Backward Decoding Strategies for the Relay Channel

Mehul Motani

National University of Singapore
Electrical & Computer Engineering
motani@nus.edu.sg

Joint work with Hon-Fah Chong & HK Garg, NUS

MSRI Workshop on Mathematics of Relaying and Cooperation in Communication Networks, Berkeley, April 12, 2006

What are some important questions?

Problem setup - A system of distributed terminals wishing to communicate information with each other.

What are some important questions?

Problem setup - A system of distributed terminals wishing to communicate information with each other.

- Persians - used system of relay messengers 2 thousand years ago

What are some important questions?

Problem setup - A system of distributed terminals wishing to communicate information with each other.

- Persians - used system of relay messengers 2 thousand years ago
- Cooperation - Yes but how? And with whom?

What are some important questions?

Problem setup - A system of distributed terminals wishing to communicate information with each other.

- Persians - used system of relay messengers 2 thousand years ago
- Cooperation - Yes but how? And with whom?
- What is relaying? - Multi-hop routing? Richer interaction?

What are some important questions?

Problem setup - A system of distributed terminals wishing to communicate information with each other.

- Persians - used system of relay messengers 2 thousand years ago
- Cooperation - Yes but how? And with whom?
- What is relaying? - Multi-hop routing? Richer interaction?
- Competition - Why should I help?

What are some important questions?

Problem setup - A system of distributed terminals wishing to communicate information with each other.

- Persians - used system of relay messengers 2 thousand years ago
- Cooperation - Yes but how? And with whom?
- What is relaying? - Multi-hop routing? Richer interaction?
- Competition - Why should I help?
- Cross-layer design - Can we separate physical layer and routing?

What are some important questions?

Problem setup - A system of distributed terminals wishing to communicate information with each other.

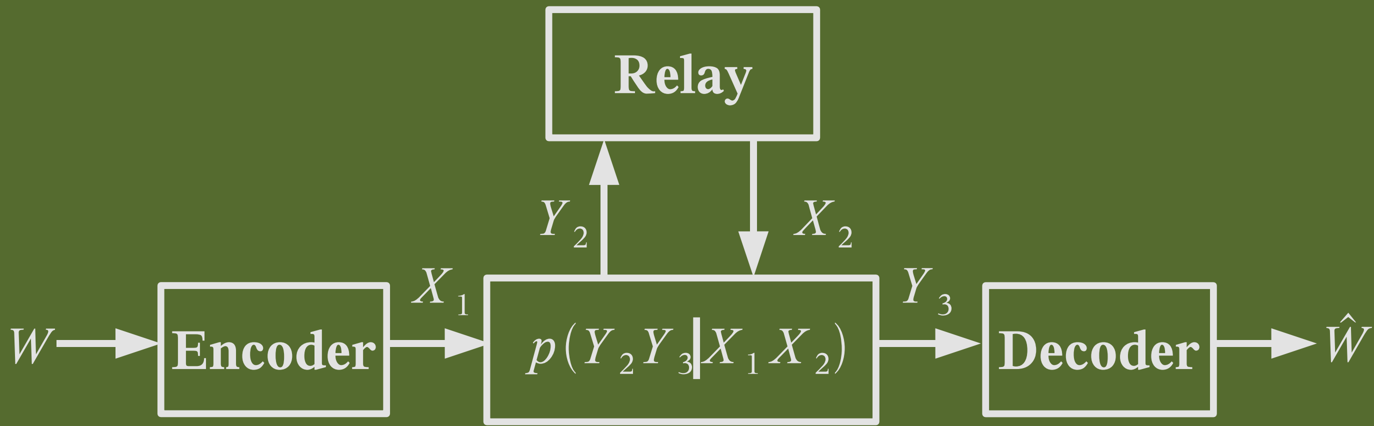
- Persians - used system of relay messengers 2 thousand years ago
- Cooperation - Yes but how? And with whom?
- What is relaying? - Multi-hop routing? Richer interaction?
- Competition - Why should I help?
- Cross-layer design - Can we separate physical layer and routing?
- Three body problem - Van der Meulen 1971, Cover & El Gamal 1979

