Midterm #1 Solutions – EECS 145L Fall 2006

1a

$V_0 = (k/f)(V_1 - V_2) \qquad V_2 = V_0 R_1 / (R_1 + R_2)$

$V_0 = (k/f)V_1 - (k/f)V_0 R_1 / (R_1 + R_2)$

$V_0\left[1 + (k/f)R_1 / (R_1 + R_2)\right] = V_1(k/f)$

$$G = V_0 / V_1 = \frac{k/f}{1 + (k/f)R_1/(R_1+R_2)} = \frac{1}{(f/k) + R_1/(R_1+R_2)} = \frac{R_1+R_2}{(f/k)(R_1+R_2)+R_1}$$

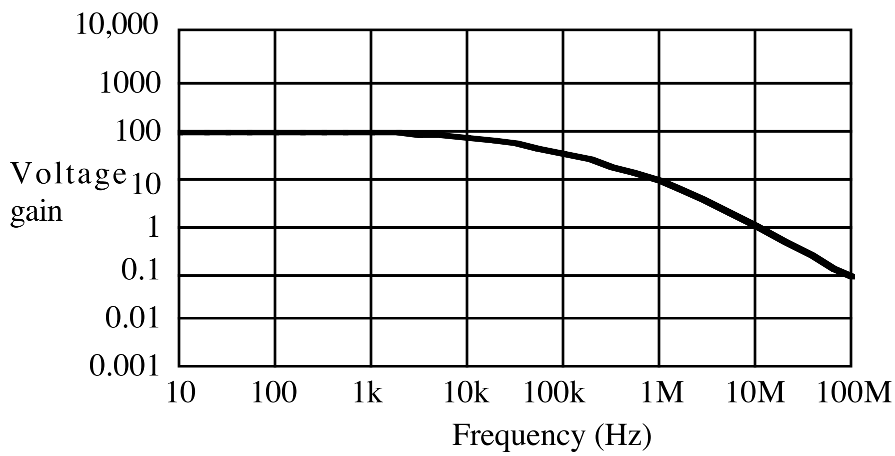
[10 points off for $G = (R_1 + R_2)/R_1$]

1b

$$\boxed{G = V_0 / V_1 = \frac{100}{1 + f/10^5\,\text{Hz}}}$$

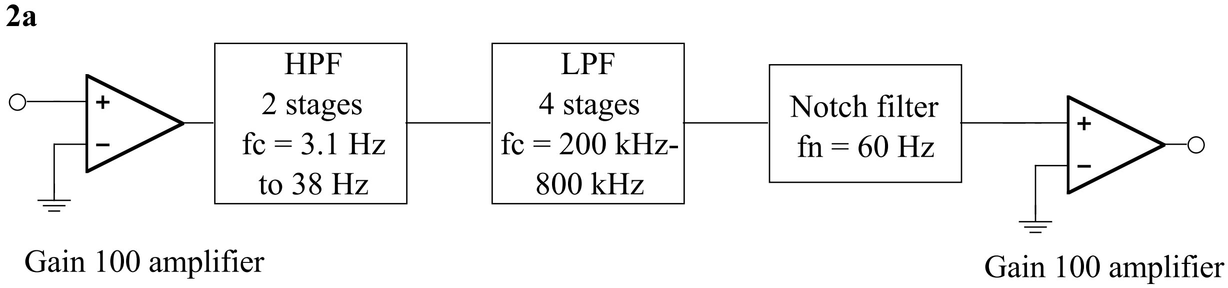
$G = 100$ for $f \ll 10^5$ Hz $\qquad G = 91$ for $f = 10^4$ Hz $\qquad G = 50$ for $f = 10^5$ Hz

$G = 9.1$ for $f = 10^6$ Hz $\qquad G = 1$ for $f = 10^7$ Hz $\qquad G = 0.1$ for $f = 10^8$ Hz



[-5 points if gain not 100 at low frequency- this was the circuit you constructed and measured in Lab 4]

2a



Differential gain:

The 1 mV signal in the 100 to 100 kHz range must be amplified by a factor of 10,000 to bring it to the desired 10 V amplitude.

If a gain of 10,000 is used before filtering, the 10 mV interferences will be amplified to 100V and cause saturation. Some amplification before filtering is desirable to minimize the effect of additive noise in the filter stages. The final amplification is done after the filters. Putting the 10,000x amplifier after the filter stages was also acceptable.

Since the signal is to be amplified with an accuracy of 1%, any filters used to remove unwanted frequencies must have gains >0.99 over the 100 to 100 kHz frequency range.

