THROUGHPUT OPTIMAL CONTROL OF COOPERATIVE RELAY NETWORKS

MSRI WORKSHOP
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Control of Stochastic Wireless Networks

- Network/MAC layer - stochastic traffic, throughput, delay.
- Physical layer - channel modelling, modulation, coding.
- Integrated power control, rate allocation, scheduling, and routing.
- Previous work: Tassiulas and Ephremides (92, 93), Yeh (02), Neely, Modiano, and Rohrs (02, 03), Yeh and Cohen (03, 04), Yeh (04).

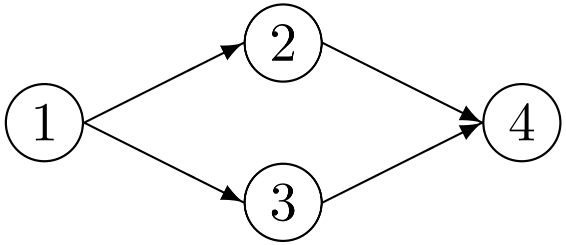
Cooperative Wireless Networks

- Previously: Nodes send independent packets.
- Cooperative relay: multiple nodes form distributed antenna array to relay packet.
- Gains in achievable rate and diversity.
- Physical-layer viewpoint: Schein and Gallager (00), Sendonaris, et al. (03), Laneman et al. (04), Host-Madsen and Zhang (05).

Our Approach

- Cooperative communication with stochastic traffic arrivals and queueing dynamics.
- Focus on parallel Gaussian relay channel model.
- Network stability region.
- Throughput optimal rate allocation and routing strategy.
- New queue coupling effects.
- Cooperative gains vs. temporarily increased traffic.
- Results for 2-parallel relay channel in *Proc. ISIT 2005*.

Parallel 2-Relay Channel



- Traffic originates at 1, 2, 3. Destination is 4.
- Each node has power constraint P.
- $1 \to 2, 3$: broadcast channel.
- $2, 3 \to 4$: multiple-access channel.
- No direct transmission from 1 to 4.
- Complete capacity region unknown.

Broadcast and Multiple-access Channels

- Gaussian broadcast channel $(1 \rightarrow 2, 3)$

$$Y_i(t) = \sqrt{h_{1i}} X_1(t) + Z_i(t), \quad i = 2, 3$$

  $Z_i(t)$ WGN with density $N_0/2$.

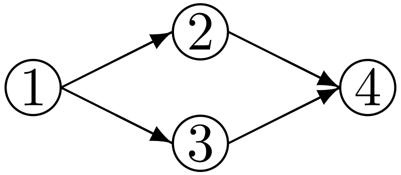
- Gaussian multiple-access channel $(2, 3 \rightarrow 4)$

$$Y_4(t) = \sum_{i=2,3} \sqrt{h_{i4}} X_i(t) + Z_4(t),$$

  $Z_4(t)$ WGN with density $N_0/2$.

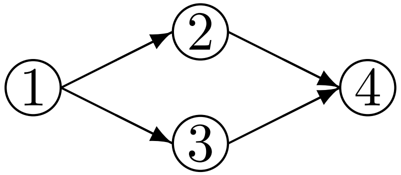
- Normalize: $W = 1, N_0 W = 1$.
- Assume symmetric network: $h_{12} = h_{13} = h_{24} = h_{34} = 1$.

Half Duplex and Coherent Gain



- Half-duplex constraint: cannot transmit and receive at same time.
- If $X_1(t) > 0$ then $X_2(t) = 0$, $X_3(t) = 0$. If $X_2(t) > 0$ or $X_3(t) > 0$ then $X_1(t) = 0$.
- Network in either broadcast or multiple-access mode.
- Nodes 2 and 3 decode and forward.
- Cooperative relay by coherent beamforming: nodes 2, 3 transmit at power $P \Rightarrow$ received power at 4 is $4P$.

Stochastic Traffic and Relaying



- Exogenous stochastic traffic arrives at 1, 2, 3 according to $A_i(t)$.
- Packet lengths Z_i i.i.d. $E[Z_i] < \infty$, $E[Z_i^2] < \infty$.
- Traffic at 1 treated as either cooperative traffic or direct traffic.
- Cooperative traffic: node 1 sends same packet to both 2 and 3, which forward packet to 4 by coherent beamforming.
- Direct traffic: node 1 sends traffic to either 2 or 3, which relays to node 4.
- All exogenous traffic arriving at 2 and 3 are direct traffic.

