Cooperation and Competition

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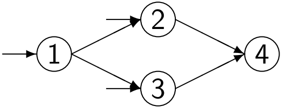
Cooperative communications



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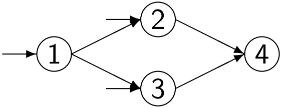

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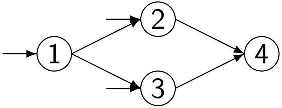

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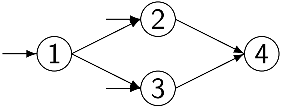

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  - Compete for network resources.
- Cooperation can increase global performance.
- However, diverting resources for cooperation may degrade individual performance.
- Natural approach is to somehow compensate nodes for cooperation.

Cooperation

- **Question:** Does there exist a way to accomplish this so that all nodes have an incentive to participate.
- Cooperative (coalitional) game theory provides a way to think about this.
- **Goal:** Look at simple models to illustrate role of cooperative games in cooperative communications.
  - Mostly preliminary results/toy problems.

Related work

- Rodoplu/Meng '02 - finite energy "delay tolerant" multi-hop networks.
- La/Anatharam '03 - multiple access channels.
- Mathur, Sankaranarayanan, Mandayam '06 - receiver cooperation.
- Jiang, Baras '06 (Mon. poster) - multi-hop networks.
- Also work on max-flow/multicommodity flow games (wire-line nets).

Cooperative Games with Transferable Pay-offs

Basic model:

- A set of users N.
- Users form *coalitions* $S \subseteq N$.
- *Characteristic function* $v(S)$ = total value of coalition.
  - Minimum value guaranteed regardless of action of users in $N\S$.
- Transferable pay-offs $\Rightarrow$ value can be arbitrarily divided among coalition.
  - e.g. via side payments (assumes existence of common valuation metric).

Cooperative games

Basic question: Given a cooperative game (N, v), what coalitions (if any) would rational agents form?

Key idea - rational agents would not form a coalition S if there exists another coalition T under which the users in T are all better off.

- Requires specifying an allocation of $v(S)$ to users in S: e.g. $\{x_i\}_{i \in S}$ s.t. $\sum_{i \in S} x_i = v(S)$.

Supper additive games

- Generally assume that v is *supper-additive*:

$$v(S \cup T) \geq V(S) + V(T), \ \forall S \cap T = \emptyset.$$

  Reasonable when S and T can do anything together they can do separately.

  $\Rightarrow$ Only need to consider the "grand coalition" N.

- In this case, a basic solution concept is the core of the game.

The core

Definition: The *core* of a cooperative game (N, v) is the set of allocations $(x_1, \ldots, x_N)$ such that:

1. $\sum_i x_i = v(N)$, and
2. for all $S \subseteq N$, $\sum_{i \in S} x_i \leq v(S)$.

i.e. the set of feasible allocations that are not dominated.

First order question: "is the core non-empty"?

Simple Example

- $N = \{1, 2, 3\}$
- Characteristic function:

$$v(S) = \begin{cases} 0, & \text{if } |S| = 1 \\ \alpha, & \text{if } |S| = 2 \\ 1, & \text{if } |S| = 3 \end{cases}$$

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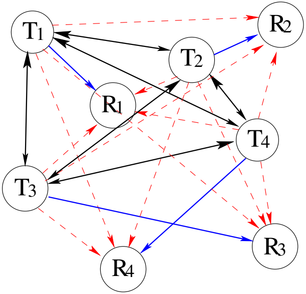
- Core is non-empty iff $\alpha \leq 2/3$.
- When $\alpha < 2/3$, any feasible allocation such that $x_i > \alpha/2$, $\forall i$ is in the core.

Cooperative games and cooperative communication

Modeling issues:

- What is the right characteristic function?
  - Consider examples where $v(S) = \max.$ sum rate for users in S.
- $v(S)$ depends on cooperative communication model.
  - Lots of ways to cooperate (not just relaying).
  - Consider several examples.
- Also depends on actions of users not in the coalition.
  - Here, treat as noise.
  - Could also treat as worst-case jammers (La/Anatharam).

Single hop networks



- Let N be set of transmitter receiver pairs in a given geographic area.
- Each pair only want to send to its own receiver.
- Assume each pair uses entire bandwidth of WHz.
- Static model.

Simple Cooperation

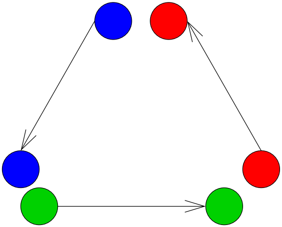
- Single-user receivers:

$$R_{ij} = \log\left(1 + \frac{h_{ii}P_i}{\sum_{j\neq i} h_{ji}P_j + \sigma^2}\right)$$

- Simple form of cooperation is power control:
  - A coalition jointly decides on their power allocation $\{P_i\}_{i\in S}$, $P_i \in [0, P^{max}]$.
- $v(S)$ = sum rate for S when users $j \in N\S$ transmit at P^{max}.
- Given such a network is the core always non-empty?

Single hop networks

Observation: The core of a single hop network with power control may be empty.



$$v(S) \approx \begin{cases} 0 & \text{if} |S| = 1 \\ \alpha & \text{if} |S| = 2 \\ 1 & \text{if} |S| = 3 \end{cases}$$

and can choose gains so that $\alpha > 2/3$.

More elaborate cooperation

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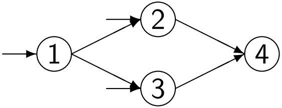
- *Stronger forms of cooperative communication better induce cooperation*.

General Networks?

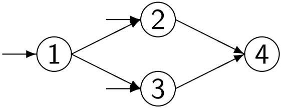
- If every receiver sends received signal to a single "joint decoding point" then [Mandayam et al.] show core is always non-empty (MIMO MAC).
- Requires a lot of cooperation, high-rate back-haul.
- In general - how much cooperation is needed to ensure core is non-empty?

Relay model



- Gaussian parallel relay model.
- All traffic to node 4.
- No direct path from $1 \rightarrow 4$.
- (Strong) half duplex:
  operate in broadcast mode of multi-access mode.
- Symmetric case: assume all $h_{ij} = 1$
  (noise power/BW normalized to 1).

Relay cooperative game



- $N = \{1, 2, 3\}$.
- $v(S)$ = total traffic to node 4.
- Cooperation:
  - 2 and 3 - joint decoding.
  - 1 and 2 or 3 - traffic forwarding.
- First consider just multi-hop routing (no cooperative gains from 2 and 3).

Core

Observation: The core of the parallel relay game is non-empty.

- Due to half duplex constraint node one is a *dummy node*

$$v(S \cup 1) = v(S) \quad \forall S$$

- $V(N)$ achieved when 2,3 transmit only their own data.

$$v(N) = \log(1 + 2P).$$

- Core allocations:
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  $x_1 = 0$, $x_2 + x_3 = V(N)$, $x_i > \log(1 + P)$, $i = 2, 3$.
- Directly generalizes to $M > 2$ relays and non-symmetric channel gains.

Cooperative forwarding

- Suppose instead that 2 and 3 cooperate to forward 1's traffic:.
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  - May want to save for a rainy day.
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- Include other costs for cooperation?
  - complexity, overhead..