Overview

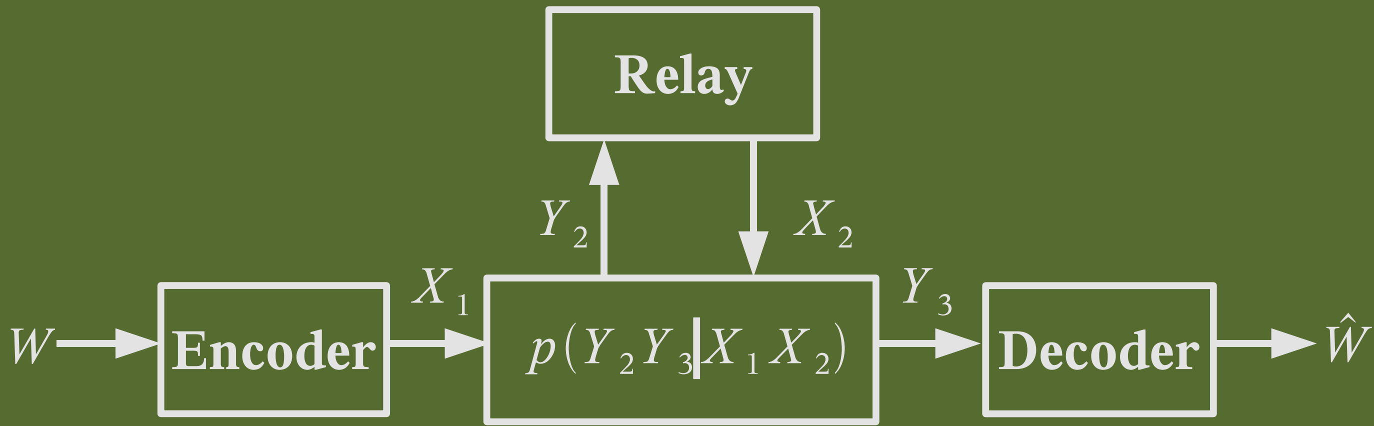
- General Relay Channel [VDM 1971]
- Decode-and-Forward, Compress-and-Forward, and Combination [CE 1979]
- Simultaneous backward decoding [CMG 2005]
- Improved simultaneous backward decoding [CMG 2006]

Model of the Relay Channel

Model of the Relay Channel



Model of the Relay Channel



- X_{1i} is a function of W
- X_{2i} is a function of the relay's past outputs $\mathbf{Y}_2^{i-1}$
- Memoryless and time-invariant network

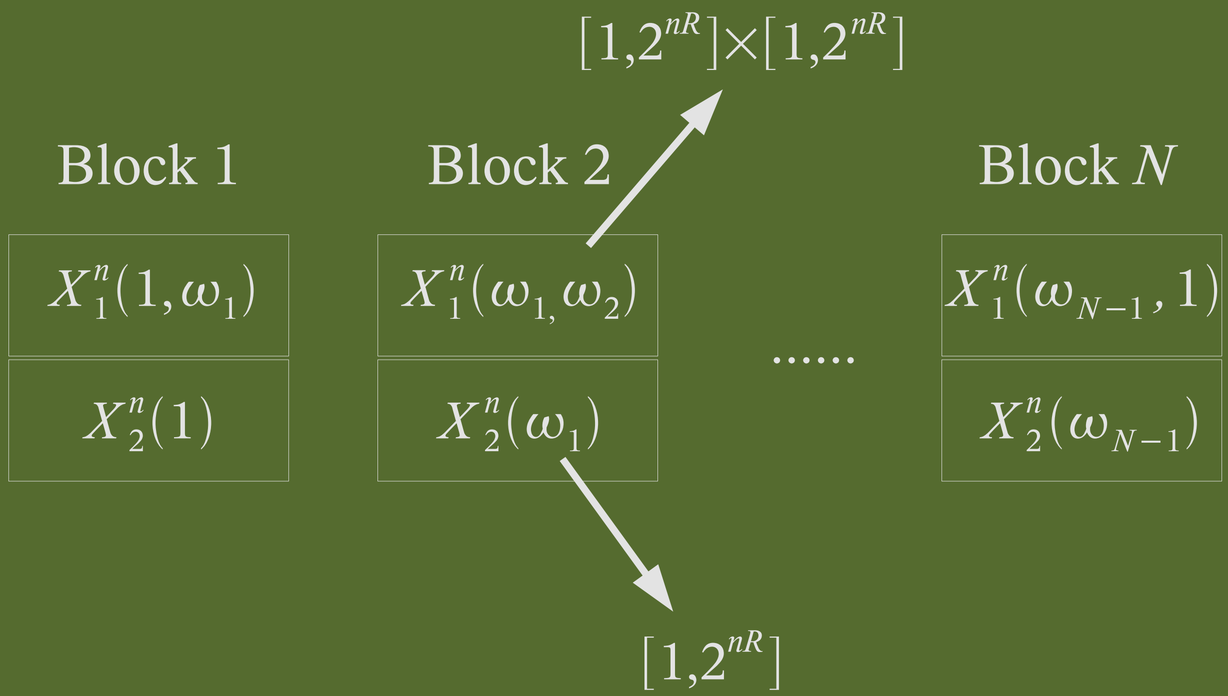
Decode-and-Forward Strategy

Receiver decodes all information from transmitter.

