RESOURCE ALLOCATION FOR ORTHOGONAL FADING RELAY CHANNELS

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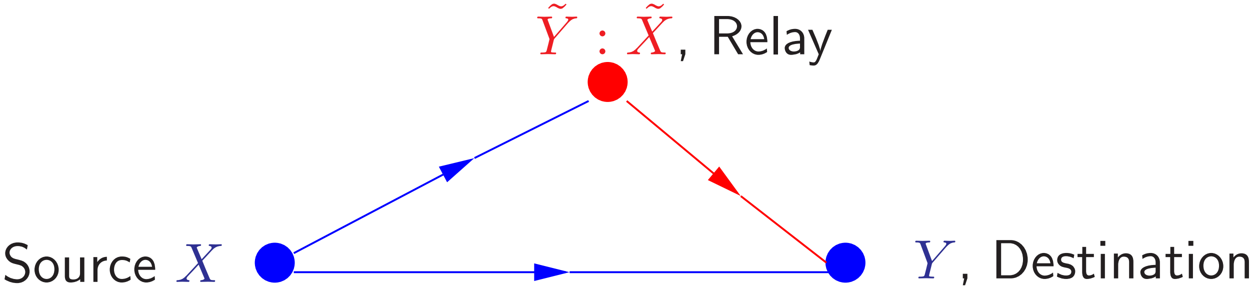
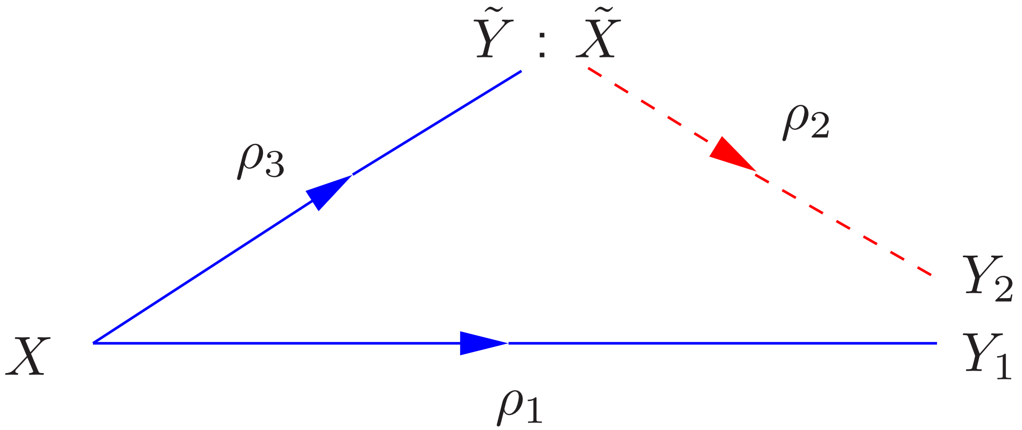
Orthogonal Relay Channels



Three-Terminal Relay Channel

- Classical Relay Channel
    - Allow relay to transmit and receive simultaneously
- Practical Constraints
    - Perfect echo cancellation at relay is not easy to implement
    - Orthogonalize transmitted and received signals at relay
- Our Work
    - Orthogonal relay channel

Gaussian Orthogonal Relay Model

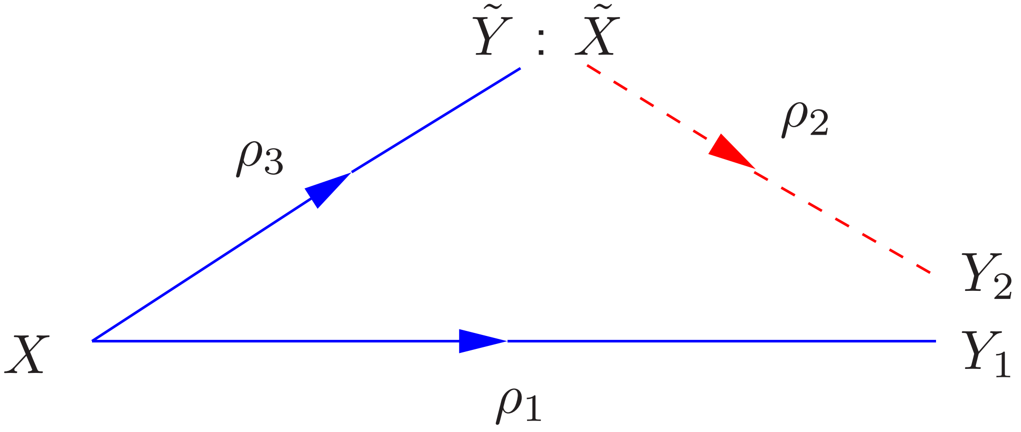


- Total channel resource $2W$ dim/sec is split into channels 1 and 2
- Realized by splitting time, bandwidth, or joint time-frequency space
- Source transmits in channel 1 and relay transmits in channel 2
- Parameter θ: fraction of channel resource allocated to channel 1

Previous Work on Orthogonal Relay Models

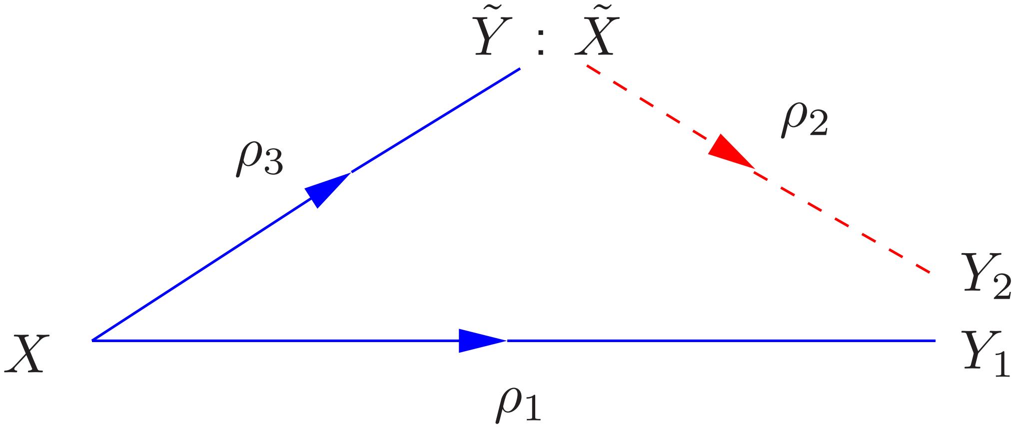
- Laneman, Tse and Wornell, IT'04: Cooperative Diversity
- El Gamal, Mohseni and Zahedi, IT'submitted: Bounds on Capacity
- Avestimehr and Tse, ISIT'05: Outage Capacity at Low SNR
- Nabar, Bolcskei and Kneubuhler, JSAC'04: Space-Time Signal Design
- Azarian, El Gamal and Schniter, IT'05: Achieving Diversity-Multiplexing Tradeoff
- Many Others ......

Our Results for Gaussian Orthogonal Relay Channel



- Published in IT, Sept 05
- Allow channel resource allocation parameter θ to be variable that can be optimized
- Optimize lower bound (based on decode-and-forward strategy) over θ
- Lower bound equals max-flow min-cut upper bound at optimizing θ $\Longrightarrow$ common value is capacity at optimizing θ
- Capacity at optimizing θ is also maximum capacity when relay to destination SNR (ρ_2) is less than a threshold

Optimizing θ (details)

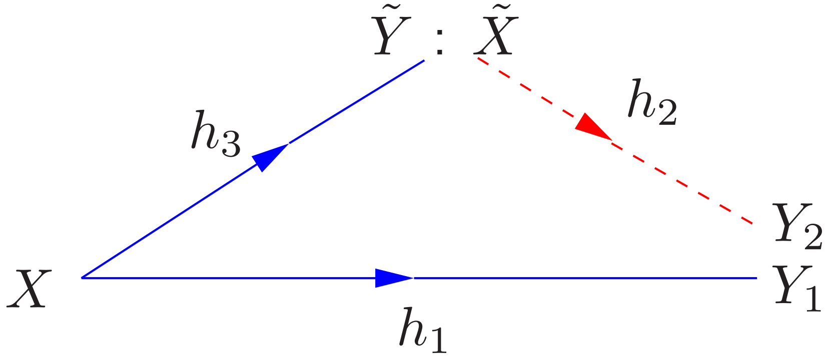


$$C_{\texttt{low}}(\theta) = \min\left\{\theta\mathcal{C}\left(\frac{\rho_1}{\theta}\right) + \bar{\theta}\mathcal{C}\left(\frac{\rho_2}{\bar{\theta}}\right),\ \theta\mathcal{C}\left(\frac{\rho_3}{\theta}\right)\right\} \text{ bps/Hz}$$

$$C_{\texttt{up}}(\theta) = \min\left\{\theta\mathcal{C}\left(\frac{\rho_1}{\theta}\right) + \bar{\theta}\mathcal{C}\left(\frac{\rho_2}{\bar{\theta}}\right),\ \ \theta\mathcal{C}\left(\frac{\rho_3+\rho_1}{\theta}\right)\right\} \text{ bps/Hz}$$

