

## Two-stage Wiener filter based cancellation receivers for DS-CDMA

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*Indexing terms:* Code division multiple access, Receivers

The authors present the two-stage Wiener filter based cancellation receiver for DS-CDMA, i.e. a receiver using Wiener filters to regenerate the interferers and to make the final despreading. Theoretical calculation as well as simulation results will show that the two-stage Wiener filter performs considerably better than any other two-stage receiver.

**Introduction:** Of the three sources of distortion in a DS-CDMA system, namely Gaussian noise, time dispersion due to multipath propagation and multiple access interference (MAI), the third one is the most structured and should therefore be the easiest to overcome. A fairly successful idea to achieve this was the cochannel interference cancellation proposed in [1]. There, the suggested way was to use a bank of digital matched filters (DMF) to regenerate the interferers, subtract their sum (cancellation) and finally use another DMF to get the desired user. The analysis of a simple Wiener filter bank to extract the different users (without cancellation) was presented in [2]. Another paper then analysed the case of a Wiener filter bank to regenerate and subsequently cancel with a DMF as the final despreader [3]. Several MMSE interference suppression schemes were presented and analysed in [4]. Our paper now analyses the use of a Wiener FIR filter for both stages of a two-stage DS-CDMA cancellation receiver. We will show that the use of Wiener filters throughout the system leads to a remarkably better system performance than using a DMF in any position of the system. Obviously, it is then possible to approximate the Wiener filters in a practical system by using adaptive filter algorithms.

**The considered system:** We are interested in the decomposition of a baseband (unmodulated) signal  $y$  that is the sum of the spread outputs of  $M$  sources, the users in the system, plus additive white Gaussian noise. We assume binary  $\{+1, -1\}$  for both the data and the spreading sequences. Each of the  $M$  involved users has a spreading sequence

$$\mathbf{c}_m = (c_{m1}, c_{m2}, \dots, c_{mN}) \quad (1)$$

The spreading sequences are randomly generated to have no special crosscorrelation properties. Furthermore, the sources are assumed to be mutually linearly independent, and so is the additive white Gaussian noise with respect to the signals. One source data bit is spread to exactly  $N$  chips on the channel. The system reconstructs the  $M-1$  interferers and subtracts their sum from  $y$  (cancellation); then the desired user is retrieved.

**Analysis of the primary signal:** We deconvolve the signal  $y$  only every  $N$ th chip. Therefore, the optimisation criterion for user  $d$  is given by

$$MSE^{(N)} = E[(x_d[\nu N] - \hat{x}_d[\nu N + (N-1)])^2] \quad (2)$$

where  $\hat{x}_d$  is the estimation made with the FIR filter used. By  $\nu$ , we count the (much slower) sequence of data bits, i.e.  $x_d[\nu N]$  is the  $\nu$ th data bit of user  $d$ . The autocorrelation matrix of  $y$  with respect to eqn. 2 is

$$(\mathbf{R})_{ij} = E[y[\nu N + (i-1)]y[\nu N + (j-1)]] \quad (3)$$

which evaluates to

$$(\mathbf{R})_{ij} = \sum_{m=1}^M c_{mi}c_{mj} + \sigma^2\delta_{ij} \quad (4)$$

where  $\sigma^2$  is the noise variance. The crosscorrelation vector for user  $d$  is then given by

$$(\mathbf{r}_d)_i = c_{di} \quad (5)$$

Thus, according to general Wiener FIR theory, the optimal (Wiener) FIR filter of length  $N$  for user  $d$  is given by

$$\mathbf{w}_d = \mathbf{R}^{-1}\mathbf{r}_d \quad (6)$$

and the corresponding minimum mean-square error by

$$MMSE_d^{(N)} = E[x_d^2] - \mathbf{w}_d^T \mathbf{r}_d \quad (7)$$

Note that according to the assumptions made on the data signals, the quadratic expectation for the source output signal evaluates to 1. Assuming a Gaussian shape of the interference distribution at the output of the FIR filter, we find a probability of error of

$$P_{e,d} = \text{erf} \left( \sqrt{\frac{1 - MMSE_d^{(N)}}{MMSE_d^{(N)}}} \right) \quad (8)$$