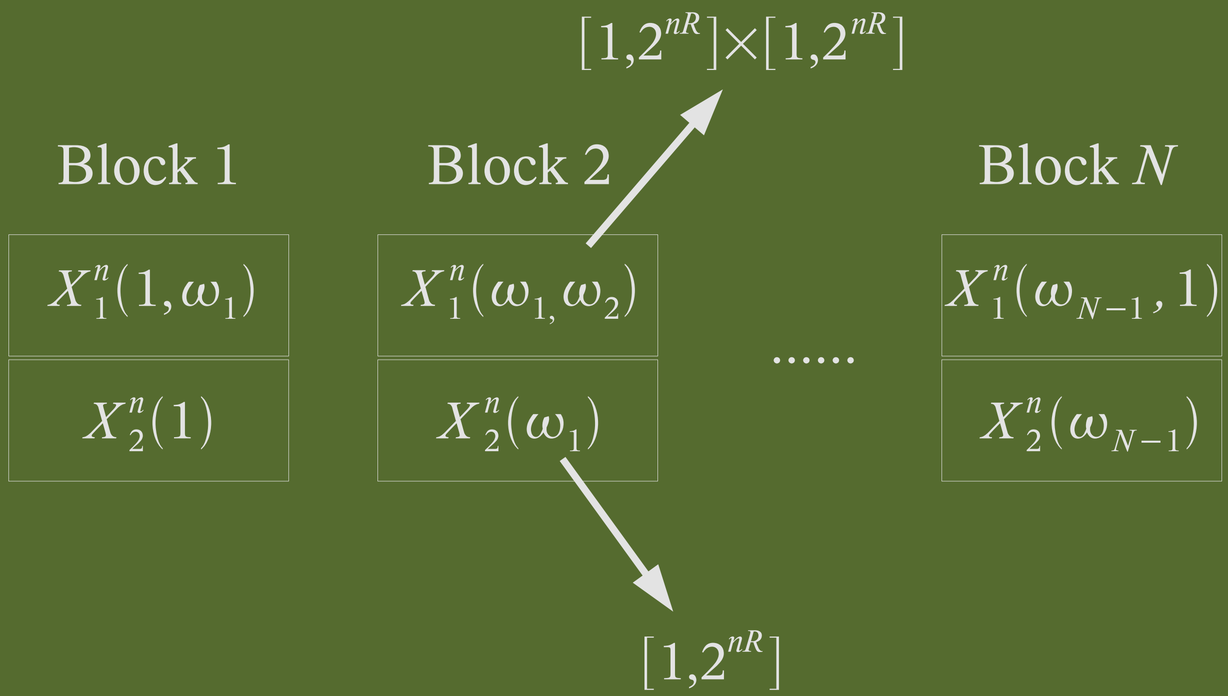
$$R_{\mathrm{DF}} = \sup_{p(x_1,x_2)} \left\{ \min \left\{ I\left(X_1; Y_2 | X_2\right), I\left(X_1 X_2; Y_3\right) \right\} \right\}$$

- Irregular Markov Encoding/Successive Decoding [Cover & El Gamal 1979]
- Regular Encoding/Backward Decoding [Willems 1992]
- Regular Encoding/Sliding Window Decoding [Xie & Kumar 2002]