Cooperative and Direct Traffic

- No incoming traffic at 2 and 3 $\Rightarrow$ 1 send only cooperative traffic.
- If traffic at 2 $\gg$ traffic at 3, 1 sends some traffic directly through 3.

Queue States and Rate Allocation

- $U_1(t)$ = unfinished work at node 1.
- $U_{id}(t)$ and $U_{ic}(t)$ are unfinished work of direct and cooperative traffic at node $i = 2, 3$.
- $U_{2c}(t) = U_{3c}(t) \equiv U_c(t)$.
- Joint queue state $\boldsymbol{U}(t) = (U_1(t), U_c(t), U_{2d}(t), U_{3d}(t))$.
- Given $\boldsymbol{U}(t)$, network controller specifies rate allocation $\boldsymbol{R}(t) = (R_1^c, R_{12}^d, R_{13}^d, R_4^c, R_{24}^d, R_{34}^d)$.

Broadcast Channel Capacity Region

- $\mathcal{C}_{BC}$ = information-theoretic capacity region of the 2-user Gaussian (degraded) broadcast channel.
- Cooperative traffic from 1 to 2, 3 represents common information in broadcast channel.
- $(R_1^c, R_{12}^d, R_{13}^d)$ achievable if $(R_{12}^d + R_1^c, R_{13}^d) \in \mathcal{C}_{BC}$.
- Cooperative broadcast region $\mathcal{C}_{CBC}$ = set of nonnegative rates $(R_1^c, R_{12}^d, R_{13}^d) \equiv (R_1, R_2, R_3)$ satisfying

$$\sum_{i=1}^{3} R_i \leq \log\left(1 + P\right).$$

Multiple-access Channel Capacity Region

- Nodes 2, 3 each devotes αP to cooperative traffic, and $(1-\alpha)P$ for direct traffic, $\alpha \in [0,1]$.
- Can achieve any $(R_4^c, R_{24}^d, R_{34}^d) \equiv (R_4, R_5, R_6)$ satisfying

$$\sum_{i \in \mathcal{S}} R_i \leq \log \left(1 + \sum_{i \in S} P_i(\alpha) \right) \quad \forall S \subseteq \{4, 5, 6\},$$

where $P_4(\alpha) = 4\alpha P$, $P_5(\alpha) = (1-\alpha)P$, $P_6(\alpha) = (1-\alpha)P$.

- This defines capacity region $\mathcal{C}_{CMAC}(\alpha)$ for fixed α.
- Cooperative multiple-access region $\mathcal{C}_{CMAC} = \bigcup_{\alpha \in [0,1]} \mathcal{C}_{CMAC}(\alpha)$.
- $\mathcal{C}_{CMAC}$ is convex.

Overall Physical Layer Capacity Region

- $\overline{\mathcal{C}}_{CBC}, \overline{\mathcal{C}}_{CMAC} = \mathcal{C}_{CBC}, \mathcal{C}_{CMAC}$ embedded in $\mathbb{R}_+^6$.
- $(R_1^c, R_{12}^d, R_{13}^d) \in \mathcal{C}_{CBC}$ iff $(R_1^c, R_{12}^d, R_{13}^d, 0, 0, 0) \in \overline{\mathcal{C}}_{CBC}$.
- $(R_4^c, R_{24}^d, R_{34}^d) \in \mathcal{C}_{CMAC}$ iff $(0, 0, 0, R_4^c, R_{24}^d, R_{34}^d) \in \overline{\mathcal{C}}_{CMAC}$.
- Half duplex constraint ⇒ overall capacity region
  $\mathcal{C} = \text{conv}(\overline{\mathcal{C}}_{CBC}, \overline{\mathcal{C}}_{CMAC})$.

Network Stability Region

- $\lambda_i = \lim_{t\to\infty} A_i(t)/t$ = packet arrival rate at node i.
- $\rho_i = \lambda_i \mathsf{E}[Z_i]$ = bit arrival rate.
- Queue i is stable if $\limsup_{t\to\infty} \frac{1}{t}\int_0^t 1_{[U_i(\tau)>\xi]}d\tau \to 0$ as $\xi \to \infty$.
- Network stability region $\mathcal{S}$ = closure of set of (ρ_1, ρ_2, ρ_3) for which there exists feasible rate allocation policy $\mathcal{R}$ defined by $\boldsymbol{R} = \mathcal{R}(\boldsymbol{u}) \in \mathcal{C}$, which keeps all queues stable.

