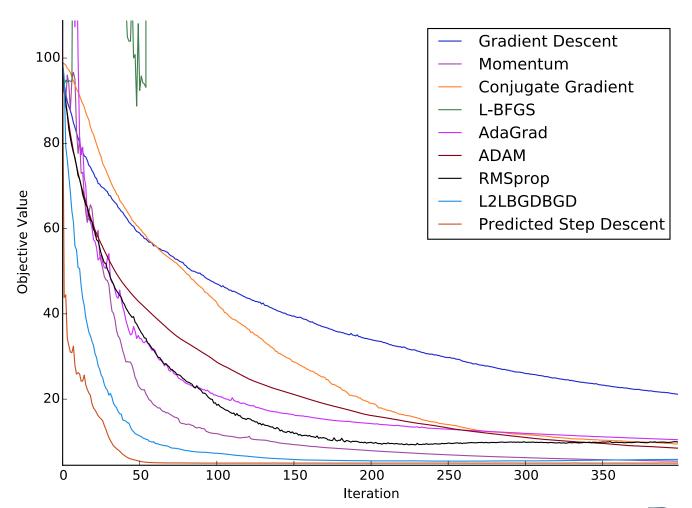


# Noisier Gradients (CIFAR-10)



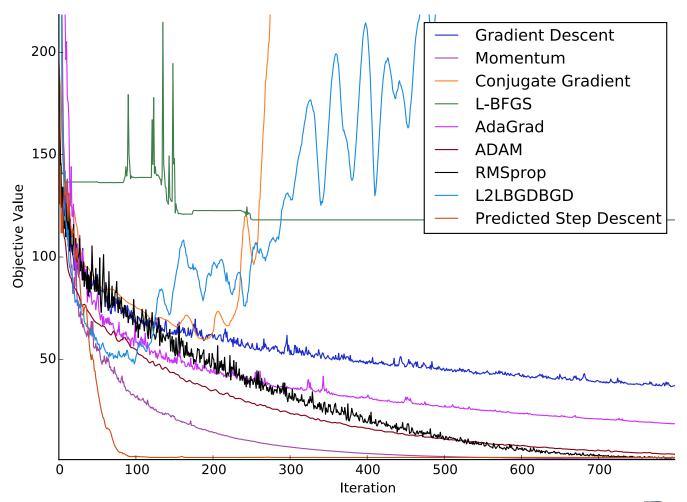


# Noisier Gradients (CIFAR-100)



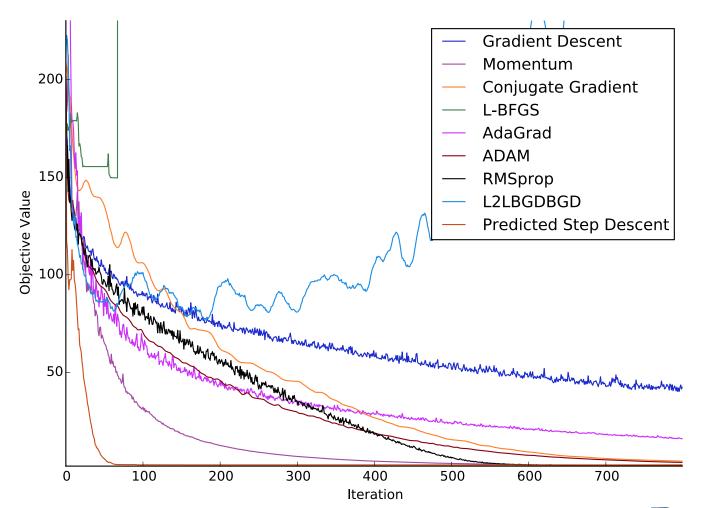


# Longer Time Horizon (TFD)



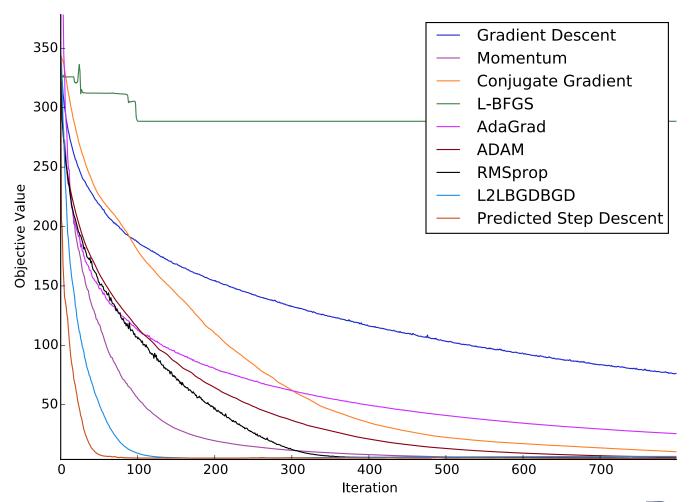


# Longer Time Horizon (CIFAR-10)



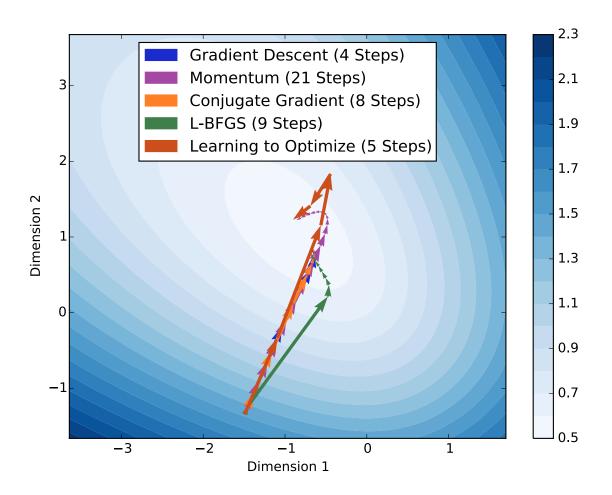


### Longer Time Horizon (CIFAR-100)



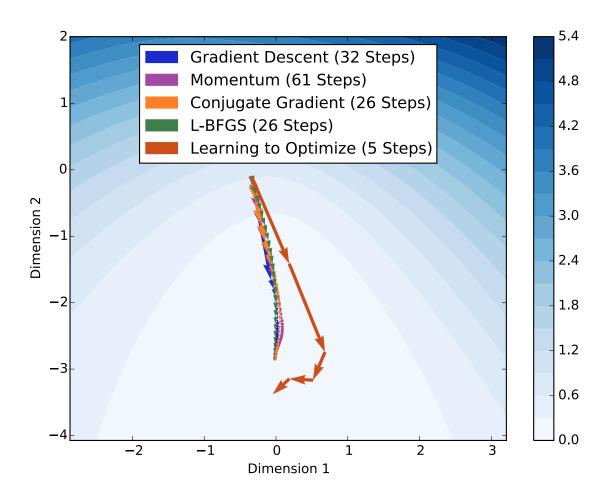


# 2D Logistic Regression





# 2D Logistic Regression



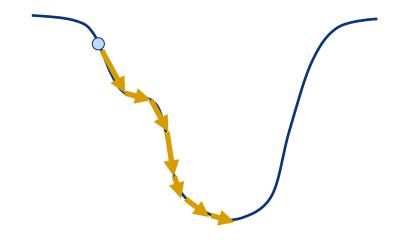


#### Generalization



# Importance of Generalization

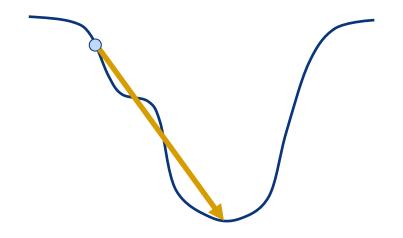
- Suppose we evaluate the performance of the optimizer on the training set.
- To learn an optimizer, we can simply run a traditional optimizer and memorize the solution.
- This is the best optimizer, since it gets to the optimum in one step.





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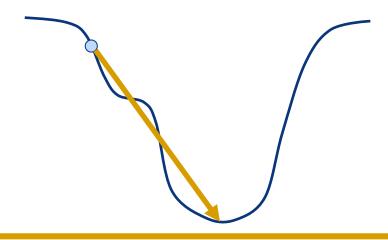




# Importance of Generalization

- Suppose we evaluate the performance of the optimizer on the training set.
- To learn an optimizer, we can simply run a traditional optimizer and memorize the solution.
- This is the hest ontimizer

It would be pointless to learn the optimizer if we didn't care about generalization.





- Generalization to similar base-models on similar tasks
  - Learned optimizer could memorize parts of the optimal parameters that are common across tasks and base-models.
    - E.g.: Weights of the lower layers in neural nets
  - Essentially the same as learning what to learn.