where $\mathcal{C}(x) := \frac{1}{2}\log(1+x)$

Orthogonal Relay Model with Fading



$$Y_1 = \sqrt{\rho_1}\, h_1 X + Z_1$$

$$Y_2 = \sqrt{\rho_2}\, h_2 \tilde{X} + Z_2$$

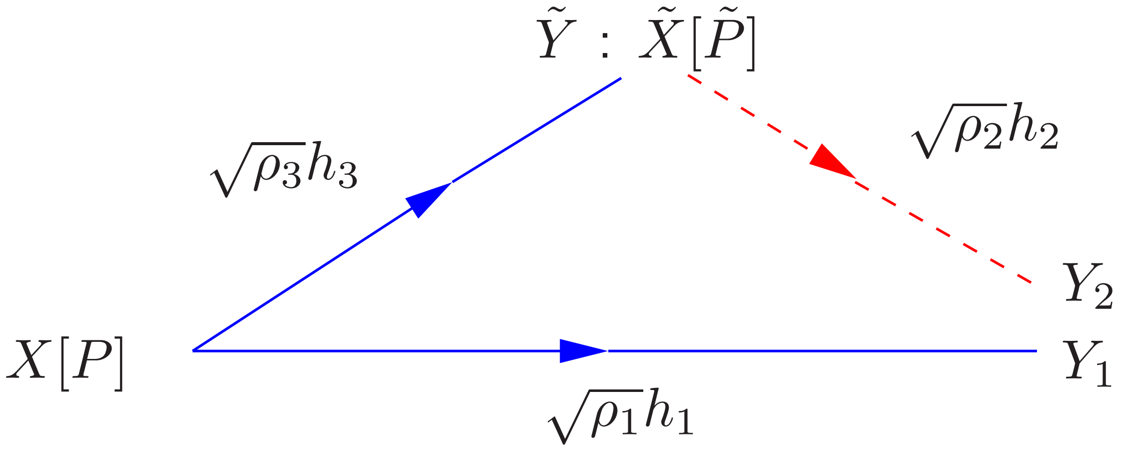
$$\tilde{Y} = \sqrt{\rho_3}\, h_3 X + \tilde{Z}$$

- h_1, h_2, h_3: independent proper complex, zero mean, variance 1
- $\underline{h} = (h_1, h_2, h_3)$: ergodic and stationary vector fading process over time
- $Z_1, Z_2, \tilde{Z}$: independent $\mathcal{CN}(0,1)$
- ρ_1, ρ_2, ρ_3: link gain to noise ratios
- $X, \tilde{X}$: subject to average power constraints $\mathbb{P}, \tilde{\mathbb{P}}$

Background

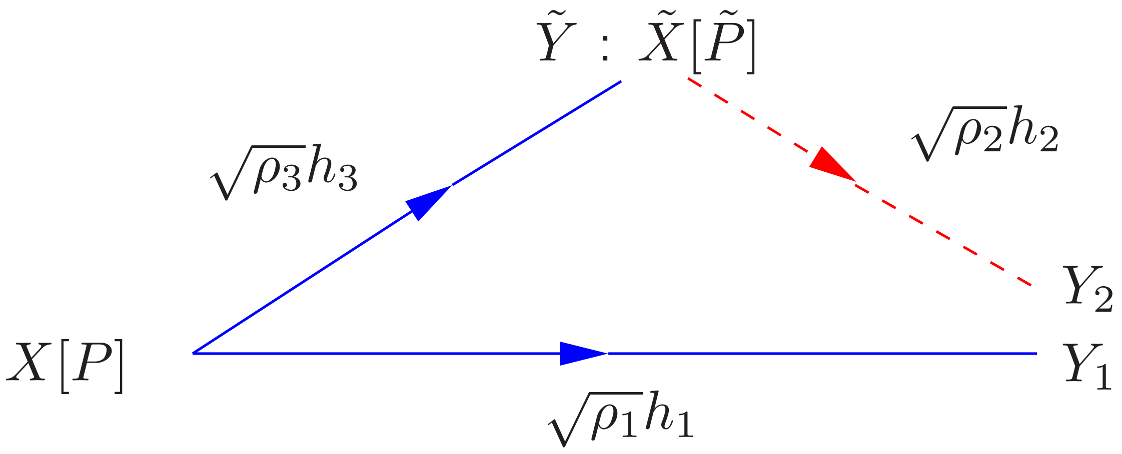
- Previous work on resource allocation for relay networks:
  - Reznik, Kulkarni and Verdu, IT'04: degraded gaussian multirelay channel
  - Maric and Yates, ISIT'04, Asilomar'04: Gaussian relay network
  - Host-Madsen and Zhang, IT'05: orthogonal relay channel
  - Gunduz and Erkip, Wirelesscomm'05: minimization outage
  - Many Others ......
- Assumption in above work: nodes are subject to sum power constraint
- Our Assumption: source and relay subject to separate power constraints

Lower Bound on Capacity for Fixed $\underline{h}$, P, $\tilde{P}$, θ



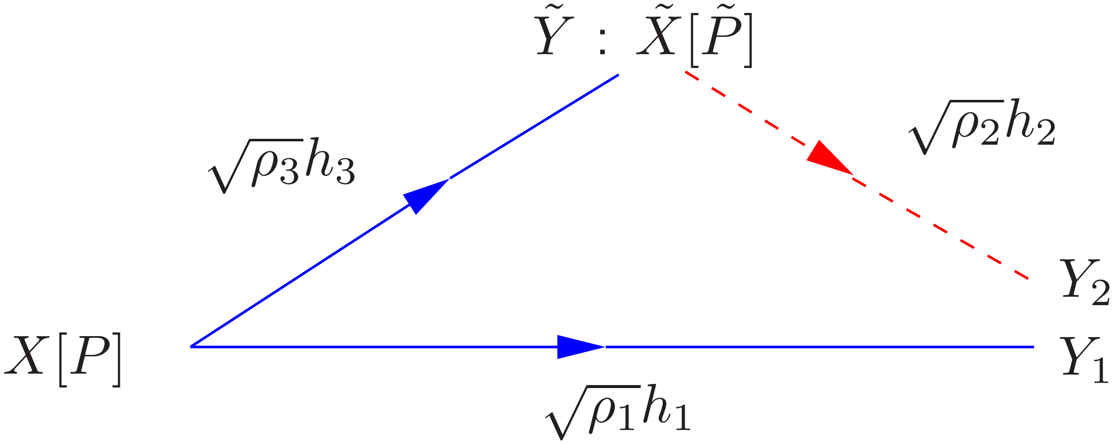
$$C_{\texttt{low}} = \min\left\{2\theta\mathcal{C}\left(\frac{P\rho_1|h_1|^2}{\theta}\right) + 2\bar{\theta}\mathcal{C}\left(\frac{\tilde{P}\rho_2|h_2|^2}{\bar{\theta}}\right),\; 2\theta\mathcal{C}\left(\frac{P\rho_3|h_3|^2}{\theta}\right)\right\}$$

Scenarios for Optimization



- Channel state information $\underline{h}$ is known at both transmitter and receiver
- Source and relay adapt power allocation according to instantaneous $\underline{h}$
- Goal: find optimal power allocation rules for source and relay
  - Scenario I: Channel resource parameter θ is fixed at $1/2$
  - Scenarios II and III: Jointly optimize θ with power allocation

Achievable Rate for Scenario I (fixed θ)



- Relaying Scheme: Decode-and-forward with $\theta = 1/2$

$$C_{\texttt{low}}(P, \tilde{P}) = \min\left\{R_1(P, \tilde{P}),\ R_2(P, \tilde{P})\right\}$$

$$R_1(P, \tilde{P}) = \mathrm{E}\left[\mathcal{C}\left(2P(\underline{h})\rho_1|h_1|^2\right) + \mathcal{C}\left(2\tilde{P}(\underline{h})\rho_2|h_2|^2\right)\right]$$

$$R_2(P, \tilde{P}) = \mathrm{E}\left[\mathcal{C}\left(2P(\underline{h})\rho_3|h_3|^2\right)\right]$$

