Efficient Rumor Spreading: Cooperative Multicasting in Wireless Networks

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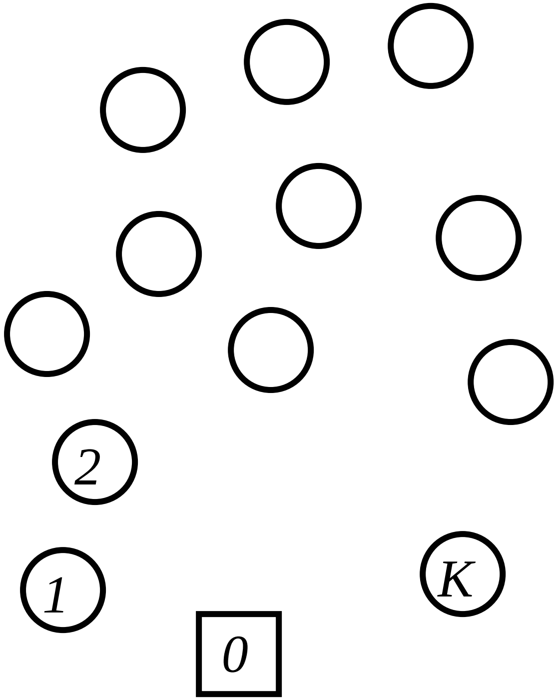
Ad-hoc Wireless Networks

- more attractive than they should be
- performance characteristics a function of
  - protocol constraints
  - topology
  - propagation models
  - nature of traffic
- traffic of interest in this talk: multicasting
  - one sender, one message, many recipients

Multicasting Model

- network of $K+1$ nodes
- node 0 is message source
- nodes $1,2,\ldots,K$ are recipients
- transmission successful if all recipients can decode
- cf. unicasting: one of the recipients can decode



Channel Model

- narrowband, slow-fading channel model
- length n message of $\log M$ bits
- node k at time $1 \le i \le n$ receives

$$y_k(i) = \sum_{j \in \mathcal{T}(i)} h_{jk} x_j(i) + z_k(i)$$

  where $x_j(i)$ is the signal sent by node j
- multipath rich: h_{jk} i.i.d. $N(0,1)$
- noise $z_k(i)$ i.i.d. $N(0, N_0)$
- channels known at receivers, not at transmitters: node k knows h_{jk}

System Model – Protocol Constraints

- sum power constraint:

$$E\left[\frac{1}{n}\sum_{i=1}^{n}\sum_{k\in\mathcal{T}(i)}\left|X_k(i)\right|^2\right]\leq P$$

- half-duplex constraint
- nodes cannot revert to receive mode after entering transmit mode
  – precludes protocols in which transmitters learn their channels
- node *0* starts in transmit mode, others in receive mode
- cooperation-free protocol: only source node *0* transmits

Main Results

Definition: Rate R is achievable if there exists a sequence of rate R protocols, indexed by network size K such that

$$\Pr[\text{all nodes can decode}] \to 1 \text{ as } K \to \infty$$

Definition: The capacity is $C = \sup R$

Theorem: The multicasting capacity is $C = \log\left(1 + \frac{P}{N_0}\right)$

Corollary: The unicasting capacity is the same.

Corollary: The capacity without cooperation is $C_{\text{nc}} = 0$.

Coding Theorem Converse

Lower bound on outage probability from MISO channel:

- genie conveys message to nodes $1,2,\ldots,K-1$
- nodes $0,1,\ldots,K-1$ act as a virtual array to send to node K

Theorem (Boche-Jorswieck): outage capacity achieved with input covariance

$$\mathrm{diag}(P_1, P_2, \ldots, P_K),$$

$$P_1 = P_2 = \cdots = P_{K_0} = \frac{P}{K_0}, \quad P_{K_0+1} = P_{K_0+2} = \cdots = P_K = 0, \quad \text{some } K_0$$

