Spectrum Sharing with Distributed Interference Compensation

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DySPAN’05
Motivation

- Several bands available for spectrum sharing.
  - Commercial bands offered in the secondary markets.
  - Government owned bands available to other service providers.
- Several users want to share the bands.
  - Can be end-users or service providers.
  - Each user can use only one band.
  - Concurrent transmissions in the same band cause mutual interferences.
- We consider the problem of joint channel selection and power control.
Each user is represented by a transmitter-receiver node pair.

- Single-hop and half-duplex transmissions.
- Channel gains are fixed (slow fading).
- No centralized controller.
- How to select channel and control power in a distributed way with limited (scalable) information exchange?
Related Work

- Channel selection in cellular networks has been studied extensively (e.g., Katzela and Naghshineh’96, Aardal et. al.’03)
- Recently extended to IEEE 802.11 WLANs (e.g., Kyasanur and Vaidya’05)
Related Work

- Channel selection in cellular networks has been studied extensively (e.g., Katzela and Naghshineh’96, Aardal et. al.’03)
- Recently extended to IEEE 802.11 WLANs (e.g., Kyasanur and Vaidya’05)
- We consider joint channel selection and power control in an ad hoc setting:
  - Allow different users to use the same channel.
  - Mitigate interference by exchanging limited information.
  - Fast convergence and substantial performance improvements compared with other heuristics.
Talk Outline

- Network model and performance metric.
- Price-based channel selection and power control algorithm.
- Convergence analysis.
- Numerical performance study.
Network Model

- $I$ transmitter-receiver pairs (users).
- $K$ parallel channels for all users.
Network Model

- $I$ transmitter-receiver pairs (users).
- $K$ parallel channels for all users.
- User $i$ chooses to transmit in one channel, $\varphi(i)$, with power $p_i^{\varphi(i)}$:  
  - Transmission power constraint
    \[ P_{i,\text{min}} \leq p_i^{\varphi(i)} \leq P_{i,\text{max}} \]
  - Received SINR in channel $\varphi(i)$
    \[ \gamma_i^{\varphi(i)} = \frac{p_i^{\varphi(i)} h_i^{\varphi(i)}}{n_0^{\varphi(i)} + \sum_{j \neq i} p_j^{\varphi(i)} h_j^{\varphi(i)}} \]
Utility Functions

User $i$’s QoS preference is given by utility $U_i \left( \gamma_i^{\phi(i)} \right)$.

- $U_i$ is increasing and strictly concave in $\gamma_i^{\phi(i)}$.
- Rate-adaptive applications with elastic demands.
- Private information, only known to the user.

Network performance = total network utility
Total Utility Maximization Problem

- **Goal**: select channel and allocate power in a distributed way to maximize total utility.

- **Challenges**:
  - Channel selection is a discrete (combinatorial) optimization problem.
  - Power assignments across users are coupled due to mutual interference.
  - Objective function may not be concave in power.

- **Our approach**: distributed cooperation by exchange of interference prices.
Single-Channel Asynchronous Distributed Pricing (SC-ADP) Algorithm

- **Price Announcement**: user $i$ announces an interference price $\pi_i^{\varphi(i)}$ in the currently selected channel $\varphi(i)$

  \[
  \pi_i^{\varphi(i)} = \left| \frac{\partial U_i(\gamma_i^{\varphi(i)})}{\partial \left( \sum_{j \neq i} p_j^{\varphi(i)} h_{ji}^{\varphi(i)} \right)} \right|. 
  \]

- **Channel Selection and Power Update**: user $i$ chooses channel $\varphi(i)$ and power $p_i^{\varphi(i)}$ to maximize surplus

  \[
  s_i = U_i \left( \gamma_i^{\varphi(i)} \left( p^{\varphi(i)} \right) \right) - p_i^{\varphi(i)} \sum_{j \neq i} \pi_j^{\varphi(i)} h_{ji}^{\varphi(i)}
  \]

  - Repeat two steps asynchronously across users.
  - Only need to announce a **single** price and measure local channel gains ($h_{ij}^k$ for all $j$ and $k$).
Convergence of SC-ADP

- Depends on the concavity of utility functions.
  - Define the coefficient of relative risk aversion of a utility $U_i$ to be
    \[
    Q_i(\gamma_i) = -\frac{\gamma_i U''_i(\gamma_i)}{U'_i(\gamma_i)}.
    \]
  - larger $Q_i(\gamma_i)$ ⇒ “more concave” $U_i$. 