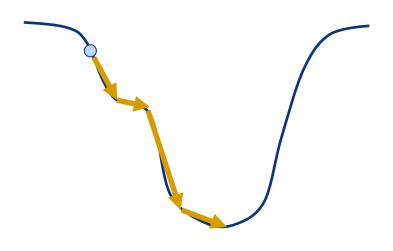
- Stronger notion: Generalization to similar basemodels on dissimilar tasks
  - The optimal parameters for dissimilar tasks are likely completely different.
  - An optimizer that memorizes any part of the optimal parameters will fail.
  - An optimizer that works in this setting must have learned not what the optimum is, but how to find it.



- Even stronger notion: Generalization to *dissimilar* base-models on *dissimilar* tasks
  - The objective functions at test time could be arbitrarily different from the objective functions seen during training.
  - This is impossible there is no optimizer that works well on all possible objective functions.

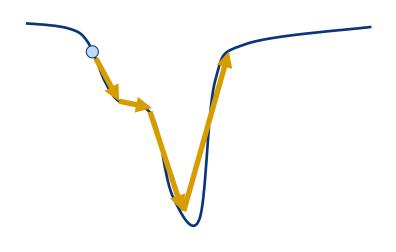


- Given any optimizer, we can always find an objective function that it performs poorly on.
- We can simply change the objective function so that the final objective value is large.



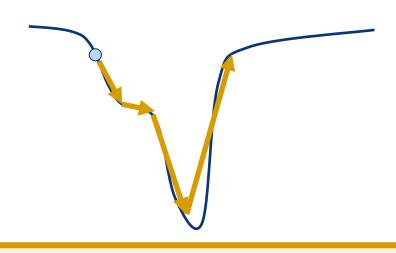


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- Given any optimizer, we can always find an objective function that it performs poorly on.
- We can simply change the objective function so that the final objective value is large.



It is not possible for the learned optimizer to generalize to all possible objective functions.



- Supervised learning requires one of the following:
  - Observations at each time step are i.i.d., or
  - The dependence of the future observation on the current observation is known.
- In our setting, neither is true:
  - The step the optimizer takes affects future gradients.
  - How the current step affects the next gradient, i.e.
     the local geometry, is not known at test time.



- When backpropagating through time, supervised learning essentially assumes the local geometry of an unseen objective function is the same as the local geometry of *one* of the training objective functions at *all* time steps.
  - In other words, it assumes  $p(s_{t+1}|s_t, a_t)$  is known and models it using the Hessians of the training objective functions.
  - This is incorrect, since the Hessians of an unseen objective function will be different.
- Hence, supervised learning overfits to the geometries of training objective functions.



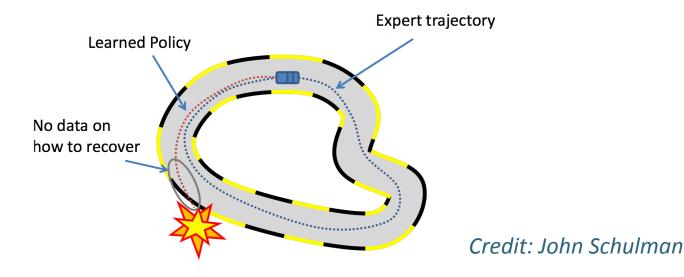
- When an optimizer trained with supervised learning is applied to an unseen objective function:
  - It takes a step,
    - > sees an unexpected gradient at the next iteration,
    - takes a step that is slightly off,
    - finds out the next gradient is even more unexpected,
    - → takes another step that is more off,

• • •

eventually diverges.



- This is known as the problem of compounding errors.
  - Supervised learning leads to a cumulative error that grows quadratically in the time horizon, rather than linearly. (Ross & Bagnell, 2010)





#### Why RL Solves This Problem

- Reinforcement learning algorithm does not assume knowledge of  $p(s_{t+1}|s_t,a_t)$ , which characterizes the geometries of training objective functions.
  - So, conditions at meta-training and meta-test times match.
  - The learned policy must account for the uncertainty in  $p(s_{t+1}|s_t,a_t)$ , and must know how to recover from mistakes.



#### Reinforcement Learning Method



# **Guided Policy Search**

- An (approximate) policy search algorithm for continuous state and action spaces. (Levine et al., 2015)
- Maintains two policies,  $\psi$  and  $\pi$  .
  - $\psi$  lies in a time-varying linear policy class.
    - Optimal policy can be found in closed form.
  - $-\pi$  lies in a stationary non-linear policy class.
- Alternates between solving for  $\psi$  and  $\pi$  .