Implication:

$$R > \log\left(1 + \frac{P}{N_0}\right) \quad \Rightarrow \quad \lim_{K\to\infty} \Pr\left(\text{error}\right) > 0$$

Coding Theorem Achievability

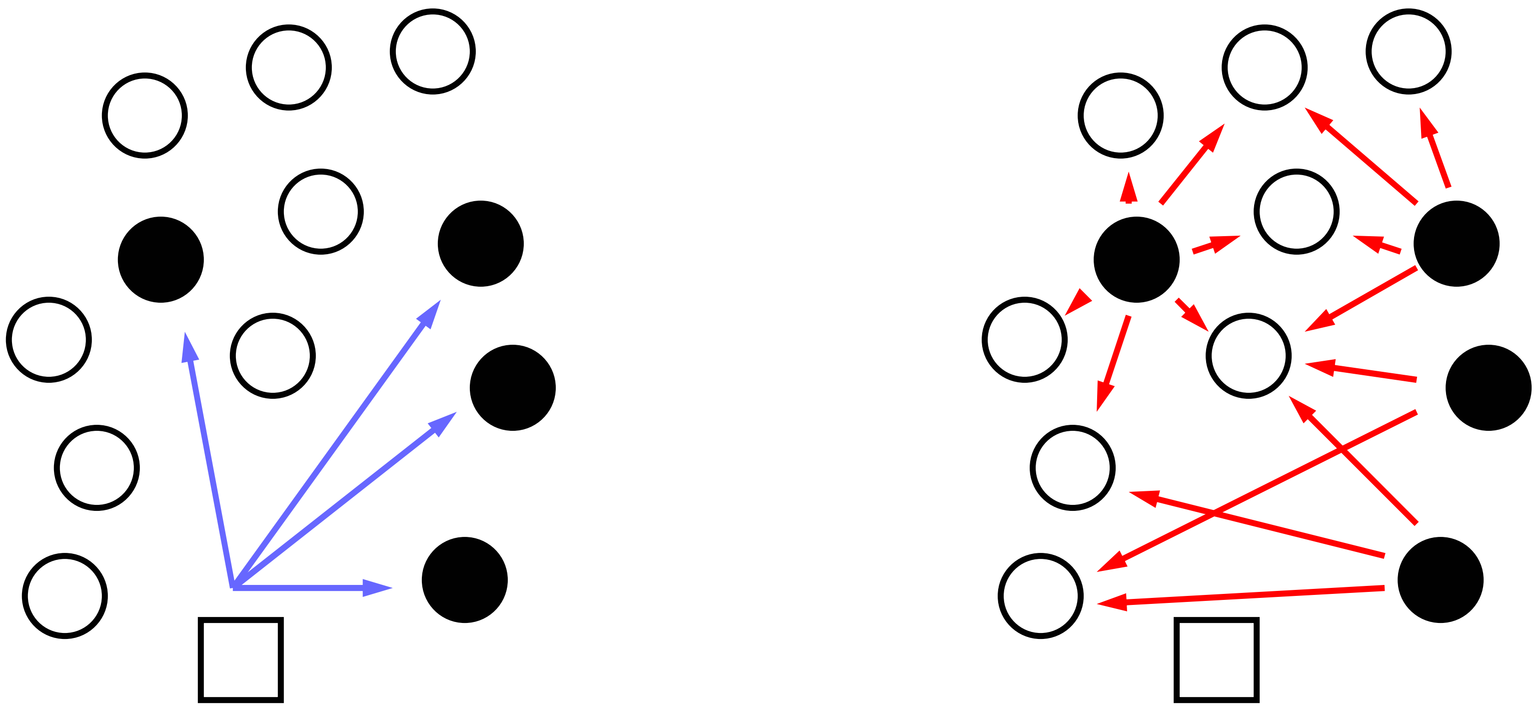
Simple 2-phase protocol:

1. source uses n_1 channel uses to broadcast message; all other nodes attempt to decode.
2. the K_1 nodes that can decode form a virtual array, retransmitting over remaining $n_2 = n - n_1$ channel uses; remaining nodes attempt to decode.