Max-Min Optimization Problem

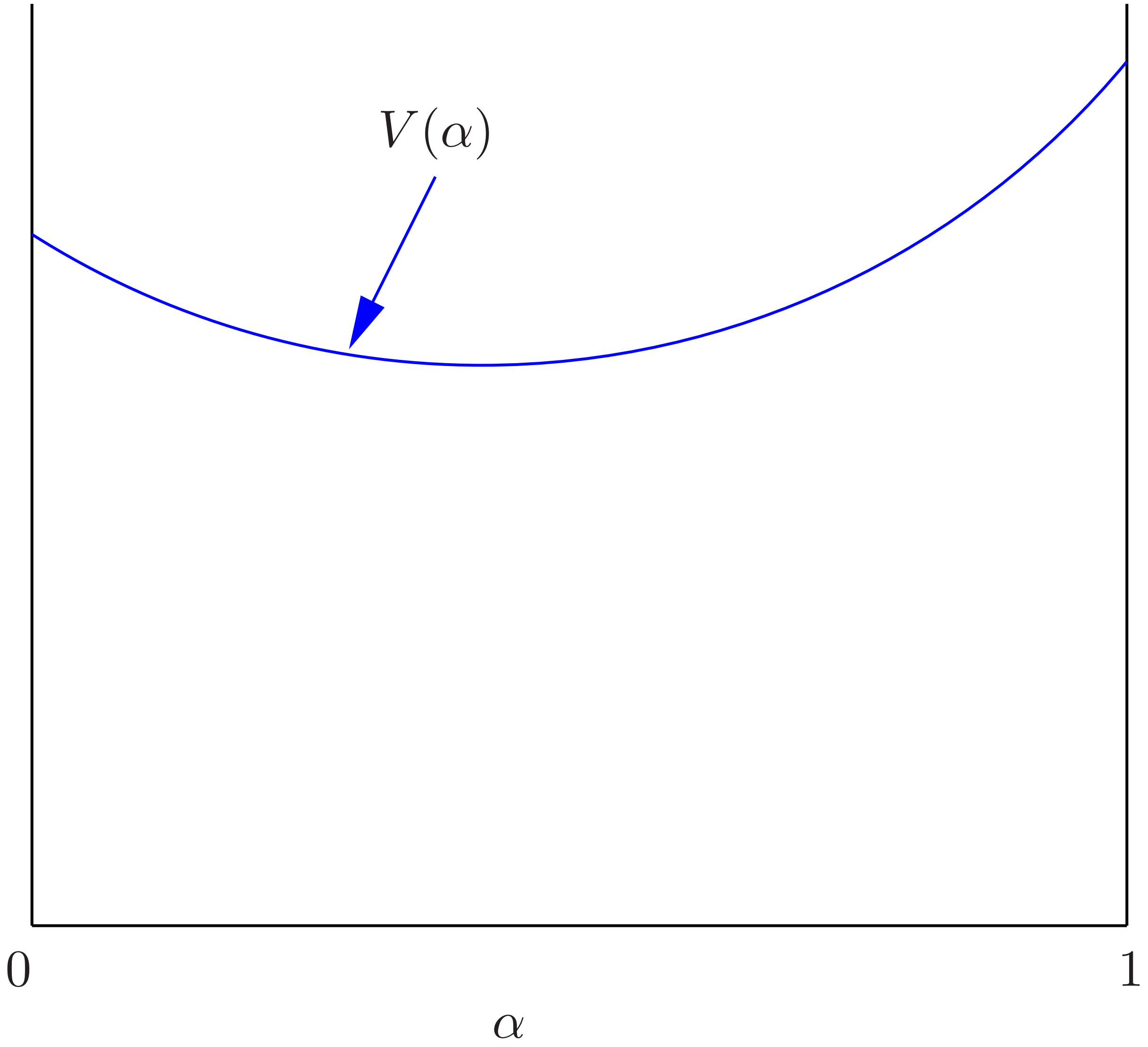
- $\mathcal{P} := (P(\underline{h}), \tilde{P}(\underline{h}))$ satisfy $\mathrm{E}[P(\underline{h})] \leq \mathbb{P}$, $\mathrm{E}[\tilde{P}(\underline{h})] \leq \tilde{\mathbb{P}}$

$$\boxed{C_{\texttt{low}} = \max_{\mathcal{P}} \ \min \{R_1(\mathcal{P}),\ R_2(\mathcal{P})\}}$$

- Define $R(\alpha, \mathcal{P}) := \alpha R_1(\mathcal{P}) + (1-\alpha) R_2(\mathcal{P}), \qquad 0 \leq \alpha \leq 1$
  - $R(\alpha, \mathcal{P})$ (as function of α): straight line from $R_2(\mathcal{P})$ to $R_1(\mathcal{P})$
  - Max-min problem corresponds to:

    maximizing the minimum of end points of $R(\alpha, \mathcal{P})$ over $\mathcal{P}$
- Define $V(\alpha) := \max_{\mathcal{P}} R(\alpha, \mathcal{P}) = R(\alpha, \mathcal{P}_\alpha)$
- $V(\alpha)$ and $R(\alpha, \mathcal{P})$ satisfy two conditions
  - $V(\alpha)$ is continuous and convex for $\alpha \in [0, 1]$
  - For any $\mathcal{P}$, $R(\alpha, \mathcal{P})$ is below or tangent to $V(\alpha)$

Illustration of Max-Min Solution



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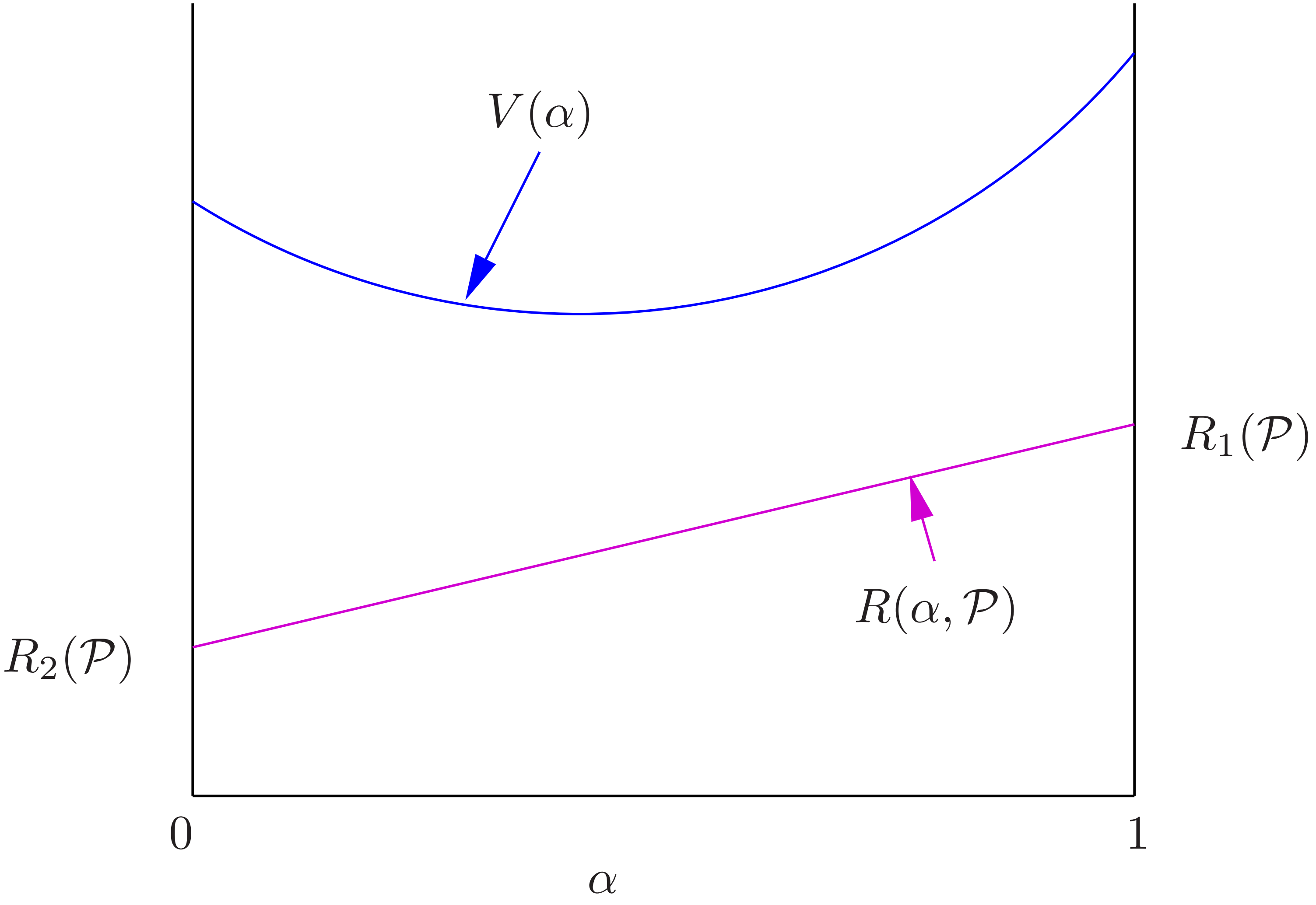

