EE227BT Discussion Section #6

Exercise 1 (Relative Entropy Minimization) Let $q \in \Delta_n$, and consider the problem of finding the closest probability distribution to q, in the *Kullback-Liebler divergence* sense, under some linear constraints

$$\begin{aligned} \min_{p \in \mathbb{R}^n_+} \ & D(p \parallel q) \\ \text{s.t. } & Ap = b \\ & e^\top p = 1 \end{aligned}$$

where $A = \begin{bmatrix} a_1 & \dots & a_n \end{bmatrix} \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m$.

Derive the Lagrange dual of this problem and simplify it to get

$$\max_{\lambda \in \mathbb{R}^m} \ b^\top \lambda - \log \sum_{i=1}^{n} q_i e^{a_i^\top \lambda}$$

Exercise 2 (Minimizing A Quadratic) Find the optimal solution of the following optimization problem

$$p^* = \min_{x \in \mathbb{R}^n} \ x^\top A x + 2 b^\top x + c$$

where $A \in \mathbb{S}^n, b \in \mathbb{R}^n, c \in \mathbb{R}$. We do **not** assume that $A \succeq 0$, so in general we have to deal with a non convex optimization problem.