Regular Encoding/Backward Decoding

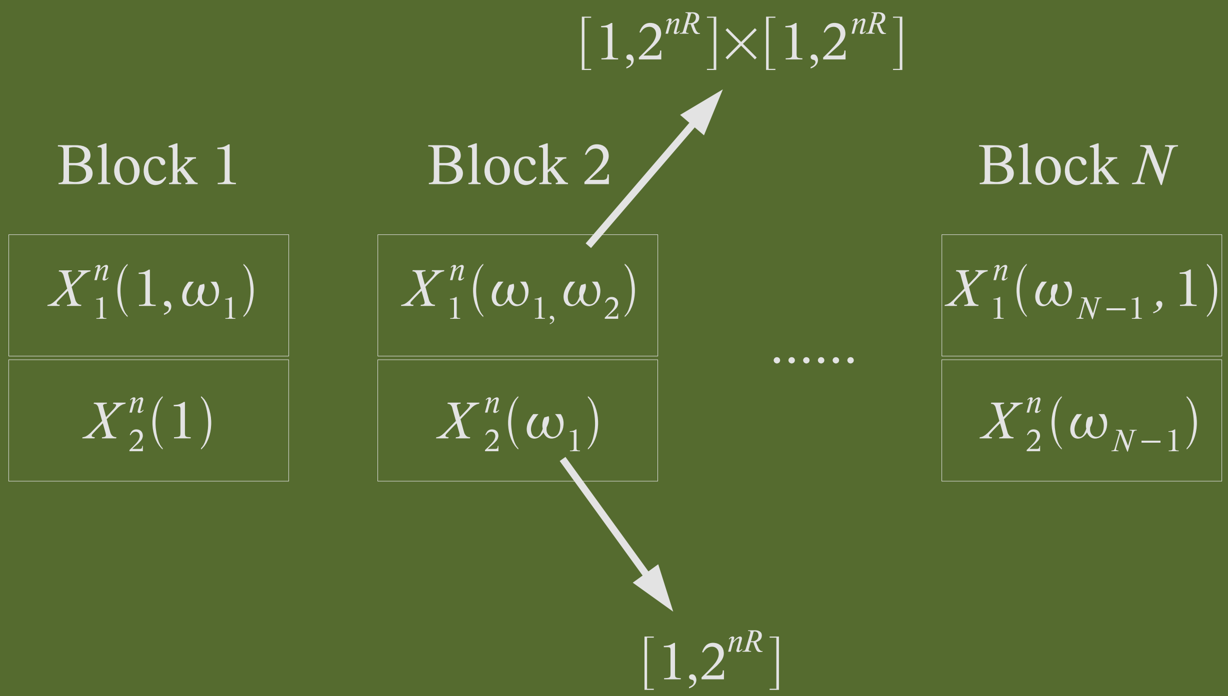


Regular Encoding/Backward Decoding



Relay can decode each block if $R \leq I\left(X_1; Y_2 | X_2\right)$

Regular Encoding/Backward Decoding



Relay can decode each block if $R \leq I\left(X_1; Y_2 | X_2\right)$

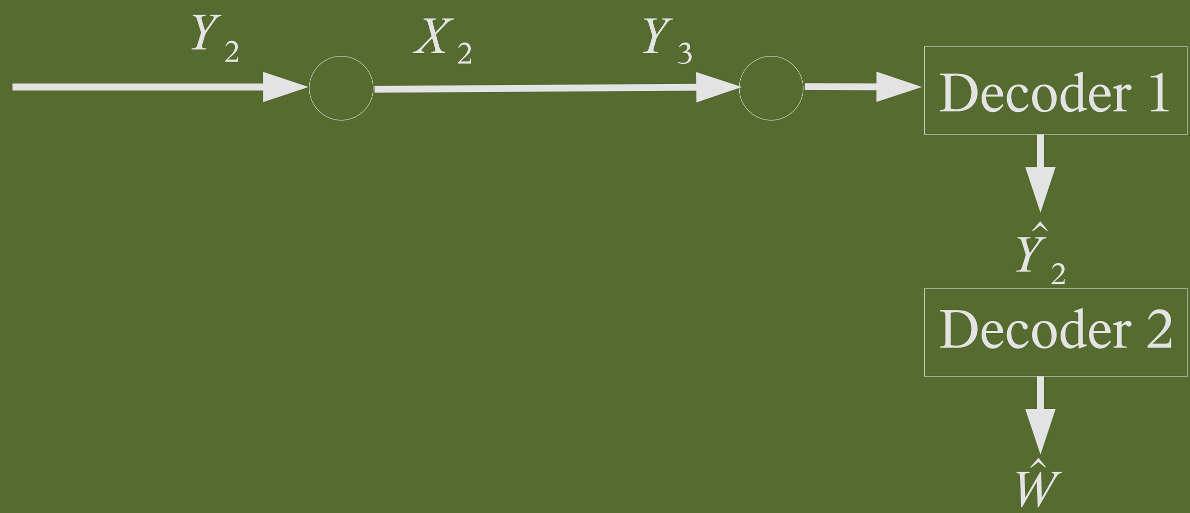
Destination can decode each block if $R \leq I\left(X_1 X_2; Y_2\right)$

Compress-and-Forward

Relay helps by forwarding quantized version of what it hears.