Network Stability Region

Theorem 1 *$\mathcal{S}$ = set of bit arrival rates $(\rho_1, \rho_2, \rho_3) \in \mathbb{R}_+^3$ for which there exist non-negative flow variables $f^c, f_{12}^d, f_{13}^d, f_{24}^d, f_{34}^d$ which support (ρ_1, ρ_2, ρ_3) relative to the weighted graph defined by $\mathcal{C}$.*

- $\rho_1 = f^c + f_{12}^d + f_{13}^d$, $\rho_{2d} = f_{24}^d - f_{12}^d$, $\rho_{2d} = f_{34}^d - f_{13}^d$.
- $(f^c, f_{12}^d, f_{13}^d, f^c, f_{24}^d, f_{34}^d) \in \mathcal{C}$.

Throughput Optimal Rate Allocation and Routing

- Find adaptive rate allocation and routing policy to stabilize network even if $\boldsymbol{\rho} = (\rho_1, \rho_2, \rho_3)$ not known, as long as $\boldsymbol{\rho} \in \mathrm{int}(\mathcal{S})$.
- For non-cooperative multi-hop networks, Maximum Differential Backlog (MDB) policy suffices (Tassiulas and Ephremides ’92, Neely et al. ’03).
- We show a modified version, Cooperative Maximum Differential Backlog (CMDB) policy, is throughput optimal for parallel relay network.

Throughput Optimal Rate Allocation and Routing

- Look at instants separated by $T > 0$.
- T large enough for large coding lengths.
- $\boldsymbol{A}_k = (A_{1k}, A_{2k}, A_{3k})$ = arrival vector for kth T-slot.
- $\{\boldsymbol{A}_k : k \in \mathbb{Z}_+\}$ i.i.d. $\sim \pi_{\boldsymbol{A}}$.
- $\mathsf{E}[\boldsymbol{A}] = \boldsymbol{\lambda} T$, $\mathsf{E}[A_i^2] < \infty$.
- Holds for independent Poisson arrival processes.

Throughput Optimal CMDB Policy

Theorem 2 *A throughput optimal rate allocation and routing policy for parallel relay network is given by*

$$\begin{aligned}\max_{\boldsymbol{R}\in\mathcal{C}} \quad & (u_1 - 2u_c)R_1^c + (u_1 - u_{2d})R_{12}^d + (u_1 - u_{3d})R_{13}^d \\ & + u_{2d}R_{24}^d + u_{3d}R_{34}^d + 2u_cR_4^c\end{aligned}$$

- $u_1 - 2u_c$ and $2u_c$ terms reflect queue coupling effect induced by cooperative transmission.

Calculating the CMBD Policy

- $(w_1, w_2, w_3, w_4, w_5, w_6) \equiv (u_1 - 2u_c,\ u_1 - u_{2d},\ u_1 - u_{3d},\ 2u_c,\ u_{2d},\ u_{3d})$.
- $(R_1,\ R_2,\ R_3,\ R_4,\ R_5,\ R_6) \equiv (R_1^c, R_{12}^d, R_{13}^d, R_4^c, R_{24}^d)$.
- CMDB policy: $\max_{\boldsymbol{R} \in \mathcal{C}} \sum_{i=1}^{6} w_i R_i$.
- $\boldsymbol{R}^* \in \mathrm{conv}(\overline{\mathcal{C}}_{CBC}, \overline{\mathcal{C}}_{CMAC}(\alpha^*))$ for some $\alpha^* \in [0, 1]$.
- $\overline{\mathcal{C}}_{CBC}$ and $\overline{\mathcal{C}}_{CMAC}(\alpha^*)$ both convex polytopes
  $\Rightarrow$ $\mathrm{conv}(\overline{\mathcal{C}}_{CBC}, \overline{\mathcal{C}}_{CMAC}(\alpha^*))$ also convex polytope.
- $\boldsymbol{R}^*$ is (WLOG) an extreme point of $\mathrm{conv}(\overline{\mathcal{C}}_{CBC}, \overline{\mathcal{C}}_{CMAC}(\alpha^*))$, thus an extreme point either of $\overline{\mathcal{C}}_{CBC}$ or of $\overline{\mathcal{C}}_{CMAC}(\alpha^*)$.