Examples:
- $U_i(\gamma_i) = \log(1 + \gamma_i) \Rightarrow Q_i(\gamma_i) \in (0, 1]$.
- $U_i(\gamma_i) = \theta_i \gamma_i^{\alpha_i} (\text{with } \alpha_i \in [-1, 0) \cup (0, 1]) \Rightarrow Q_i(\gamma_i) \in (0, 2]$. 

Proof: show users will not oscillate in channel selection.
Convergence of SC-ADP

- Depends on the concavity of utility functions.
  - Define the coefficient of relative risk aversion of a utility $U_i$ to be
    
    $$Q_i(\gamma_i) = -\frac{\gamma_i U_i''(\gamma_i)}{U_i'(\gamma_i)}.$$

  - larger $Q_i(\gamma_i) \Rightarrow$ “more concave” $U_i$.

- Prop: For a two-user $K$-channel network, the SC-ADP algorithm with sequential updates converges if
  
  (1) Both users have $Q_i(\gamma_i) \in (0, 1]$, or
  (2) Both users have $Q_i(\gamma_i) \in (0, 2]$, and $0 < P_i^{\text{min}} = P_i^{\text{max}}$.

- Examples:
  
  - $U_i(\gamma_i) = \log(1 + \gamma_i) \Rightarrow Q_i(\gamma_i) \in (0, 1]$.
  - $U_i(\gamma_i) = \theta_i \gamma_i^\alpha / \alpha$ (with $\alpha \in [-1, 0) \cup (0, 1]$) $\Rightarrow Q_i(\gamma_i) \in (0, 2]$.

- Proof: show users will not oscillate in channel selection.
utility $\log(1 + \gamma_i^{\varphi(i)})$, 4 channels, 10 users
Convergence of SC-ADP: Channel Selection

utility $\log(1 + \gamma_i^{\phi(i)})$, 4 channels, 10 users
Performance Comparison

- **SC-ADP Max Power**: user $i$ transmits with maximum power in the channel that maximizes surplus.

- **Best SINR**: user $i$ transmits with maximum power in the channel that yields the highest SINR:
  \[
  \varphi(i) = \arg \max_k \frac{h_{ii}^k}{n_0 + \sum_{j \neq i} p_j^k h_{ji}^k}.
  \]

- **Best Channel**: user $i$ transmits with maximum power in the channel with the largest channel gain
  \[
  \varphi (i) = \arg \max_k h_{ii}^k.
  \]
Performance Comparison

utility \log(1 + \gamma_i^{\varphi(i)})$, 4 channels, $10m \times 10m$ area
Performance Comparison

- **Multi-channel ADP (MC-ADP):** user $i$ allocates power across $K$ channels to maximize surplus

\[
\sum_{k=1}^{K} \log \left( 1 + \gamma_i^k \right) - \sum_{k=1}^{K} p_i^k \sum_{j \neq i} \pi_j^k h_{ij}^k,
\]

subject to total power constraint $\sum_k p_i^k \leq P_i^{\text{max}}$.

- **Iterative Water-filling (IWF):** user $i$ allocates power across $K$ channels to maximize rate

\[
\sum_{k=1}^{K} \log \left( 1 + \gamma_i^k \right),
\]

subject to total power constraint $\sum_k p_i^k \leq P_i^{\text{max}}$. 
4 channels, $10m \times 10m$ area
Conclusions

- Presented distributed channel selection and power control algorithm for multi-channel wireless networks.
- Users exchange prices, which reflect their sensitivities to interference.
- Each user chooses a channel, and adjusts power to maximize its surplus.
- Proved convergence of the SC-ADP algorithm in some special cases.
- Showed substantial performance improvement over other heuristics.
Q & A