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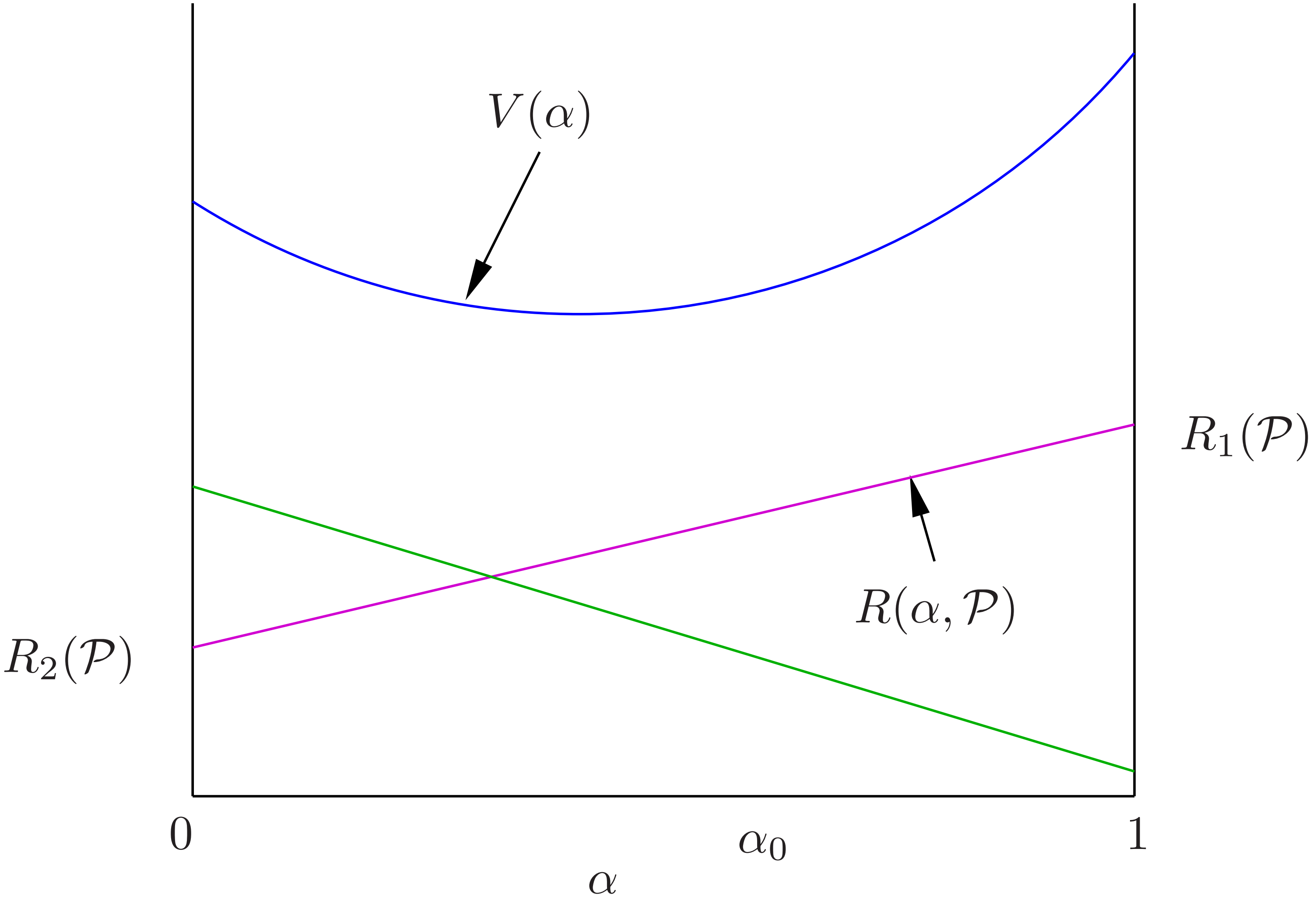

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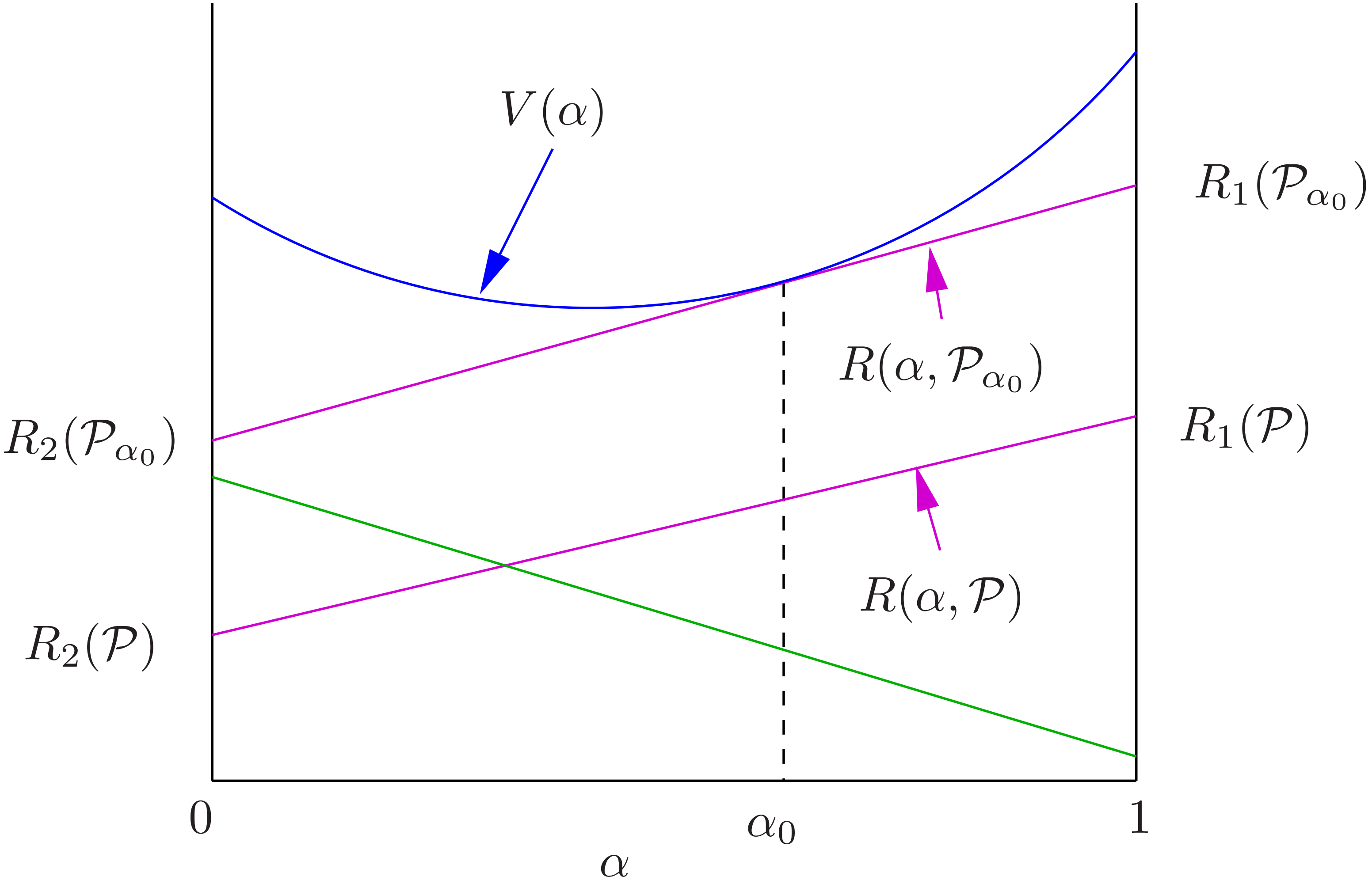