5 MHz interference:

The unwanted 5 MHz interference has an amplitude of 10 mV and must be amplified by < 10 to produce an output <0.1 V. A notch filter is not recommended for a frequency this high.

A low pass filter is needed to drop the system gain from 10,000 at 100 kHz to <10 at 5 MHz. Therefore the LPF gain must be >0.99 at 100 kHz and <0.001 at 5 MHz, a factor of 50 in frequency.

Looking at the Butterworth LPF gain table:
Initial guess; look down column 0.001 and find where f/fc < f2/f1 = 50; result is n =2
Using n = 2, in column 0.99 f/fc = 0.377; fc = f1/0.377 = 265 kHz
Using n = 2 and fc = 265 kHz, in column 0.001 f/fc = 31.62; f2 = (265 kHz)(31.62) = 8.38 MHz
The LPF with n =2 that passes through gain = 0.99 at 100 kHz passes through gain = 0.001 at 8.38 MHz. This LPF has a gain > 0.001 at 5 MHz and does not meet the requirements.

Using n = 4, in column 0.99 f/fc = 0.614; fc = f1/0.614 = 163 kHz
Using n = 4 and fc = 163 kHz, in column 0.001 f/fc = 5.623; f2 = (163 kHz)(5.623) = 917 kHz
So n = 4 meets the requirements.

[-3 points for using a 5 MHz notch filter, since it is beyond the limits of what is practical (stray capacitance)] [A 5 MHz notch filter was accepted if an n =2 LPF was also used]

0 to 0.1 Hz temperature drift:

The unwanted 0 to 0.1Hz temperature drift has an amplitude of 10 mV and must be amplified by < 10 to produce an output <0.1 V.

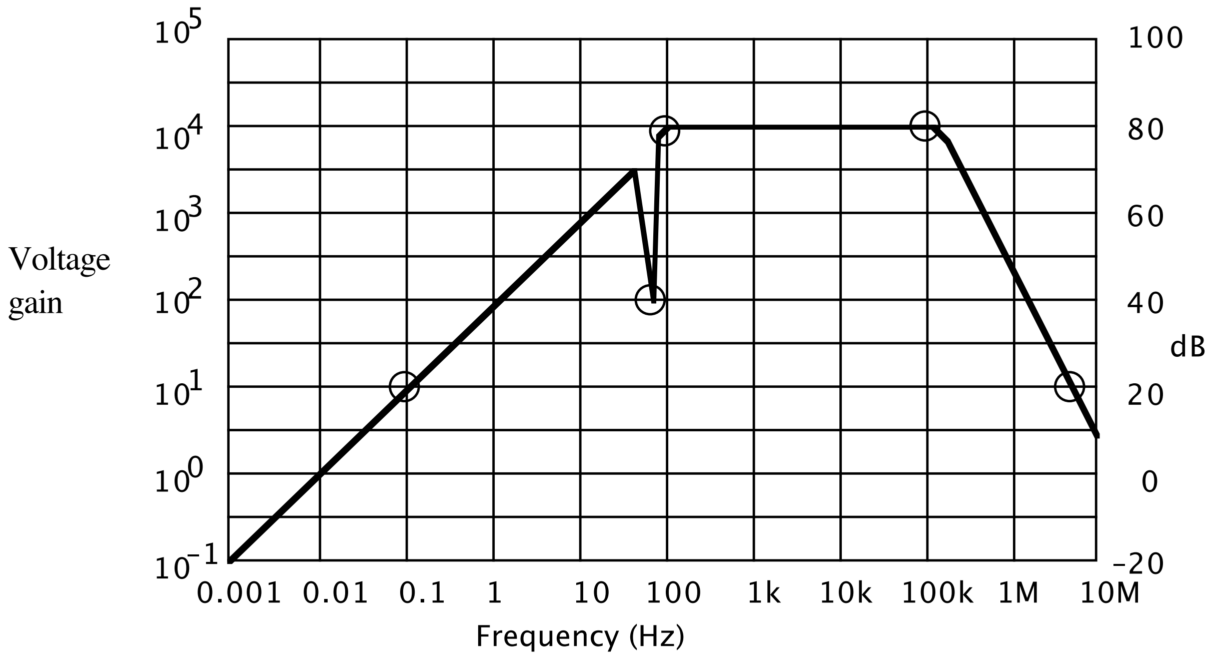
A high pass filter is needed to drop the system gain from 10,000 at 100 Hz to <10 at 0.1 Hz. Therefore the HPF gain must be >0.99 at 100 Hz and <0.001 at 0.1 Hz, a factor of 1000 in frequency.