#### **ADMM**

 Alternating direction method of multipliers (Boyd et al., 2011) solves the following problem:

$$\min_{\theta \in \Theta, \eta \in H} f(\theta) + g(\eta) \text{ s.t. } A\theta + B\eta = c$$

where f and g are convex functions, and  $\Theta$  and H are closed convex sets.

It alternates between the following updates:

$$\theta^{(t+1)} \leftarrow \arg\min_{\theta \in \Theta} f(\theta) + \langle \lambda^{(t)}, A\theta + B\eta^{(t)} - c \rangle + \frac{\rho}{2} \left\| A\theta + B\eta^{(t)} - c \right\|_{2}^{2}$$

$$\eta^{(t+1)} \leftarrow \arg\min_{\eta \in H} g(\eta) + \langle \lambda^{(t)}, A\theta^{(t+1)} + B\eta - c \rangle + \frac{\rho}{2} \left\| A\theta^{(t+1)} + B\eta - c \right\|_{2}^{2}$$

$$\lambda^{(t+1)} \leftarrow \lambda^{(t)} + \rho \left( A\theta^{(t+1)} + B\eta^{(t+1)} - c \right)$$



# Bregman ADMM

 Bregman ADMM (Wang & Banerjee, 2014) generalizes ADMM and uses Bregman divergence as penalty. It solves:

$$\min_{\theta \in \Theta, \eta \in H} f(\theta) + g(\eta) \text{ s.t. } A\theta + B\eta = c$$

where f and g are convex functions, and  $\Theta$  and H are closed convex sets.

It alternates between the following updates:

$$\theta^{(t+1)} \leftarrow \arg\min_{\theta \in \Theta} f(\theta) + \langle \lambda^{(t)}, A\theta + B\eta^{(t)} - c \rangle + \rho B_{\phi}(c - A\theta, B\eta^{(t)})$$

$$\eta^{(t+1)} \leftarrow \arg\min_{\eta \in H} g(\eta) + \langle \lambda^{(t)}, A\theta^{(t+1)} + B\eta - c \rangle + \rho B_{\phi}(B\eta, c - A\theta^{(t+1)})$$

$$\lambda^{(t+1)} \leftarrow \lambda^{(t)} + \rho \left( A\theta^{(t+1)} + B\eta^{(t+1)} - c \right)$$



#### Reinforcement Learning Problem

Recall the reinforcement learning problem:

$$\min_{\theta} \mathbb{E}_{s_0, a_0, s_1, \dots, s_T} \left[ \sum_{t=0}^{T} c(s_t) \right]$$

where the expectation is taken w.r.t.

$$q(s_0, a_0, s_1, \dots, s_T) = p_i\left(s_0\right) \prod_{t=0}^{T-1} \pi\left(a_t | s_t; \theta\right) p\left(s_{t+1} | s_t, a_t\right)$$
 State Action Initial State Distribution Policy Parameters Distribution



#### Reinforcement Learning Problem

Recall the reinforcement learning problem:

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 State Action Initial State Distribution Policy Parameters State Transition Distribution



#### **Guided Policy Search**

Guided Policy Search performs dual decomposition:

$$\min_{\theta,\eta} \mathbb{E}_{\psi} \left[ \sum_{t=0}^{T} c(s_t) \right] \text{ s.t. } \psi \left( a_t | s_t, t; \eta \right) = \pi \left( a_t | s_t; \theta \right) \ \forall a_t, s_t, t$$

 It relaxes the problem by only enforcing equality on the first moments\*:

$$\min_{\theta,\eta} \mathbb{E}_{\psi} \left[ \sum_{t=0}^{T} c(s_t) \right] \text{ s.t. } \mathbb{E}_{\psi} \left[ a_t \right] = \mathbb{E}_{\psi} \left[ \mathbb{E}_{\pi} \left[ \left. a_t \right| s_t \right] \right] \ \forall t$$



<sup>\*</sup>The Bregman divergence penalty is applied on the original distributions.