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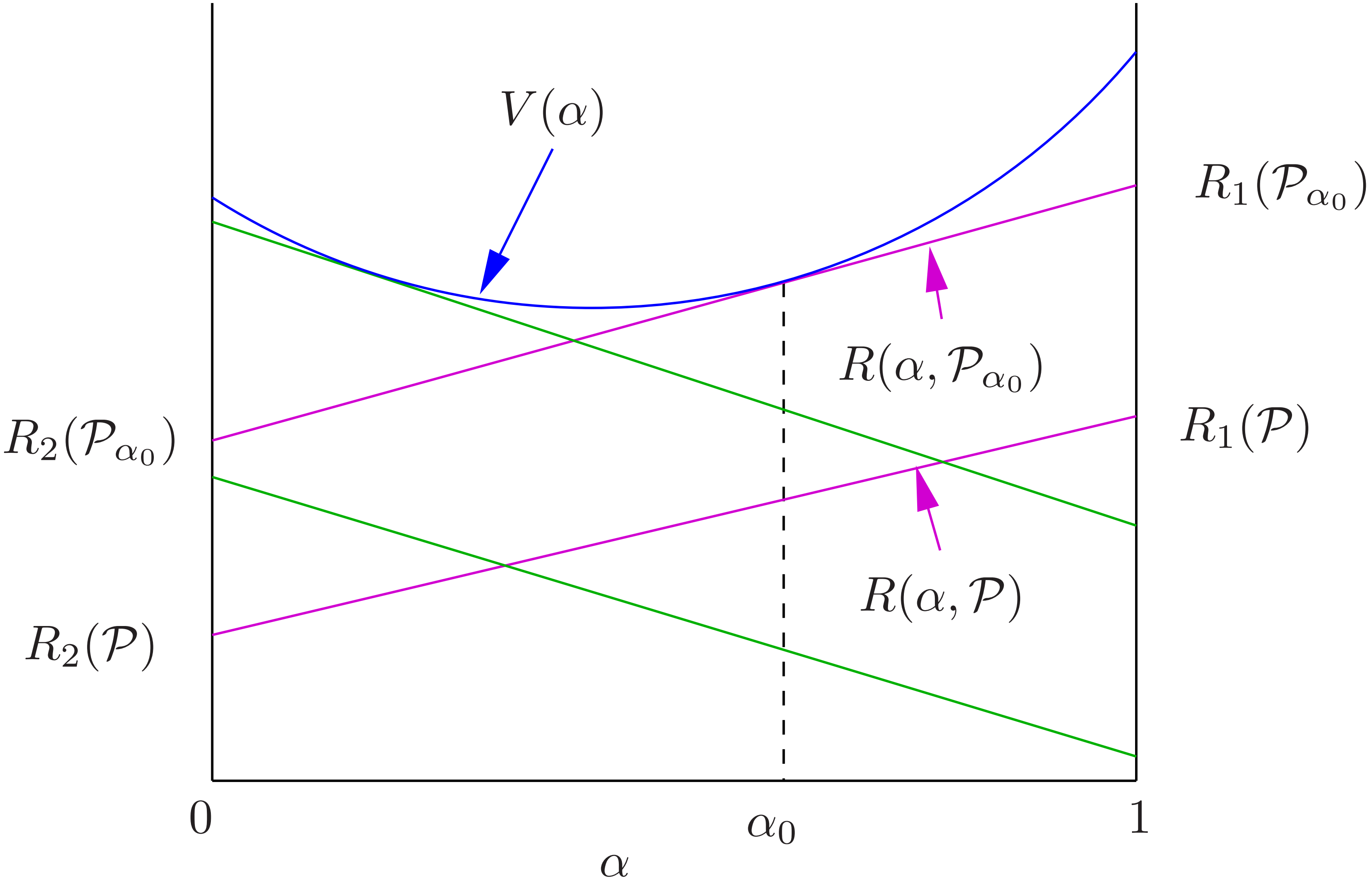

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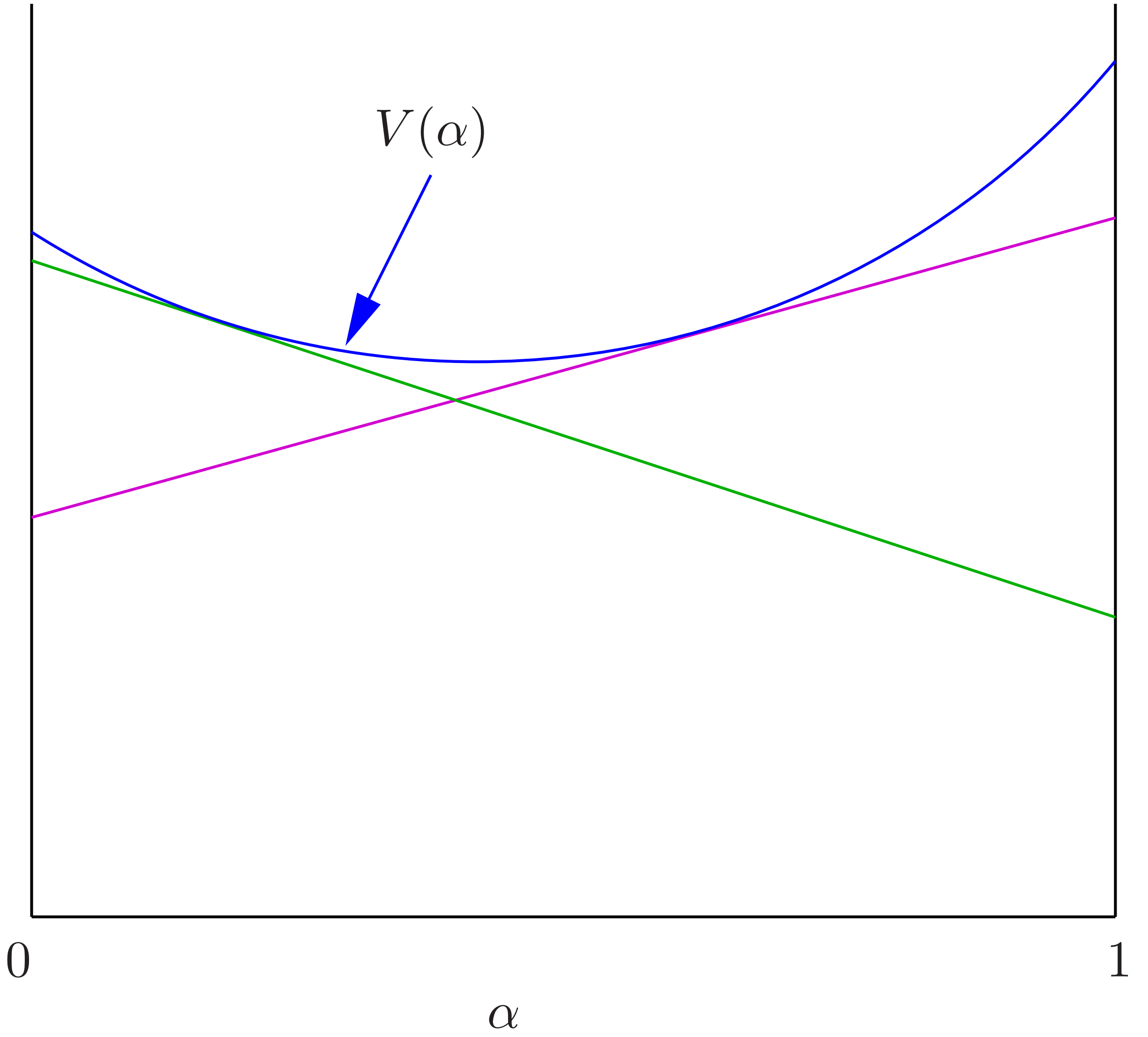