Codebook generation:

- source node generates $(\log M, n_1)$ code using codebook with i.i.d. $N(0, P_1)$ entries
- other nodes generate independent $(\log M, n_2)$ codes using codebook with i.i.d. $N(0, P_2)$ entries

2-Phase Protocol



Phase 1 Phase 2

Rate Analysis

1. Choose $R_1(\alpha) = \log\left(1 + \frac{P_1}{N_0}\log\frac{1}{\alpha}\right)$

   where α is target fraction of nodes

   - $P_1 = P$ meets power constraint

2. Choose

   $$R_2(\beta) = \log\left(1 + (1-\beta)\frac{P_2}{N_0}\right)$$

   where β is a design parameter

   - $E[K_1] = \alpha K \;\Rightarrow\; P_2 = \frac{P}{\alpha K}$ meets power constraint

Effective Rate: $\frac{1}{R} = \frac{1}{R_1(\alpha)} + \frac{1}{R_2(\beta)}$

Outage Probability Analysis

- From MISO analysis, outage event is

$$\frac{1}{\alpha K}\sum_{j=1}^{K_1}\left|h_{jk}\right|^2 < 1-\beta$$

- obtain Chernoff bound:
  - outage probability decays exponentially with αK
  - identical behavior for unicasting (union bound)

Parameter Selection

1. choose α small so $R_1 \to \infty$ and $R \to R_2$

2. choose β small so $R_2 \to C$

3. choose αK large so that $\Pr[\text{outage}] \to 0$

Example: $\alpha \sim \frac{1}{\log K}, \ \beta \sim \frac{1}{K}$

Intuition: for large network, can find many nodes with large gains so that can be served with negligible channel uses, then give large MISO diversity gain

Multiple Antenna Extensions

Case of interest: Node k has T_k antennas

Unicasting (destination node $k=K$): $C_{\mathrm{uc}} \geq T_K \log\left(1+\frac{P}{N_0}\right)$