# **Guided Policy Search**

 To solve the problem, it uses Bregman ADMM, which alternates between the following updates:

$$\eta \leftarrow \arg\min_{\eta} \sum_{t=0}^{T} \mathbb{E}_{\psi} \left[ c(s_{t}) - \lambda_{t}^{T} a_{t} \right] + \nu_{t} \mathbb{E}_{\psi} \left[ D_{KL} \left( \psi \left( a_{t} | s_{t}, t; \eta \right) \| \pi \left( a_{t} | s_{t}; \theta \right) \right) \right]$$

$$\theta \leftarrow \arg\min_{\theta} \sum_{t=0}^{T} \lambda_{t}^{T} \mathbb{E}_{\psi} \left[ \mathbb{E}_{\pi} \left[ a_{t} | s_{t} \right] \right] + \nu_{t} \mathbb{E}_{\psi} \left[ D_{KL} \left( \pi \left( a_{t} | s_{t}; \theta \right) \| \psi \left( a_{t} | s_{t}, t; \eta \right) \right) \right]$$

$$\lambda_{t} \leftarrow \lambda_{t} + \alpha \nu_{t} \left( \mathbb{E}_{\psi} \left[ \mathbb{E}_{\pi} \left[ a_{t} | s_{t} \right] \right] - \mathbb{E}_{\psi} \left[ a_{t} \right] \right) \ \forall t$$

 The optimization in the first update can be solved in closed form using a modification of linear-quadratic regulator (LQR).



# Landscape of Meta-Learning Methods



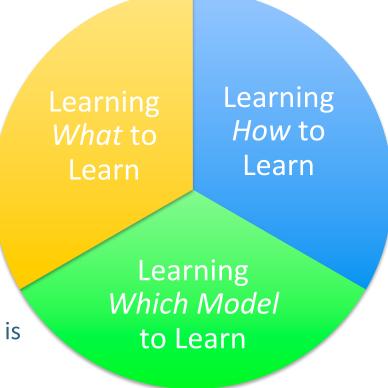
### Forms of Learning to Learn

Learn parameter values of the base-model that are useful across tasks.

- Transfer Learning
- Multi-Task Learning
- Few-Shot Learning

Learn which base-model is best suited for a task.

HyperparameterOptimization



Learn how to train the base-model.



### Learning What to Learn

Goal

• Learn what parameter values of the base-model are useful across tasks.

Meta-knowledge

Intermediate features that are shared by tasks across the family,
 e.g. Gabor filters for vision tasks.

Extent of Generalization

• Need to generalize across similar tasks.

Parameterization Challenges

 Need to parameterize the space of intermediate features – this is straightforward.

Examples

- Transfer & multi-task learning, e.g. (Suddarth & Kergosien, 1990)
- Few-shot learning, e.g. (Finn et al., 2017), (Snell et al., 2017)



### Learning Which Model to Learn

Goal

• Learn which base-model is best suited for a task.

Meta-knowledge

 Correlations between different base-models and their performance on different tasks.

Extent of Generalization

• Need to generalize across base-models, and *ideally*, across tasks.

Parameterization Challenges

 Need to parameterize the space of base-models – unclear how we can do this.

Examples

- Hyperparameter optimization does not generalize across tasks
- (Bradzil et al., 2003), (Schmidhuber, 2004), (Hochreiter et al., 2001)



### Learning Which Model to Learn

- (Bradzil et al., 2003): Enumerate a small set of base-models not expressive.
- (Schmidhuber, 2004): Search over the space of all possible programs takes exponential time.
- (Hochreiter et al., 2001): Search over base-models represented by a single step of a recurrent neural net not expressive.
- Hyperparameter optimization: Search over a predefined set of hyperprameters – not expressive.

Generalization

Me

Parameterization Challenges

• Need to parameterize the space of base-models – unclear how we can do this.

Examples

- Hyperparameter optimization does not generalize across tasks
- (Bradzil et al., 2003), (Schmidhuber, 2004), (Hochreiter et al., 2001)



Learning to Optimize

#### Learning How to Learn

#### Goal

- Learn how to train the base-model.
- Learn about the *process*, rather the *outcome* of learning.

#### Meta-knowledge

• Commonalities in the behaviours of learning algorithms that achieve good performance.

#### Extent of Generalization

 Need to generalize across dissimilar tasks and/or similar basemodels.

#### Parameterization Challenges

- Need to parameterize the space of learning algorithms.
- Key Idea: Parameterize the update formula in optimizer.

#### Examples

- (Bengio et al., 1991) learned algorithm indep. of tasks/base-models
- (Li & Malik, 2016), (Andrychowicz et al., 2016), etc.



#### For More Details...

#### **Learning to Optimize**

Ke Li, Jitendra Malik

arXiv:1606.01885, 2016 and ICLR, 2017

#### **Learning to Optimize Neural Nets**

Ke Li, Jitendra Malik

arXiv:1703.00441, 2017