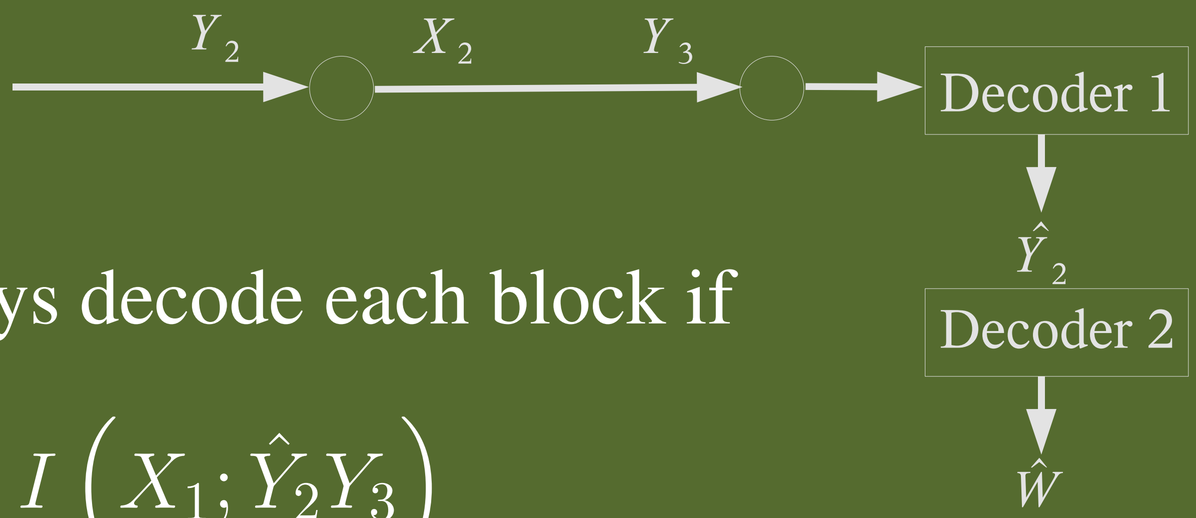
Compress-and-Forward

Relay helps by forwarding quantized version of what it hears.



Compress-and-Forward

Relay helps by forwarding quantized version of what it hears.



Destination can always decode each block if

$$R_{\mathrm{CF}} \leq I\left(X_1; \hat{Y}_2 Y_3\right)$$

subject to the constraint

$$I\left(X_2; Y_3\right) \geq I\left(Y_2; \hat{Y}_2 | X_2 Y_3\right)$$

Combining DF and CF

[Cover & El Gamal 1979]
Irregular Block Markov Encoding & Succesive Decoding

$$R = \sup \left\{ \min \left\{ \begin{array}{l} I\left(X_1; \hat{Y}_2 Y_3 | X_2 U\right) + I\left(U; Y_2 | X_2 V\right), \\ I\left(X_1 X_2; Y_3\right) - I\left(Y_2; \hat{Y}_2 | U X_1 X_2 Y_3\right) \end{array} \right\} \right\}$$

subject to the constraint

$$I\left(\hat{Y}_2; Y_2 | U X_2 Y_3\right) \leq I\left(X_2; Y_3 | V\right)$$

Combining DF and CF

[Chong, Motani & Garg ISIT 2005]
Regular Encoding & Simultaneous Backward Decoding

$$R = \sup \left\{ \min \left\{ \begin{array}{l} I\left(X_1; \hat{Y}_2 Y_3 | X_2 U\right) + I\left(U; Y_2 | X_2 V\right), \\ I\left(X_1 X_2; Y_3\right) - I\left(Y_2; \hat{Y}_2 | U X_1 X_2 Y_3\right) \end{array} \right\} \right\}$$

subject to the constraint

$$I\left(\hat{Y}_2; Y_2 | U X_2 Y_3\right) \leq I\left(X_2; Y_3 | U V\right)$$

Comparing CE1979 and CMG2005

$$\mathcal{R} = \sup \left\{ \min \left\{ \begin{matrix} I\left(X_1; \hat{Y}_2 Y_3 | U X_2\right) + I\left(U; Y_2 | V X_2\right), \\ I\left(X_1 X_2; Y_3\right) - I\left(Y_2; \hat{Y}_2 | U X_1 X_2 Y_3\right) \end{matrix} \right\} \right\}$$

Comparing CE1979 and CMG2005

$$\mathcal{R} = \sup \left\{ \min \left\{ \begin{array}{l} I\left(X_1; \hat{Y}_2 Y_3 | U X_2\right) + I\left(U; Y_2 | V X_2\right), \\ I\left(X_1 X_2; Y_3\right) - I\left(Y_2; \hat{Y}_2 | U X_1 X_2 Y_3\right) \end{array} \right\} \right\}$$

$$R_{\mathrm{CE79}} = \mathcal{R}$$

$$\text{subject to}: \; I\left(\hat{Y}_2; Y_2 | U X_2 Y_3\right) \leq I\left(X_2; Y_3 | V\right)$$