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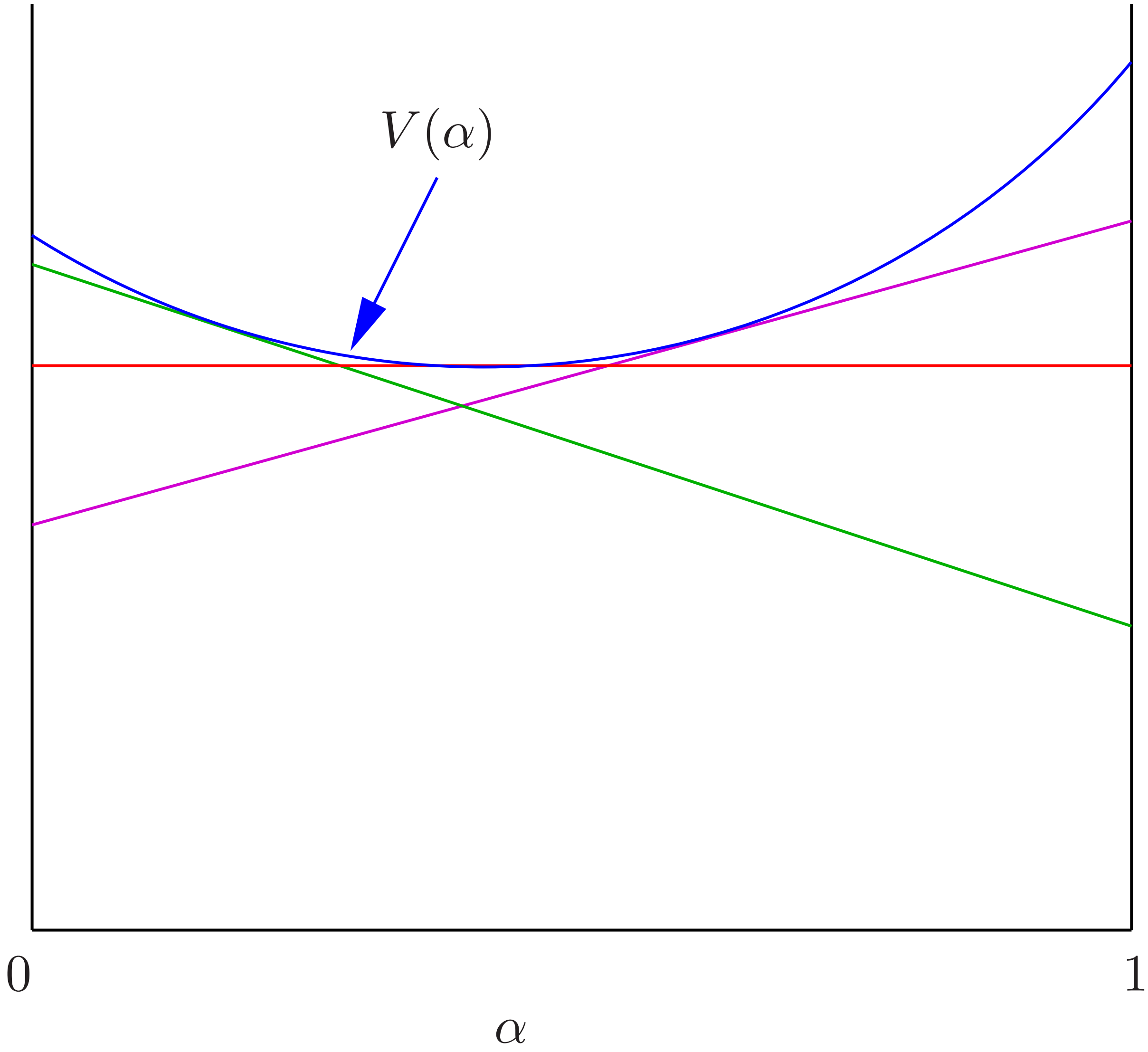
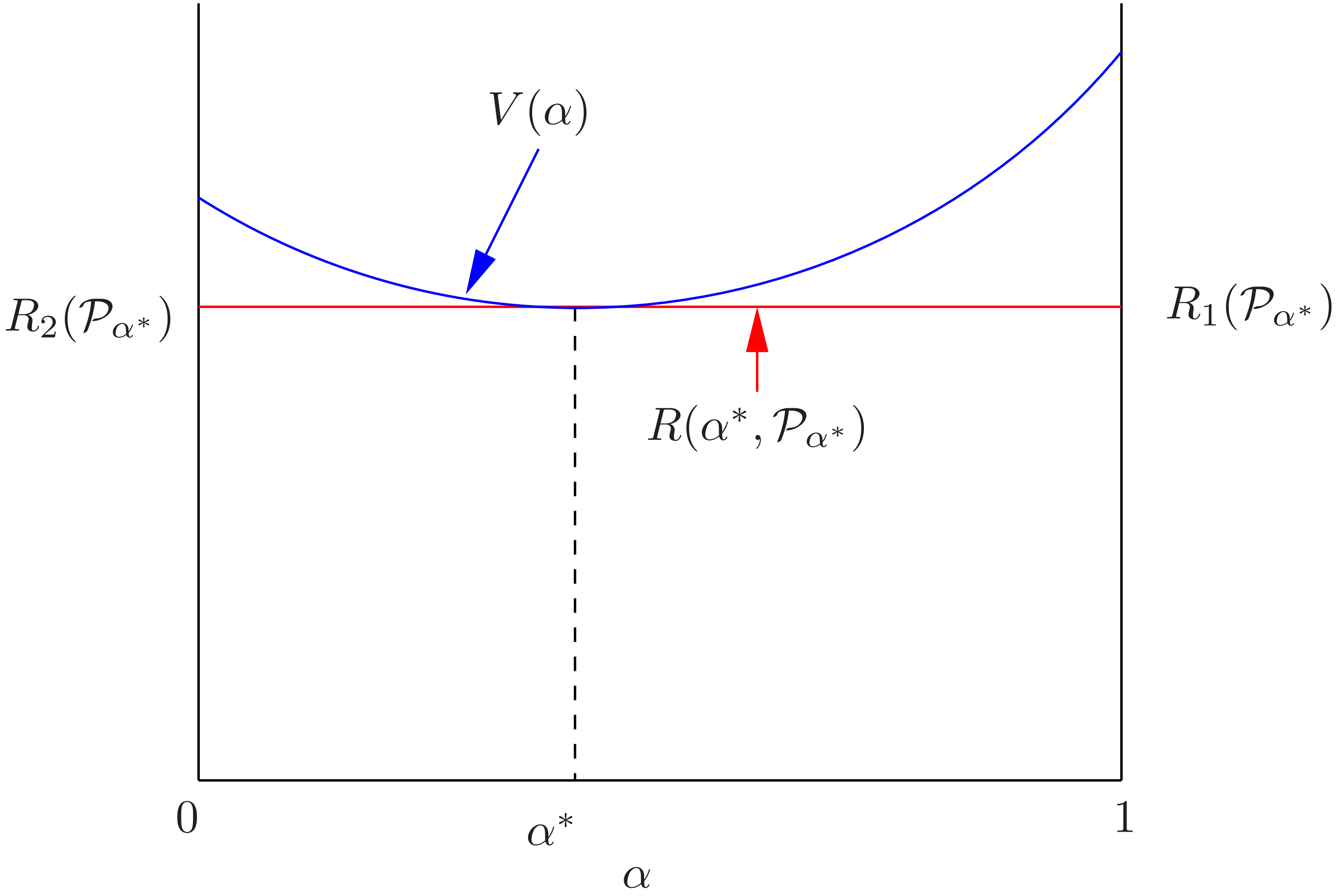
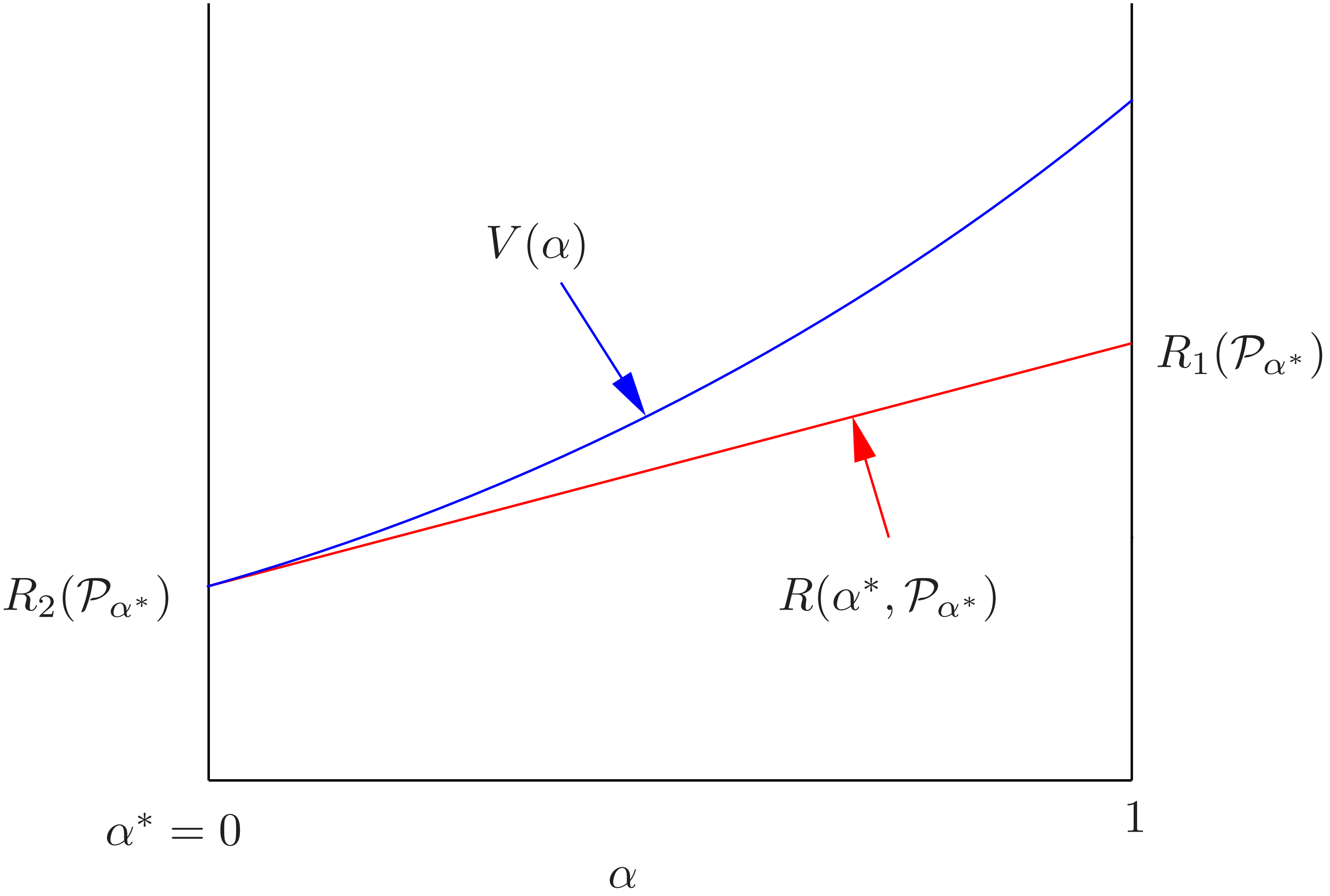