Using n = 2, in column 0.99 f/fc = 2.649; fc = 37.8 Hz
Using n = 2 and fc = 37.8 Hz, in column 0.001 f/fc = 0.032; f2 = (37.8 Hz)(0.032) = 1.21 Hz
The HPF that passes through gain = 0.99 at 100 Hz passes through gain = 0.001 at 1.21Hz and will be even lower at 0.1 Hz. This satisfies the requirements.

[-2 points for each missing corner or notch frequency]
[-2 points for each missing filter order]
[-2 points for gain of 10,000 before filtering since the interference will saturate before reaching (±10 mV)(10,000) = ±100 volts]
[-2 points for using a HPF with a large number of poles for 60 Hz, rather than a notch filter]
[-2 points for using an excessive value of n for the LPF or HPF, e.g. n>6]
[-1 point for HPF n =1 (need n = 2)]
[-1 point for LPF n = 2 (need n = 4)]
[-2 points if filters are correct but no work is shown]

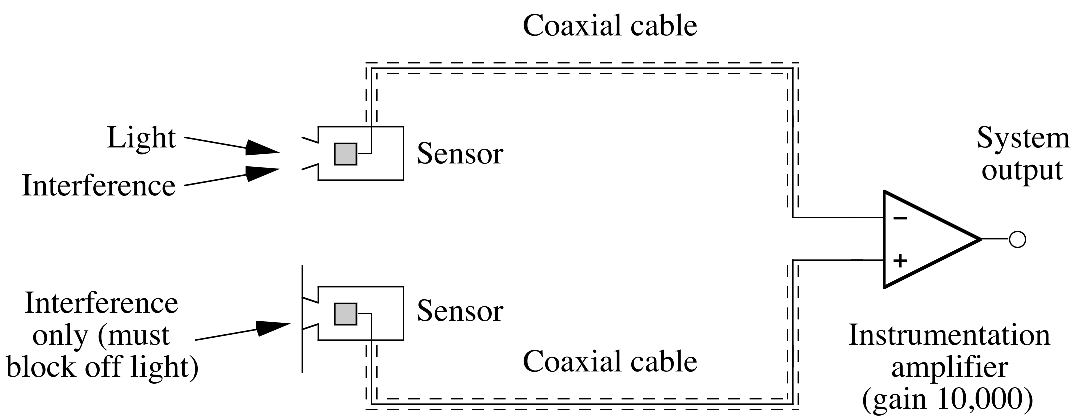
[-5 points for missing an entire element in the system, such as amplification or one of the three required filters]

2b



[-2 points if the gain is not 10^4 in the 100 Hz to 100 kHz band]
[-2 points each if the gains at 0.1 Hz, 60 Hz, and 5 MHz are too high]
[-2 if curve shows gain = 10,000 in the 100Hz to 100 kHz range but 0.001 at 0.1 Hz and 5 MHz; this mixes system gain and filter gain in the same curve]

3a



[-10 points if both sensors see the same signal- differential amplification will then yield zero]
[-10 points if one sensor is blocked from both light and interference]

3b

Differential gain $G_{\pm}$ = 10V/1 mV = 10,000.
Common mode gain G_c < 0.1 V/10 mV = 10 at 0 to 0.1 Hz

Common mode gain G_c < 0.1 V/1 mV = 100 at 60 Hz
Common mode gain G_c < 0.1 V/10 mV = 10 at 1 MHz
So Common Mode Rejection requirements of the instrumentation amplifier are

f = 0 to 0.1 Hz 10^3 or 60 dB

f = 60 Hz 10^2 or 40 dB

f = 5 MHz 10^3 or 60 dB

The most difficult requirement will be at 5 MHz, because stray capacitive coupling makes G_c large at high frequency and limited amplifier gain-bandwidth product makes $G_{\pm}$ smaller at high frequency.

[-8 points for only providing the definition of CMR]
[-2 points for CMRR = 100 (40 dB)]

145L midterm #1 grade distribution:

Problem

1	26.6 (5.2 rms) (30 max)
2	37.3 (2.0 rms) (40 max)
3	26.6 (2.9 rms) (30 max)

maximum score = 100
average score = 90.4 (6.6 rms)

65-69	0	D
70-74	1	D
75-79	0	C
80-84	1	C
85-89	5	B
90-94	4	B
95-99	3	A
100	2	A

GPA 3.1