Comparing CE1979 and CMG2005

$$\mathcal{R} = \sup \left\{ \min \left\{ \begin{array}{l} I\left(X_1; \hat{Y}_2 Y_3 | U X_2\right) + I\left(U; Y_2 | V X_2\right), \\ I\left(X_1 X_2; Y_3\right) - I\left(Y_2; \hat{Y}_2 | U X_1 X_2 Y_3\right) \end{array} \right\} \right\}$$

$$R_{\text{CE79}} = \mathcal{R}$$

$$\text{subject to}: \; I\left(\hat{Y}_2; Y_2 | U X_2 Y_3\right) \leq I\left(X_2; Y_3 | V\right)$$

$$R_{\text{CMG05}} = \mathcal{R}$$

$$\text{subject to}: \; I\left(Y_2; \hat{Y}_2 | U X_2 Y_3\right) \leq I\left(X_2; Y_3 | U V\right)$$

Comparing CE1979 and CMG2005

$$\mathcal{R} = \sup \left\{ \min \left\{ \begin{array}{l} I\left(X_1; \hat{Y}_2 Y_3 | U X_2\right) + I\left(U; Y_2 | V X_2\right), \\ I\left(X_1 X_2; Y_3\right) - I\left(Y_2; \hat{Y}_2 | U X_1 X_2 Y_3\right) \end{array} \right\} \right\}$$

$$R_{\text{CE79}} = \mathcal{R}$$

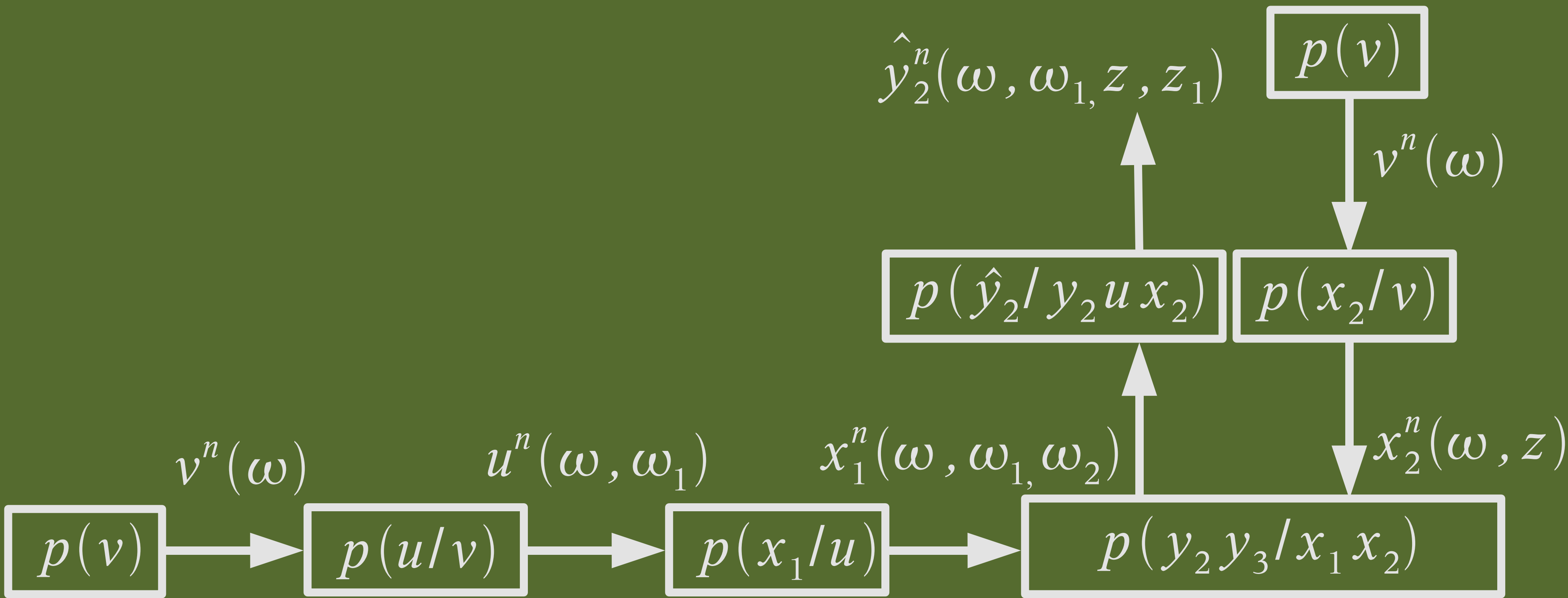
$$\text{subject to}: \; I\left(\hat{Y}_2; Y_2 | U X_2 Y_3\right) \leq I\left(X_2; Y_3 | V\right)$$

$$R_{\text{CMG05}} = \mathcal{R}$$

$$\text{subject to}: \; I\left(Y_2; \hat{Y}_2 | U X_2 Y_3\right) \leq I\left(X_2; Y_3 | U V\right)$$

