EE227BT Discussion Section #2

Exercise 1 (Congruence Transformations) Let $A \in \mathbb{S}^n, B \in \mathbb{R}^{n\times n}$. Assume that B is invertible. Then

$$A \succeq 0 \;\Leftrightarrow\; B^\top AB \succeq 0$$

Exercise 2 (Schur Complements)

Let $A \in \mathbb{R}^{n\times n}, C \in \mathbb{R}^{m\times m}, B \in \mathbb{R}^{n\times m}$. Consider the block matrix

$$X = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

.

1. Give a necessary condition on A, B, C for X to be symmetric positive-definite.
2. Compute $det(X)$ as a function of $S = D - CA^{-1}B$.

Exercise 3 (Kullback-Leibler divergence) For positive vectors $p, q \in \mathbb{R}^n_{++}$ of the same dimension, the *Kullback-Leibler divergence of* p *relative to* q is defined as:

$$D(p \parallel q) \doteq \sum_{i=1}^{n} \left[p_i(\log p_i - \log q_i) - (p_i - q_i)\right]$$

Prove that:

1. $D(p \parallel q) \geq 0$, with equality if and only if $p = q$.
2. The function $\begin{cases} D : \mathbb{R}^n_{++} \times \mathbb{R}^n_{++} \to \mathbb{R}_+ \\ D(p \parallel q) = \sum_{i=1}^n \left[p_i(\log p_i - \log q_i) - (p_i - q_i)\right] \end{cases}$ is strictly convex.