Contents

1	Introduction 1	
	1.1 Motivating examples 1	
	1.2 <i>Optimization problems</i> 5	
	1.3 Important classes of optimization problems	10
	1.4 History 14	

I Linear algebra models 19

- Vectors and functions 2 21
 - 2.1 Vector basics 21
 - 2.2 Norms and inner products 28
 - 2.3 Projections onto subspaces 37
 - 2.4 Functions 43
 - 2.5 Exercises 53

Matrices 3 55

- 3.1 Matrix basics 55
- 3.2 Matrices as linear maps 61
- 3.3 Determinants, eigenvalues, and eigenvectors 64
- 3.4 Matrices with special structure and properties 75
- 3.5 Matrix factorizations 82

vi

3.6 Matrix norms 84
3.7 Matrix functions 87
3.8 Exercises 91

4 Symmetric matrices 97

4.1 Basics 97

4.2 *The spectral theorem* 103

4.3 Spectral decomposition and optimization 107

4.4 *Positive semidefinite matrices* 110

4.5 Exercises 118

5 Singular value decomposition 123

5.1 Singular value decomposition 123

5.2 Matrix properties via SVD 127

5.3 SVD and optimization 133

5.4 Exercises 145

6 Linear equations and least squares **151**

6.1 Motivation and examples 151

6.2 *The set of solutions of linear equations* 158

6.3 Least-squares and minimum-norm solutions 160

6.4 Solving systems of linear equations and LS problems 169

6.5 Sensitivity of solutions 173

6.6 Direct and inverse mapping of a unit ball 177

6.7 Variants of the least-squares problem 183

6.8 Exercises 193

7 *Matrix algorithms* 199

7.1 Computing eigenvalues and eigenvectors 199

- 7.2 Solving square systems of linear equations 206
- 7.3 QR factorization 211
- 7.4 Exercises 215

Convex optimization models Π 221

Linear, quadratic, and geometric models

293

296

- 8 Convexity 223
 - 8.1 Convex sets 223
 - 8.2 Convex functions 230
 - 8.3 Convex problems 249
 - 8.4 *Optimality conditions* 268
 - 8.5 Duality 272
 - 8.6 Exercises 287

9

9.1 Unconstrained minimization of quadratic functions 294 9.2 Geometry of linear and convex quadratic inequalities 9.3 *Linear programs* 302 9.4 Quadratic programs 311 9.5 Modeling with LP and QP 320 9.6 LS-related quadratic programs 331 9.7 Geometric programs 335 9.8 Exercises 341 Second-order cone and robust models 10 347 10.1 Second-order cone programs 347 10.2 SOCP-representable problems and examples 353 10.3 Robust optimization models 368

10.4 Exercises 377 viii

- 11Semidefinite models38111.1 From linear to conic models38111.2 Linear matrix inequalities38311.3 Semidefinite programs39311.4 Examples of SDP models39911.5 Exercises418
- *12 Introduction to algorithms* 425
 - 12.1 Technical preliminaries 427
 12.2 Algorithms for smooth unconstrained minimization 432
 12.3 Algorithms for smooth convex constrained minimization 452
 12.4 Algorithms for non-smooth convex optimization 472
 12.5 Coordinate descent methods 484
 12.6 Decentralized optimization methods 487
 12.7 Exercises 496
 - III Applications 503
- Learning from data 505
 13.1 Overview of supervised learning 505
 13.2 Least-squares prediction via a polynomial model 507
 13.3 Binary classification 511
 13.4 A generic supervised learning problem 519
 13.5 Unsupervised learning 524
 13.6 Exercises 533
- 14Computational finance53914.1 Single-period portfolio optimization53914.2 Robust portfolio optimization546

14.3 Multi-period portfolio allocation54914.4 Sparse index tracking55614.5 Exercises558

15 Control problems 567

15.1 Continuous and discrete time models 568
15.2 Optimization-based control synthesis 571
15.3 Optimization for analysis and controller design 579
15.4 Exercises 586

16Engineering design59116.1 Digital filter design59116.2 Antenna array design60016.3 Digital circuit design60616.4 Aircraft design60916.5 Supply chain management61316.6 Exercises622

Index 629