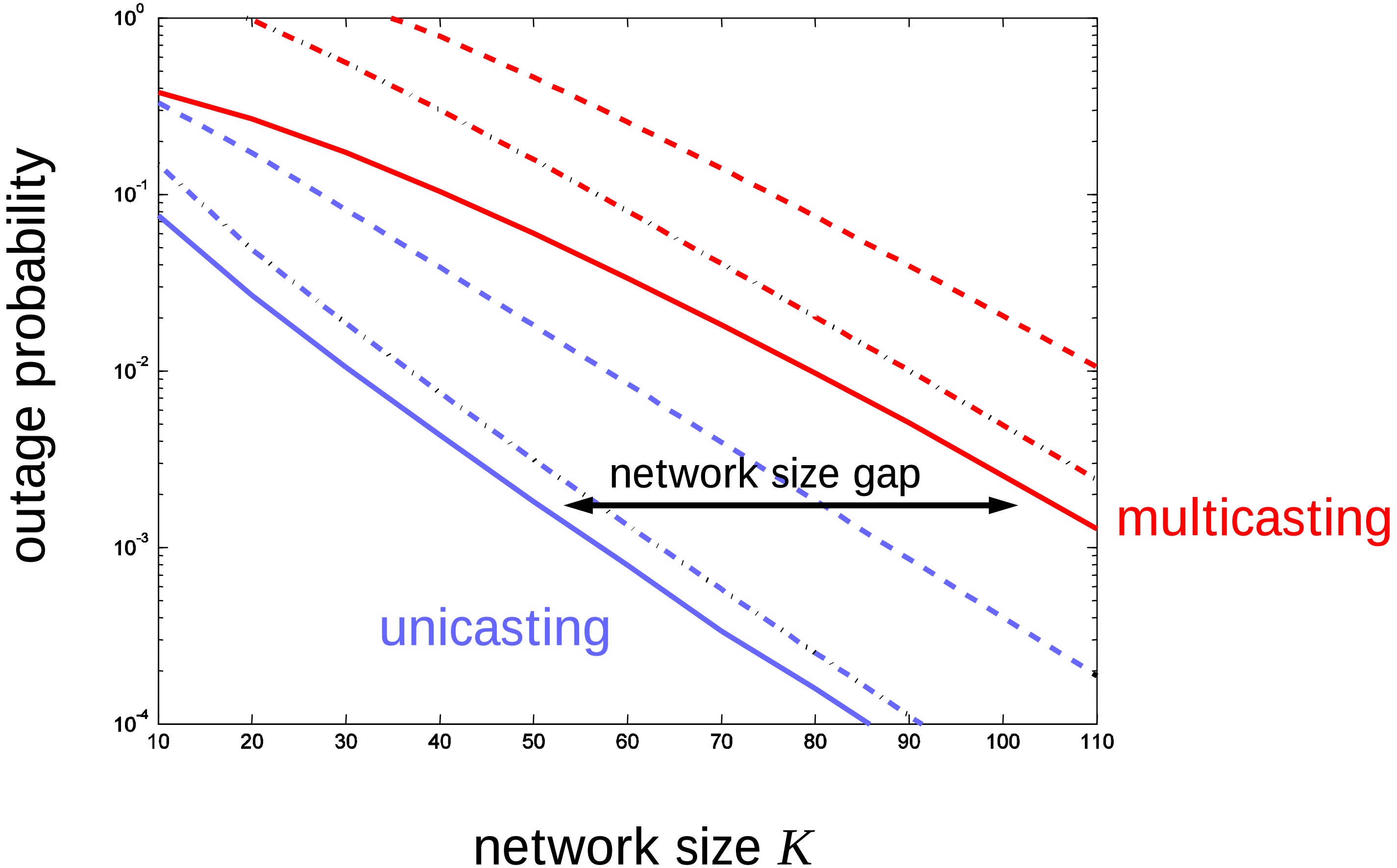
- use MIMO on Phase 2; source and relay node antenna size unimportant

Multicasting: $C_{\mathrm{mc}} \geq \min(T_1, T_2, \ldots, T_K) \log\left(1+\frac{P}{N_0}\right)$

- node with fewest antennas limits rate
- lower bounds tight if Teletar conjecture true in general (MIMO)

Scaling Behavior of Protocol

$$R = \frac{1}{2}\log\left(1+\frac{P}{2N_0}\right) = \frac{1}{2}R_1(\alpha) = \frac{1}{2}R_2(\beta)$$