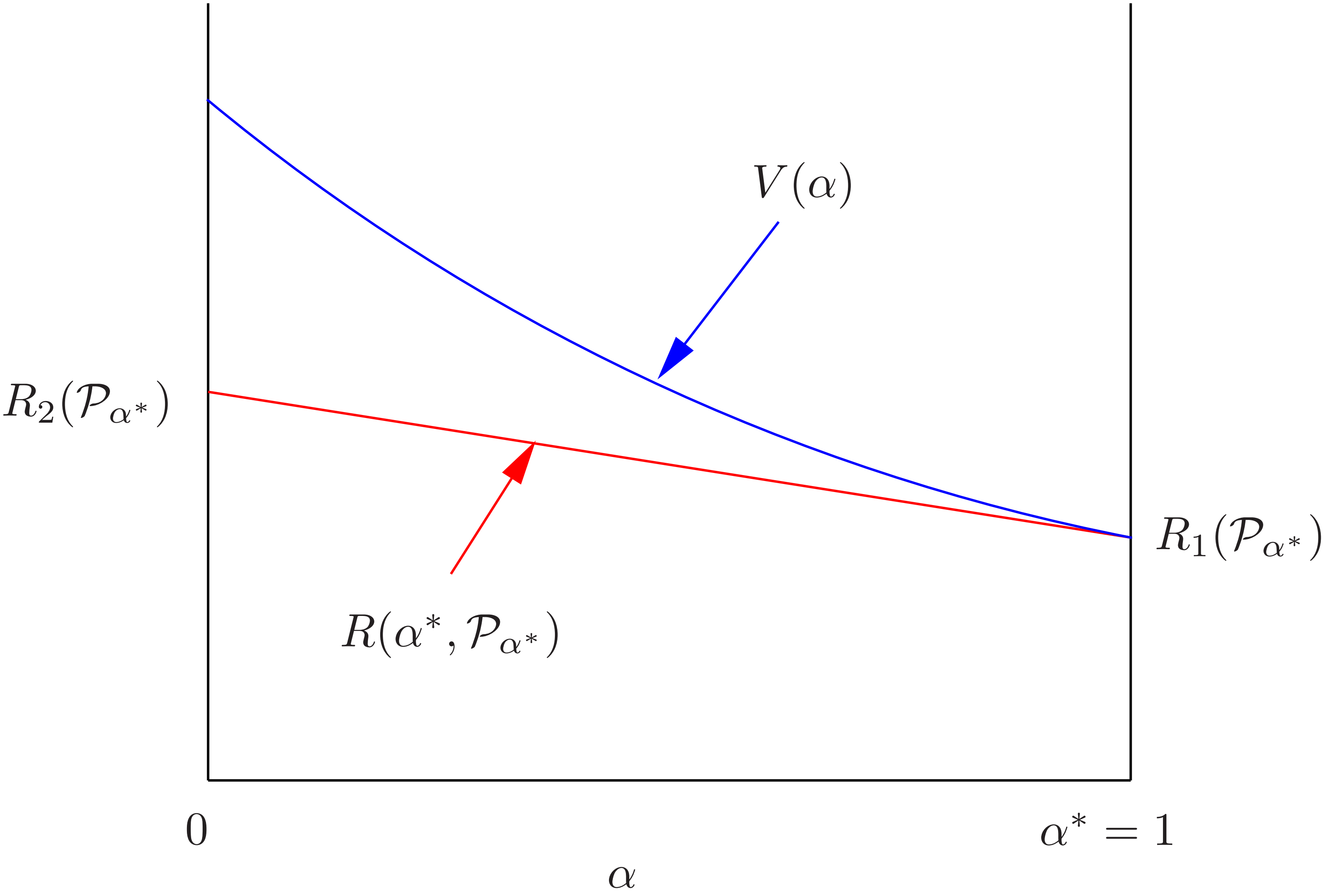
Max-Min Rule: Case III



Max-Min Rule: Case I



Max-Min Rule: Case II



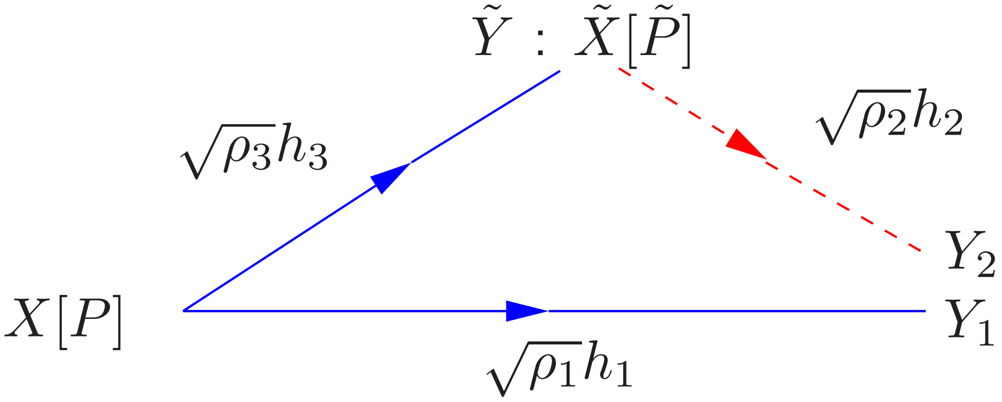
General Solution for Max-Min Rule

- Suppose α^* is a solution to $V(\alpha^*) = \min_{\alpha \in [0,1]} V(\alpha)$. Then $\mathcal{P}_{\alpha^*}$ is a max-min rule. ($\mathcal{P}_{\alpha^*}$ is defined to maximize $R(\alpha^*, \mathcal{P})$ over $\mathcal{P}$)
- Relationship between $R_1(\mathcal{P}_{\alpha^*})$ and $R_2(\mathcal{P}_{\alpha^*})$ falls into three cases:
  - Case I: If $\alpha^* = 0$, $R_1(\mathcal{P}_{\alpha^*}) \geq R_2(\mathcal{P}_{\alpha^*})$
  - Case II: If $\alpha^* = 1$, $R_1(\mathcal{P}_{\alpha^*}) \leq R_2(\mathcal{P}_{\alpha^*})$
  - Case III: If $0 < \alpha^* < 1$, $R_1(\mathcal{P}_{\alpha^*}) = R_2(\mathcal{P}_{\alpha^*})$

$$R_1(\mathcal{P}) = \mathrm{E}\left[\mathcal{C}\left(2P(\underline{h})\rho_1|h_1|^2\right) + \mathcal{C}\left(2\tilde{P}(\underline{h})\rho_2|h_2|^2\right)\right]$$

$$R_2(\mathcal{P}) = \mathrm{E}\left[\mathcal{C}\left(2P(\underline{h})\rho_3|h_3|^2\right)\right]$$

Power Allocations: Case I



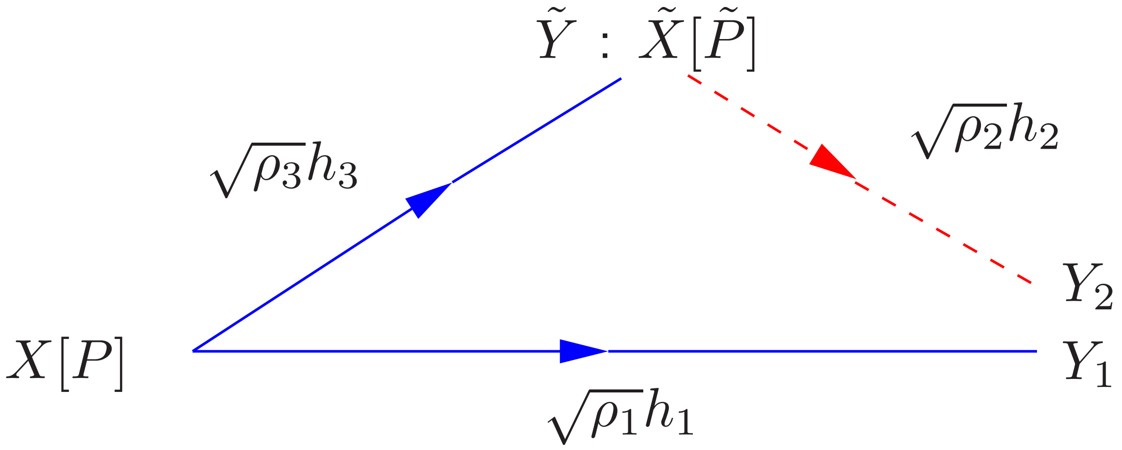
Case I: $\tilde{P} \geq \tilde{P}_u(P)$

$$P(\underline{h}) = \left(\frac{1}{\lambda \ln 2} - \frac{1}{\rho_3 |h_3|^2}\right)^+,$$

$$\tilde{P}(\underline{h}) = \left(\frac{1}{\mu \ln 2} - \frac{1}{\rho_2 |h_2|^2}\right)^+$$