**Postcancellation statistics:** For the signal after the cancellation  $y'$  we then calculate the statistics with respect to the decimation-by- $N$  as in eqn. 2,

$$(\mathbf{R}'_d)_{ij} = E[y'_d[\nu N + (i-1)]y'_d[\nu N + (j-1)]] \quad (9)$$

Let  $x_m$  denote  $x_m[\nu N]$ , the output of source  $m$  at time  $\nu$ ,

$$\begin{aligned} (\mathbf{R}'_d)_{ij} &= E \left[ \left( x_d c_{di} - \sum_{m \neq d} c_{mi}(\hat{x}_m - x_m) \right) \right. \\ &\quad \left. \times \left( x_d c_{dj} - \sum_{m \neq d} c_{mj}(\hat{x}_m - x_m) \right) \right] + \sigma^2 \delta_{ij} \\ &= E[x_d c_{di} x_d c_{dj}] \\ &\quad + \sum_{m \neq d} c_{mi} c_{mj} E[(\hat{x}_m - x_m)(\hat{x}_m - x_m)] + \sigma^2 \delta_{ij} \\ &= c_{di} c_{dj} + 4 \sum_{m \neq d} P_{e,m} c_{mi} c_{mj} + \sigma^2 \delta_{ij} \end{aligned} \quad (10)$$

where the probability of error for user  $m$ ,  $P_{e,m}$ , is calculated according to eqn. 8. The crosscorrelation vector (for user  $d$ ) remains unchanged. Thus we get the postcancellation Wiener FIR filter:

$$\mathbf{w}'_d = \mathbf{R}'_d^{-1} \mathbf{r}_d \quad (11)$$

The corresponding minimum mean-square error is

$$MMSE_d^{(N)'} = E[x_d^2] - \mathbf{w}'_d{}^T \mathbf{r}_d \quad (12)$$

Again, we can theoretically predict the probability of bit error. Assuming Gaussian distribution, we get

$$P_{e,d} = \text{erf} \left( \sqrt{\frac{1 - MMSE_d^{(N)'}}{MMSE_d^{(N)'}}} \right) \quad (13)$$

**Theory against simulation:** Fig. 1 shows theoretical and simulation results for three receiver structures. The top two curves are for a Wiener filter receiver with no cancellation. 'TheoWF' is eqn. 8, 'SimuWF' is the result of a simulation using a filter according to eqn. 6. The second two curves are for Wiener filter based cancellation with a DMF for final despreading. 'TheoWFDMF' is the theoretical prediction (assuming Gaussian interference distributions throughout), and 'SimuWFDMF' is the simulated result. The final two curves are for our new receiver structures. 'SimuWFWF' is the result of a simulation using optimal (Wiener) filters throughout the system, namely our two-stage Wiener filter receiver, and 'TheoWFWF' is eqn. 13.

The remarkable difference between the theoretical calculation and the actual performance of the cancellation receivers is due to the Gaussian assumption in the prediction. The probability distribution of the sum signal of a quite large number of involved users can be approximated by a Gaussian shape according to the central limit theorem, which is illustrated by the fact that, in Fig. 1, the curves 'TheoWF' and 'SimuWF' are almost equal. However, the signal after the cancellation has a limited number of high power interferers and therefore the central limit theorem is no longer applicable.

Fig. 2 shows simulated system performance against the number of users for three different two-stage cancellation receivers. The receiver structures simulated are cancellation and despreading based on matched filters ('SimuDMFDMF'), cancellation based on Wiener filters followed by despreading using matched filters ('SimuWFDMF') and our new structure which uses Wiener filters throughout ('SimuWFWF'). This Figure clearly shows the higher user capacity offered by the new structure.

# Improvement of the majority gate algorithm for grey scale dilation/erosion

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*Indexing terms: Mathematical morphology, Computer vision*

An improvement of the majority gate algorithm suitable for grey scale morphological operations is presented in the Letter. The redundancy of temporal signals led to a simplified hardware implementation. It is shown that max/min operators can be computed by the same circuit. A new pipelined systolic array architecture based on this circuit is illustrated for dilation/erosion operations.

**Introduction:** Mathematical morphology offers a unified and powerful approach to numerous image processing problems, such as shape extraction, noise cleaning and object selection according to their size distribution [1]. The two most basic morphological operations are dilation and erosion. Grey scale dilation is defined as the maximum of the sums of the image window pixels and the corresponding structuring element pixels [2]. Similarly grey scale erosion is the minimum of the differences. Morphological image processing machines, such as the cytocomputer and the CLIP array processors have been built [3]. Speed can be significantly improved by using the threshold decomposition technique, but hardware cost becomes prohibitively expensive as the resolution of the image increases [4]. Recently, a new algorithm based on the majority gate median filtering algorithm [5] has been proposed in [6]. This is basically a bit-sliced algorithm where the different bits of each addition/subtraction result are processed by different processing elements (PEs). Also, the max/min computation is achieved by different PEs.