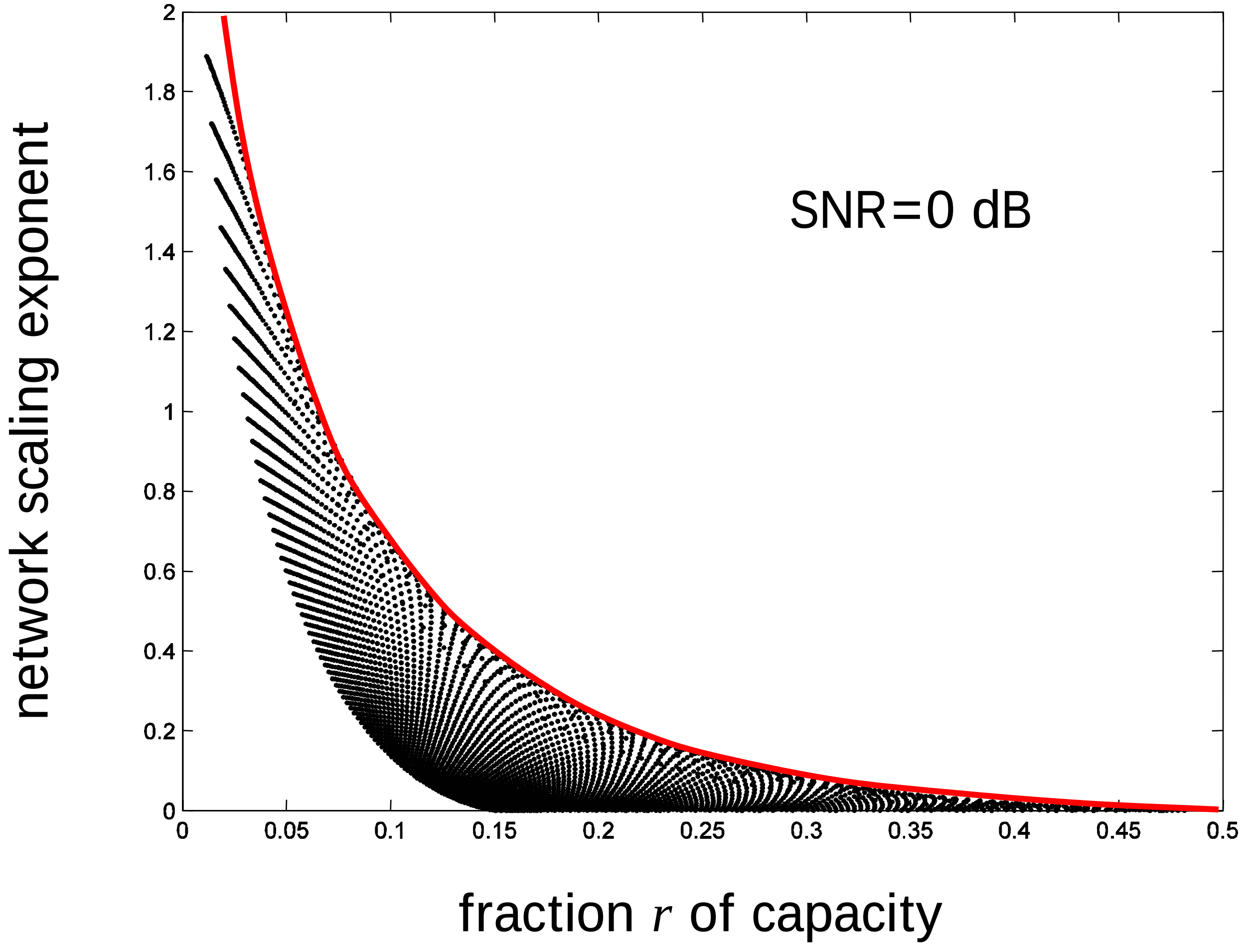
Network Scaling Exponent

- can define reliability function with respect to network size K
- can define network scaling exponent as supremum

Fact: network scaling exponent same for both unicasting and multicasting

Fact: 2-stage protocol has poor reliability function

2-Phase Protocol Reliability Function



Towards a Better Reliability Function...

- MISO channel gives upper bound on exponent
  - easy to calculate
  - corresponds to serving $\sim K$ users in Phase 1
- 2-phase protocol serves $K / \log K$ users in Phase 1 (with neglible channel uses, i.e., opportunistically)
  - Phase 2 results in exponent of zero
- 3-phase protocol: repeat Phase 2, engaging all successful users
  - Phase 2 serves $\sim K$ users, so protocol achieves MISO exponent
  - Effective rate halved: $R=C/2$
  - better than reliability-optimized 2-phase protocol
  - can avoid rate loss with simple feedback after Phase 2

(...ongoing broader collaboration including Lizhong Zheng)

Concluding Remarks

- can get reliable communication in large network size limit, any SNR
  - cf. large SNR (Zheng-Tse) analysis, any network size
- simple 2-phase protocol is capacity-achieving
  - doesn’t achieve network scaling exponent
- worth trying to understand exponent better
- worth extending to other channel models
- worth investigating impact of individual power constraints