Calculating the CMBD Policy: Case 1

- $\boldsymbol{R}^*$ an extreme point of $\overline{\mathcal{C}}_{CBC}$.
- $\boldsymbol{R}^* = (R_1^*, R_2^*, R_3^*, 0, 0, 0)$.
- $\mathcal{C}_{CBC}$ a simplex.
- Let $w_{[1]} \geq w_{[2]} \geq w_{[3]}$ be w_1, w_2, w_3 in decreasing order.
- $R_{[1]}^* = \log(1+P)$, $R_{[2]}^* = R_{[3]}^* = 0$.
- Maximum rate to stream with largest differential queue weight.
- Optimal objective:

$$\max_{\boldsymbol{R} \in \mathcal{C}} \sum_{i=1}^{6} w_i R_i = L_{CBC}^*(w_1, w_2, w_3) = \left(\max_{i=1,2,3} w_i \right) \log(1+P).$$

Calculating the CMBD Policy: Case 2

- $\boldsymbol{R}^*$ an extreme point of $\overline{\mathcal{C}}_{CMAC}(\alpha^*)$.
- $\boldsymbol{R}^* = (0, 0, 0, R_4^*, R_5^*, R_6^*)$.
- $\mathcal{C}_{CMAC}(\alpha^*)$ a polymatroid.
- Let $w_{[4]} \geq w_{[5]} \geq w_{[6]}$ be w_4, w_5, w_6 in decreasing order.
- $R_{[i]}^* = \log\left(1 + \frac{P_{[i]}(\alpha^*)}{1+\sum_{j<i} P_{[j]}(\alpha^*)}\right)$, $i = 4, 5, 6$.
- Successively decode streams in increasing order of differential queue weight.

Calculating the CMBD Policy: Case 2

- To find α^*, solve

$$\max_{\alpha\in[0,1]} \sum_{i=4}^{6} w_{[i]} \log\left(1+\frac{P_{[i]}(\alpha)}{1+\sum_{j<i} P_{[j]}(\alpha)}\right).$$

- If $w_4 \equiv 2u_c \geq w_5 \equiv u_{2d} \geq w_6 \equiv u_{3d}$, objective $L(\alpha)$ concave in α over $[0,1]$, $L'(\alpha) \geq 0\ \forall\alpha \in [0,1] \Rightarrow \alpha^* = 1$: all power allocated to cooperative transmission.

- Optimal objective:

$$L^*_{CMAC}(w_4, w_5, w_6) = \sum_{i=4}^{6} w_{[i]} \log\left(1+\frac{P_{[i]}(\alpha^*)}{1+\sum_{j<i} P_{[j]}(\alpha^*)}\right).$$

Calculating the CMBD Policy

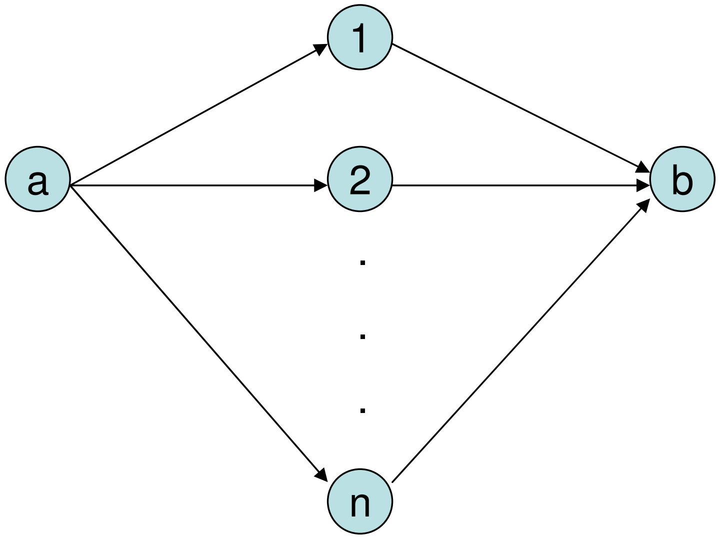
Theorem 3 *For four-node parallel relay network, If* $L^*_{CBC}(w_1, w_2, w_3) \geq L^*_{CMAC}(w_4, w_5, w_6))$*, then throughput optimal rate allocation* $\boldsymbol{R}^* = (R^*_1, R^*_2, R^*_3, 0, 0, 0)$*, where*

$$R^*_{[1]} = \log(1 + P), \quad R^*_{[2]} = R^*_{[3]} = 0.$$

Otherwise, $\boldsymbol{R}^* = (0, 0, 0,\ R^*_4, R^*_5, R^*_6)$*, where*

$$R^*_{[i]} = \log\left(1 + \frac{P_{[i]}(\alpha^*)}{1 + \sum_{j<i} P_{[j]}(\alpha^*)}\right), \quad i = 4, 5, 6.$$