$$\Longrightarrow R_{\text{CE79}} \subseteq R_{\text{CMG05}}$$

A Medley of Auxiliary Random Variables



Backward Decoding Strategies

- Regular Encoding/Backward Decoding
- Regular Encoding/Sequential Backward Decoding [CMG 2005, ISIT05]

$$w_{1b-1} \longrightarrow z_{b-1} \longrightarrow w_{2b}$$

- Regular Encoding/Simultaneous Backward Decoding [CMG 2005, ISIT05]

$$\big(w_{1b-1}, z_{b-1}\big) \text{ simultaneously } \longrightarrow w_{2b}$$

- Regular Encoding/Improved Simultaneous Backward Decoding [CMG 2006]

$$\big(w_{1b-1}, z_{b-1}, w_{2b}\big) \text{ simultaneously}$$

Transmission

$\underline{b = 1}$

$\mathbf{x}_1\,(1, w_{11}, w_{21})$ $\quad$ $\mathbf{x}_2\,(1, 1)$ $\quad$ $\hat{\mathbf{y}}_2\,(1, \hat{w}_{11}, 1, z_1)$

$\underline{b = 2, ..., B-2}$

$\mathbf{x}_1\,(w_{1b-1}, w_{1b}, w_{2b})$ $\quad$ $\mathbf{x}_2\,(\hat{w}_{1b-1}, z_{b-1})$ $\quad$ $\hat{\mathbf{y}}_2\,(\hat{w}_{1b-1}, \hat{w}_{1b}, z_{b-1}, z_b)$

$\underline{b = B-1}$

$\mathbf{x}_1\,(w_{1B-2}, 1, 1)$ $\quad$ $\mathbf{x}_2\,(\hat{w}_{1B-2}, z_{B-2})$ $\quad$ $\hat{\mathbf{y}}_2\,(\hat{w}_{1B-2}, 1, z_{B-2}, z_{B-1})$

$\underline{b = B}$

$\mathbf{x}_1\,(1, 1, 1)$ $\quad$ $\mathbf{x}_2\,(1, z_{B-1})$ $\quad$ $\hat{\mathbf{y}}_2\,(1, 1, z_{B-1}, 1)$

Decoding at the Relay

- Determine $\hat{w}_{1b}$
- Recall we know w_{b-1} and z_{b-1} from the previous block.

$$\left(\mathbf{u}^n\left(w_{1b-1}, \hat{w}_{1b}\right), \mathbf{v}^n\left(w_{1b-1}\right), \mathbf{x}_2^n\left(w_{1b-1}, z_{b-1}\right), \mathbf{y}_2^n\left(b\right)\right) \in A_\epsilon^{*(n)}$$

- $|w_{1b}| < 2^{nI(U;Y_2|VX_2)}$

Compression at the Relay

- Determine z_b
- Recall we have now decoded w_{1b} correctly.

$$\left(\mathbf{u}^n \left(w_{1b-1}, w_{1b}\right), \mathbf{x}_2^n \left(w_{1b-1}, z_{b-1}\right), \hat{\mathbf{y}}_2^n \left(w_{1b-1}, w_{1b}, z_{b-1}, z_b\right), \mathbf{y}_2^n (b) \right) \in A_\epsilon^{*(n)}$$

- $|z| > 2^{nI\left(Y_2;\hat{Y}_2|UX_2\right)}$

Transmission

$\underline{b = 1}$

$\mathbf{x}_1\left(1, w_{11}, w_{21}\right)$ $\mathbf{x}_2\left(1, 1\right)$ $\hat{\mathbf{y}}_2\left(1, \hat{w}_{11}, 1, z_1\right)$

$\underline{b = 2, ..., B - 2}$

$\mathbf{x}_1\left(w_{1b-1}, w_{1b}, w_{2b}\right)$ $\mathbf{x}_2\left(\hat{w}_{1b-1}, z_{b-1}\right)$ $\hat{\mathbf{y}}_2\left(\hat{w}_{1b-1}, \hat{w}_{1b}, z_{b-1}, z_b\right)$

$\underline{b = B - 1}$

$\mathbf{x}_1\left(w_{1B-2}, 1, 1\right)$ $\mathbf{x}_2\left(\hat{w}_{1B-2}, z_{B-2}\right)$ $\hat{\mathbf{y}}_2\left(\hat{w}_{1B-2}, 1, z_{B-2}, z_{B-1}\right)$

$\underline{b = B}$

$\mathbf{x}_1\left(1, 1, 1\right)$ $\mathbf{x}_2\left(1, z_{B-1}\right)$ $\hat{\mathbf{y}}_2\left(1, 1, z_{B-1}, 1\right)$

Decoding at the Receiver

- Determine $(\hat{w}_{1b-1}, \hat{z}_{b-1}, \hat{w}_{2b})$ simultaneously for block b.