An improvement of the majority gate algorithm suitable for grey scale morphological operations has been proposed in this Letter. The redundancy of temporal signals led to a simplified hardware implementation of the PE. Additionally, it has been shown that there is no need for different PEs to calculate max/min operators. Both operators can be computed by the same PE. This circuit, combined with an adder/subtractor circuit, can implement the operations of dilation and erosion. As dilation and erosion usually operate on the image in sequence it becomes clear that in order to implement any other morphological operation, there is no need to perform them simultaneously.

**The extension of the majority-gate algorithm:** To describe the proposed algorithm the notations and the definitions found in [6] have been adopted. Let  $y$  be the output signal of the max/min operator and  $n$  the window size of the operator. Then, the operator has  $n$  inputs  $b_i$ . Also,  $o_j$  is the  $j$ th bit of the binary representation of  $y$ .

It has been shown that [6]:

$$o_j = (t_{1,j}) \text{ OR } (t_{2,j}) \text{ OR } \dots \text{ OR } (t_{n,j}) \quad (1)$$

for the max operator and

$$o_j = (t_{1,j}) \text{ AND } (t_{2,j}) \text{ AND } \dots \text{ AND } (t_{n,j}) \quad (2)$$

for the min operator where  $t_{i,j}$  is a temporal signal defined as:

$$t_{i,j} = (f_{i,j}) \text{ AND } (b_{i,j}) \quad (3)$$

for the max operator and

$$t_{i,j} = (f_{i,j}) \text{ OR } (b_{i,j}) \quad (4)$$

for the min operator, and  $f_{i,j}$  is the flag signal indicating whether  $b_i$  remains within the set of the maxima or the minima:

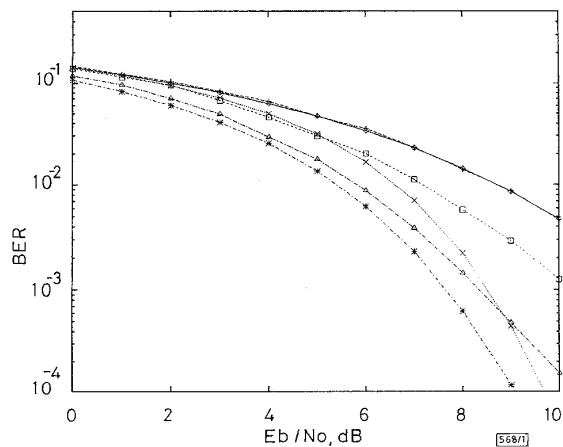
$$f_{i,j+1} = (f_{i,j}) \text{ AND } [(t_{i,j}) \text{ XNOR } (o_j)] \quad (5)$$

for the max operator and

$$f_{i,j+1} = (f_{i,j}) \text{ OR } [(t_{i,j}) \text{ XOR } (o_j)] \quad (6)$$

for the min operator.

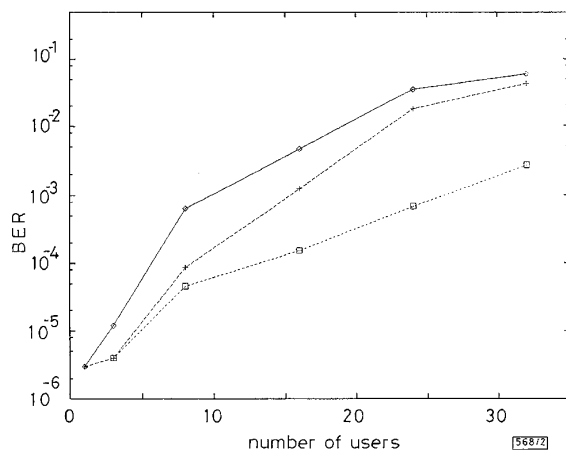
The following improvements have been made to the previous briefly described technique. By substituting eqn. 3 into eqn. 5 and using Boolean algebra,



**Fig. 1** BER against  $E_b/N_0$  for 16 active users with 31 chip spreading sequences

Each point representing a simulation result is based on at least 1000 errors

◇ TheoWFWF, + SimuWFWF, □ SimuWFDWMW, × TheoWFDWMF, △ SimuWFWF, \* TheoWFWF



**Fig. 2** BER against number of active users at  $E_b/N_0 = 10\text{dB}$  with 31 chip spreading sequences

Each point representing a simulation result is based on at least 1000 errors

◇ SimuDMFDMF, + SimuWFDWMF, □ SimuWFWF

**Conclusion:** We have shown that, in a stationary environment, the use of a two-stage Wiener filter receiver results in a considerably better system performance than any other two stage cancellation receiver. In practice, the Wiener solution can be approximated by one of the adaptive algorithms.

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1 March 1996

Electronics Letters Online No: 19960558

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