Generalization to n Relays



- $h_{a1} \leq h_{a2} \leq \ldots \leq h_{an}$.
- $\mathcal{S}$ = set of all cooperation sets $\emptyset \subset S \subseteq \{1, \ldots, n\}$.
- Direct traffic: $\{i\} \in \mathcal{S}$ for all $1 \leq i \leq n$.
- $\boldsymbol{U}(t) = (U_a(t), (U_S(t))_{S \in \mathcal{S}})$.
- $\boldsymbol{R}(t) = ((R_{aS}(t))_{S \in \mathcal{S}}, (R_{Sb}(t))_{S \in \mathcal{S}})$.

General Throughput Optimal CMDB Policy

Theorem 4 *A throughput optimal rate allocation and routing policy for the n-parallel relay network is given by*

$$\max_{\boldsymbol{R}\in\mathcal{C}} \sum_{S\in\mathcal{S}} \left[(u_a - |S|u_S)R_{aS} + |S|u_S R_{Sb}\right]$$

- $\mathcal{C} = \mathrm{conv}(\overline{\mathcal{C}}_{CBC}, \overline{\mathcal{C}}_{CMAC})$.
- Applies to general asymmetric settings.
- Optimization reduces to maximization over $\mathcal{C}_{CBC}$ and $\mathcal{C}_{CMAC}(\boldsymbol{\alpha})$.

Maximization over $\mathcal{C}_{CBC}$

- Optimization:

$$\max_{(R_{aS})_{S\in\mathcal{S}}\in\mathcal{C}_{CBC}} (u_a - |S|u_S)R_{aS} \tag{1}$$

- $(R_{aS})_{S\in\mathcal{S}} \in \mathcal{C}_{CBC} \Rightarrow (R_1, \ldots, R_n) \in \mathcal{C}_{BC}$.
- $R_i = \sum_{S\in\mathcal{S}_i} R_{aS}$, where $\mathcal{S}_i = \{S \in \mathcal{S} | i = \min S\}$.
- $\mathcal{C}_{BC}$ = capacity region of Gaussian BC with independent info.
- (1) reduces to

$$\max_{(R_1,\ldots,R_n)\in\mathcal{C}_{BC}} \sum_{i=1}^{n} \max(\{u_a - |S|u_S\}_{S\in\mathcal{S}_i})R_i \tag{2}$$

- $R^*_{aS} = R^*_i$ if $S = \arg\max_{S'\in\mathcal{S}_i}(\{u_a - |S'|u_{S'}\})$;
  $R^*_{aS} = 0$ for all other $S \in \mathcal{S}_i$
  i.e. at any time, each i participates in only one cooperative set.
- Solve (2) by greedy algorithm (Tse 97, Li & Goldsmith 01).

Maximization over $\mathcal{C}_{CMAC}(\boldsymbol{\alpha})$

- Optimization:

$$\max_{((R_{Sb})_{S\in\mathcal{S}})\in\mathcal{C}_{CMAC}(\boldsymbol{\alpha})} \sum_{S\in\mathcal{S}} |S|u_S R_{Sb}.$$

- $\mathcal{C}_{CMAC}(\boldsymbol{\alpha})$ has at most $2^{2^n-1}-1$ constraints.
- Polymatroid structure: sort $(|S|u_S)_{S\in\mathcal{S}}$, successively decode in increasing order of $|S|u_S$.
- At most 2^n-1 coefficients $\Rightarrow$ can solve in $O(n)$ time.

Summary and Conclusions

- Framework for optimal control of cooperative wireless networks with random arrivals and queueing.
- Focus on parallel Gaussian relay model.
- Half duplex between broadcast and multiple-access channels.
- Networks contains both cooperative and direct traffic.
- Throughput optimal policy is Cooperative Maximum Differential Backlog (CMDB).
- CMDB incorporates cooperative gains and exhibits queue coupling.