Math221 Homework # 5 Solutions (Fall 2009)

Instructor: James W. Demmel

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Question 1.

Matlab code is posted separately on the class web page. There are 3 files. SpMV_test.m takes m, n and d as arguments and produces a random n by m sparse matrix A of density d, stores it in CSR and CSC format, and tests the functions that perform Sparse-Matrix-Vector multiplication (SpMV) y = A*x of matrices in these formats: SpMV_CSR.m and SpMV_CSC.m SpMV_test.m also produces a random n by n symmetric sparse matrix As of density d, storing just the upper half in CSR format (see the comments in the code for details of how the diagonal is handled), and uses both SpMV_CSR.m and SpMV_CSC.m to multiply y=As*x.

Question 2.

Part 1 The (i,j) entry of $A_r \cdot A_b$ is $\sum_k A_r(i,k) \cdot A_b(k,j)$. The only nonzero terms in this sum occur when $A_r(i,k) = A_b(k,j) = 1$, i.e. when there is a red edge from i to k and a blue edge from k to j. Thus there is exactly one term in the sum (each equal to one) for each path from i to j consisting of one red edge (to k) followed by one blue edge (from k), and so the sum is the number of such paths.

Part 2 We begin by noting that Part 1 can be extended to slightly more general graphs where there can be multiple different blue edges in G_b from i to j, and $A_b(i,j)$ is the number of such edges; if we consider paths that use different edges from i to j as different paths, then the entries of $A_r \cdot A_b$ count the number of different paths as before.

We proceed by induction. The base case is $k = 1$, where there is one path of length 1 (a single red edge) in $G_r = (V, E_r)$ from i to j if and only if $A_r(i,j) = 1$. Now assume that for some k, the number of paths of length k in G_r from i to j is $(A_r^k)(i,j)$. We need to prove this for $k+1$. Build the graph $G_b = (V, E_b)$ and matrix A_b as follows: If there are m red paths of length k from i to j in G_r, let there be m different blue edges connecting i directly to j in E_b, and let $A_b(i,j) = m$. By induction $A_b = (A_r)^k$. Now consider all paths from i to j consisting of one red edge followed by one blue edge; by the above extension to Part 1, there are $(A_r \cdot A_b)(i,j) = (A_r)^{k+1}(i,j)$ such paths. But each such path corresponds to a unique path of $k+1$ red edges in G_r. This completes the induction.