Problem 1: Let $R$ and $S$ be regular expressions. Does $(RS + R)^*RS = (RR^*S)^*$?

**Proof:** Let $R = 0$ and $S = 1$ then $(RR^*S)^*$ generates $\epsilon$ and $(RS + R)^*RS$ does not because all strings it generates must end in $01$. Therefore, they cannot be equal.

Problem 2: Let $L = \{ww^R | w \in \Sigma^*\}$. Is $L$ regular?

**Proof:** No. Let $p$ be the pumping length and let $w = 0^p1^p0^p \in L$. The pumping lemma says that if $w = xyz$ with $|xy| \leq p, |y| > 0$, then $xy^iz \in L$ for all $i \in \mathbb{N}$. However, if we try to pump $w$ we know that $xy$ must consist only of the first set of $0$’s, thus $xy^iz = 0^p0^|y|1^2p0^p$. Since $|y| > 0$ this is not a palindrome; $p + i|y| \neq p$ and so it is not in the language. Therefore $L$ is not regular.

Problem 3: Show that $0^n^2$ is not regular.

**Proof:** Let $p$ be the pumping length and $v$ a number larger than $p$ that is not a square (i.e. $v \neq k^2$ for any $k \in \mathbb{N}$). Let $w = 0^v^2 = xyz$ where $|xy| \leq p$ and $|y| > 0$. Let’s pump $w$ once so we have $xy^2z = 0^{v^2 + |y|}$. Since $|xy| \leq p$ and $|y| > 0$, we know that $1 \leq |y| \leq p$. $v^2 + |y|$ can’t be a square number because the next square number larger than $v^2$ is $(v + 1)^2$. $(v + 1)^2 = v^2 + 2v + 1 \geq v^2 + p$ which is the largest that $|y|$ could be. Therefore, $xy^2z \notin L$, so $L$ is not regular.

Problem 4: Show that $0^{2^n}$ is not regular.

**Proof:** Let $p$ be the pumping length and $w = 0^{2^p} = xyz$ where $|xy| \leq p, |y| > 0$. Note that the next largest string in this language is $0^{2^p + 1}$. However, when we pump $w$ we get $0^{2^p + |y|}$. In order for this to be in the language $|y|$ must be larger than $2^p + 1 - 2^p = 2^p$. However, $2^p > p \geq |xy|$, making this impossible and the language non-regular.

Problem 5: Is $L = \{ww^Rx | w, w^R, x \in \Sigma^+\}$ regular?

**Proof:** Argue by closure properties. Intersect with $10(00)^*11(00)^*011 = 10^n110^n11$ which isn’t regular by the pumping theorem.

This example works because we have an odd number of 0’s between each set of 1’s. Therefore, we can’t say that $ww^R$ is between the first two 1’s. This forces the palindrome part to be the entire string except the last 1. You can prove this language isn’t regular by the pumping lemma (pump $10^p110^p11$).
Problem 6: Decide if these statements are true or false and explain why.

1. Suppose $L_1$ is a regular language, and $L_2 = \{w | w \notin L_1 \text{ and } w \text{ ends with } 01\}$. Then $L_2$ must be regular.
   True. $L_2 = \overline{L_1} \cap (\Sigma^*01)$

2. The class of $\epsilon$-NFAs which do not allow $q_0$ to belong to $F$ can not accept all possible regular languages.
   False. Just let there be an $\epsilon$ transition to a final state from $q_0$.

3. If $L$ is non-regular then $L^*$ must be non-regular.
   False. $L = \{0^n | n \in \mathbb{N}\}$ contains 0 so $L^* = 0^*$

4. The class of non-regular languages is closed under complementation.
   True because regular languages are.