Midterm #1 Solutions – EECS 145L Fall 1999

1a The op-amp equation is $V_0 = A(V_+ - V_-)$

If V_0 is finite and A is infinite, then $V_+ = V_-$ (virtual short rule)

$V_- = V_+ = V_2 R_2 /(R_1 + R_2)$

Since no current flows in or out of the op-amp inputs

$$\frac{V_+ - V_1}{R_1} + \frac{V_+ - V_0}{R_2} = \frac{V_+(R_1 + R_2) - V_1 R_2 - V_0 R_1}{R_1 R_2} = \frac{V_2 R_2 - V_1 R_2 - V_0 R_1}{R_1 R_2} = 0$$

$$\frac{V_2 - V_1}{R_1} = \frac{V_0}{R_2} \qquad G_\pm = V_0/(V_2 - V_1) = R_2 / R_1$$

[5 points off for correct setup followed by algebraic errors]
[10 points off for trying without using the virtual short rule]
[15 points off for missing or incorrect setup]

1b To determine common mode gain, set $V_1 = V_2$ in the equation $\frac{V_2 - V_1}{R_1} = \frac{V_0}{R_2}$, and we have $V_0 = 0$ and $G_c = 0$

2 $$4\text{nVHz}^{-1/2}\sqrt{\Delta f} = 1.29 \times 10^{-10}\ \Omega^{-1/2}\ \text{Hz}^{-1/2}\sqrt{R\Delta f}$$

$$\sqrt{R} = 40/1.29 \qquad R = 961\ \Omega$$

3a Differenial gain 1000, bandwidth 10 kHz

[3 points off for Gain = 10,000, bandwidth 1 kHz]

3b Output V_{rms} = (4 nV $\text{Hz}^{-1/2}$) (100 $\text{Hz}^{1/2}$) (1000) = 0.4 mV in 10 kHz

[2 points off for 0.4 µV]

3c We want a Butterworth low-pass filter with a gain of 0.99 at 1 kHz and 0.01 at 2 kHz. Look at the 0.99 and 0.01 columns of the equation sheet to find a filter order where the f/f_c ratios are 2 or a bit less. Order 6 gives 2.154/0.723 = 2.98 (too high). Order 8 gives 1.778/0.784 = 2.27 (still too high). Order 10 gives 1.585/0.823 = 1.93 (OK).

The order is 10 and the corner frequency is 1/0.823 = 1.22 kHz. (order 12 also accepted)

[5 points off for not giving corner frequency]

[2 points off for not specifying low pass]

[2 points off for order 8]

After amplification and filtering, the output noise would be V_{rms} = (4 nV $\text{Hz}^{-1/2}$) $\sqrt{1.22\ \text{kHz}}$ (1000) = (4 nV $\text{Hz}^{-1/2}$) (34.9 $\text{Hz}^{1/2}$) (1000) = 0.139 mV in 10 kHz

[4 points off if output noise not given]

So the filtering reduced the output noise from ±0.4 mV to ±0.14 mV

3d The best way to reduce the 60 Hz interference from the middle ot a band of frequencies of interest is to use a notch filter. The common mode rejection ratio of 60 dB means that the common mode gain is 1000/1000 = 1. So the instrumentation amplifier 60 Hz output will be ±10 mV from a common mode input of ±10 mV. The output due to a differential 60 Hz interference is (±0.01 mV) (1000) = ±10 mV. In the worst case, these are in phase, producing a total of ±20 mV. A notch filter can reduce this total by a factor of typically 30, to ±0.7 mV.

[Any value between 0.1 and 2 mV was accepted for full credit]

[3 points off for including one 10 mV and not the other]

[6 points off if input noise not given]

[10 points off for using a HPF, which removes the important earthquake frequencies below 60 Hz]

3e Seismometer followed by instr amp of gain 1000, followed by notch filter (60 Hz) followed by low pass filter of order 10 and corner frequency 1.22 kHz. [OK to reverse order of low pass and notch filters]

145L midterm #1 grade distribution:

Problem	
1	32.5 (35 max)
2	9.7 (10 max)
3	47.3 (55 max)

maximum score =	100	
average score =	89.4	
66-70	0	
71-75	1	
76-80	4	C
81-85	1	
86-90	3	B
91-95	4	B
96-100	6	A