where λ and μ are chosen to satisfy power constraints

Power Allocations: Case II



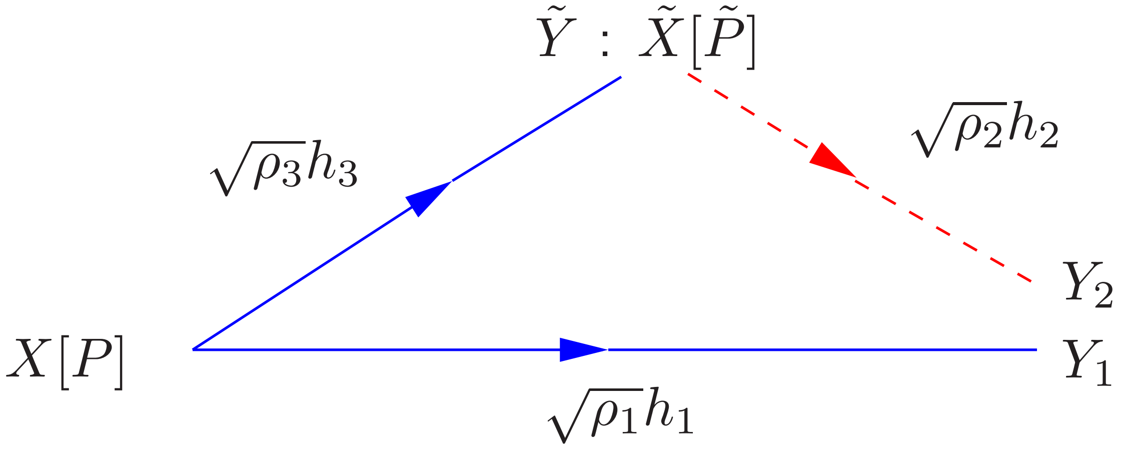
Case II: $\tilde{P} \leq \tilde{P}_l(P)$

$$P(\underline{h}) = \left(\frac{1}{\lambda \ln 2} - \frac{1}{\rho_1 |h_1|^2}\right)^+,$$

$$\tilde{P}(\underline{h}) = \left(\frac{1}{\mu \ln 2} - \frac{1}{\rho_2 |h_2|^2}\right)^+$$

where λ and μ are chosen to satisfy power constraints

Power Allocations: Case III



Case III (Equalizer Rule): $\tilde{P}_l(P) < \tilde{P} < \tilde{P}_u(P)$

- Optimal $P(\underline{h})$ equals positive root $p(\underline{h})$ of

$$\frac{\alpha}{\ln 2}\frac{1}{\frac{1}{\rho_1|h_1|^2}+p(\underline{h})}+\frac{1-\alpha}{\ln 2}\frac{1}{\frac{1}{\rho_3|h_3|^2}+p(\underline{h})}-\lambda=0 \qquad (*)$$

- α satisfies equalizer rule condition: $R_1(\mathcal{P}_\alpha)=R_2(\mathcal{P}_\alpha)$

Scenario II

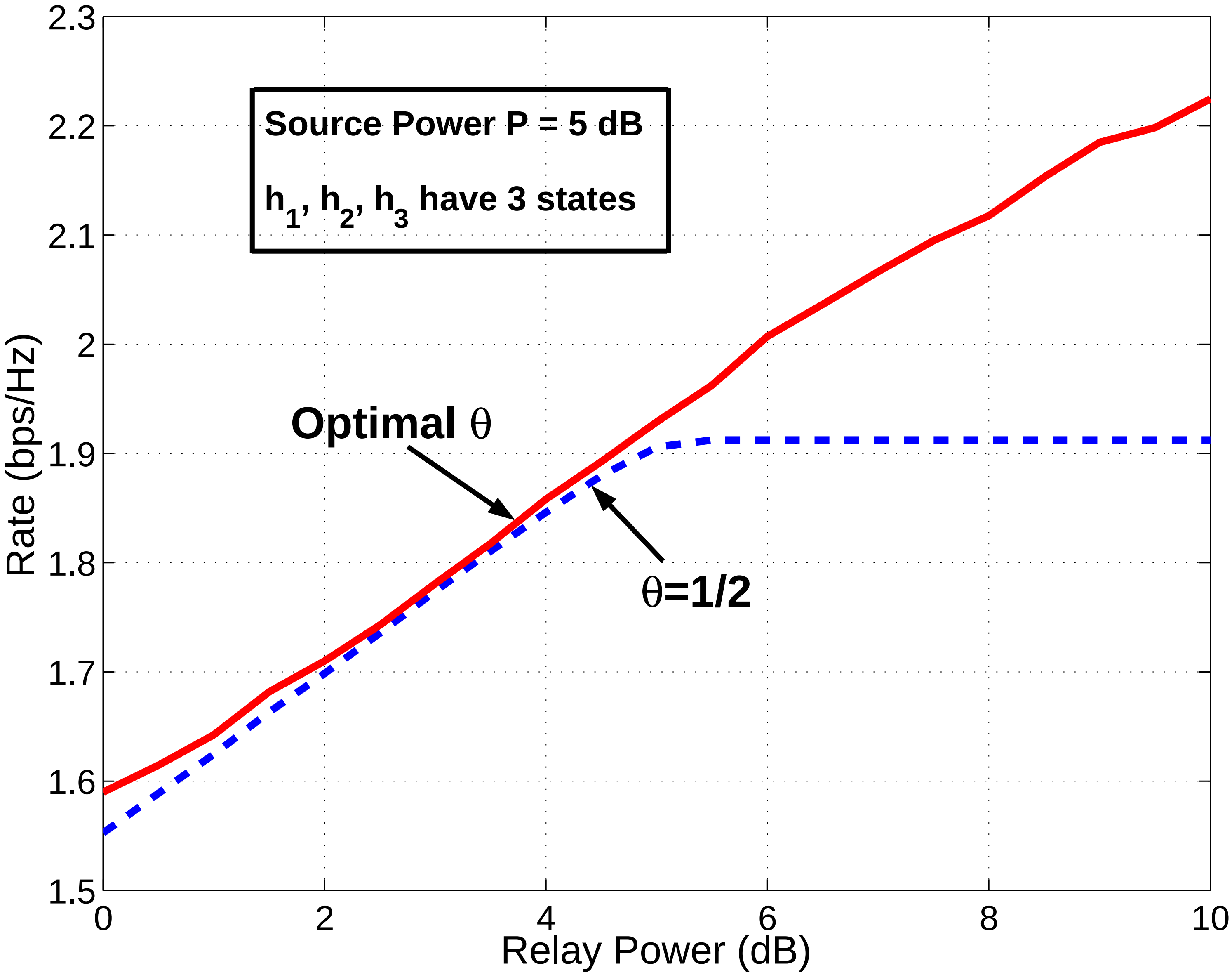
- Scenario I:
  - Fixed channel resource allocation parameter ($\theta = \frac{1}{2}$)
  - Only considered optimization of power allocations $\mathcal{P} = (P(\underline{h}), \tilde{P}(\underline{h}))$
  - Distributed resource allocation is optimum
- Scenario II:
  - θ is same for all $\underline{h}$
  - Jointly optimize $\mathcal{P} = (P(\underline{h}), \tilde{P}(\underline{h}))$ and θ
  - Technique for solving max-min problem applies
  - Joint optimization over θ and $\mathcal{P} = (P(\underline{h}), \tilde{P}(\underline{h}))$ at α^* is convex programming problem
  - Distributed resource allocation is again optimum

Lower bound terms for Scenario II

$$R_1(\mathcal{P},\theta) = 2\theta \mathrm{E}\left[\mathcal{C}\left(\frac{P(\underline{h})\rho_1|h_1|^2}{\theta}\right)\right] + 2\bar{\theta}\left[\mathcal{C}\left(\frac{\tilde{P}(\underline{h})\rho_2|h_2|^2}{\bar{\theta}}\right)\right]$$

$$R_2(\mathcal{P},\theta) = 2\theta \mathrm{E}\left[\mathcal{C}\left(\frac{P(\underline{h})\rho_3|h_3|^2}{\theta}\right)\right]$$

Performance Comparison



Scenario III

- Scenario I & II:
  - θ is not adapted to $\underline{h}$
  - Optimum resource allocation is distributed - each transmitter needs to know only channel state of only links over which it transmits
- Scenario III: most general scenario
  - θ is allowed to be adapted to $\underline{h}$
  - Joint optimization over $\theta(\underline{h})$ and $\mathcal{P} = (P(\underline{h}), \tilde{P}(\underline{h}))$
  - Capacity is obtained in some special cases
  - Optimum resource allocation is not distributed

Concluding Remarks

- Separate power constraints
  - More practical for geographically separated nodes
  - Naturally leads to distributed power allocation
- Structure of optimal power allocation functions
  - For relay: $\tilde{P}(\underline{h})$ depends only on h_2, always water-filling
  - For source: $P(\underline{h})$ depends only on h_1 and h_3; is not water-filling in general
- Technique of solving *max-min* problem is applicable to general relay networks, e.g., orthogonal relay broadcast channel (Liang&Veeravalli, SPAWC'05)
- Capacity is obtained in special cases