$$\Big(\mathbf{v}\left(\hat{w}_{1b-1}\right), \mathbf{u}\left(\hat{w}_{1b-1}, w_{1b}\right), \mathbf{x}_1\left(\hat{w}_{1b-1}, w_{1b}, \hat{w}_{2b}\right), \mathbf{x}_2\left(\hat{w}_{1b-1}, \hat{z}_{b-1}\right),$$

$$\hat{\mathbf{y}}_2\left(\hat{w}_{1b-1}, w_{1b}, \hat{z}_{b-1}, z_b\right), \mathbf{y}_3\left(b\right) \Big) \in A_\epsilon^{*(n)}\left(V, U, X_1, X_2, \hat{Y}_2, Y_3\right).$$

Constraints at the Receiver

The conditions below imply that the probability of error may be driven to zero as $n \longrightarrow \infty$.

$$
\begin{aligned}
|w| + |w_2| + |z| &\leq 2^{n\left(I\left(UVX_1X_2\hat{Y}_2;Y_3\right)+I\left(X_1;X_2\hat{Y}_2|UV\right)\right)} \\
|w_2| + |z| &\leq 2^{n\left(I\left(X_1X_2\hat{Y}_2;Y_3|UV\right)+I\left(X_1;X_2\hat{Y}_2|UV\right)\right)} \\
|z| &\leq 2^{nI\left(X_2\hat{Y}_2;X_1Y_3|UV\right)} \\
|w_2| &\leq 2^{nI\left(X_1;X_2\hat{Y}_2Y_3|UV\right)}.
\end{aligned}
$$

Improved Simultaneous Backward Decoding

$$R = \sup \left\{ \min \left\{ \begin{array}{l} I\left(UVX_1X_2\hat{Y}_2;Y_3\right) + I\left(X_1;X_2\hat{Y}_2|UV\right) \\ -I\left(Y_2;\hat{Y}_2|UX_2\right) \\ I\left(U;Y_2|VX_2\right) + I\left(X_1X_2\hat{Y}_2;Y_3|UV\right) \\ +I\left(X_1;X_2\hat{Y}_2|UV\right) - I\left(Y_2;\hat{Y}_2|UX_2\right) \\ I\left(U;Y_2|VX_2\right) + I\left(X_1;Y_3\hat{Y}_2|UX_2\right) \end{array} \right\} \right\}$$

subject to the constraint

$$I\left(\hat{Y}_2;Y_2|UX_2\right) \leq I\left(X_2\hat{Y}_2;X_1Y_3|UV\right).$$

Compress-and-forward strategy (CMG 2006)

Set $U = \phi$ and $V = \phi$. We obtain the following

$$R \leq I\left(X_1; \hat{Y}_2 Y_3 | X_2\right) \quad (1)$$

$$R \leq I\left(X_1 X_2; Y_3\right) + I\left(\hat{Y}_2; Y_3 X_1 | X_2\right) - I\left(Y_2; \hat{Y}_2 | X_2\right) \quad (2)$$

$$I\left(X_2; Y_3\right) \geq I\left(Y_2; \hat{Y}_2 | Y_3 X_2\right) - I\left(X_2 \hat{Y}_2; X_1 | Y_3\right) \quad (3)$$

Compress-and-forward strategy (CE 1979)

$$R \leq I\left(X_1; \hat{Y}_2 Y_3 | X_2\right) \tag{4}$$

$$I\left(X_2; Y_3\right) \geq I\left(Y_2; \hat{Y}_2 | Y_3 X_2\right) \tag{5}$$

Compare (3) with (5):

$$I\left(X_2; Y_3\right) \geq I\left(Y_2; \hat{Y}_2 | Y_3 X_2\right) - I\left(X_2 \hat{Y}_2; X_1 | Y_3\right)$$

CMG constraint more relaxed than CE constraint.

Comparison

(1) $\leq$ (2) iff (5) holds.

$$(1) \leq (2) \iff I\left(X_2; Y_3\right) \qquad \geq I\left(Y_2; \hat{Y}_2 | Y_3 X_2\right)$$

So improved backward decoding is better when

$$\begin{aligned} & I\left(Y_2; \hat{Y}_2 | Y_3 X_2\right) - I\left(X_2 \hat{Y}_2; X_1 | Y_3\right) \\ & \leq I\left(X_2; Y_3\right) \\ & \leq I\left(Y_2; \hat{Y}_2 | Y_3 X_2\right) \end{aligned}$$

In Summary

- For the general relay channel, CMG 2006 contains all the rate regions known to date (including CE 1979 and CMG 2005).
- Lessons learned - Simultaneous decoding is good!
- Food for thought - Is backward decoding critical?