CS 70 SPRING 2008 — DISCUSSION #8

LUQMAN HODGKINSON, AARON KLEINMAN, MIN XU

1. Probability Space: The Final Frontier

Exercise 1. Describe fully the probability space associated with the following random experiments, i.e, specify exactly what the sample space is and the probability associated with each sample point.

(1) Aaron flips two fair coins one after another
(2) Aaron flips two fair identical coins at the same time.
(3) Prof. Wagner chooses three students in CS70 uniformly at random to bestow A+.
(4) Min flips a fair coin repeatedly until he sees a head.

Exercise 2. Let the sample space $\Omega = \{0, 1, 2, 3\}$, and let the probability of each sample point be uniform. What is the probability of the events $A = \{1, 2\}$, $B = \{2, 3\}$, $C = \{1, 3\}$? Are events A and B independent? What is $Pr[A|B \cup C]$?

Exercise 3. Consider the case where Prof. Wagner is choosing 3 students from a class of 130 uniformly at random and without replacement. Describe the event that you will be chosen, i.e. specify what sample points this event contains. What is the probability of the event that you will be chosen?

Answer:

2. Luqman's Alarm and Conditional Probability

Luqman lives in a particularly dangerous part of Berkeley (for the sake of this problem) and at any given night, he has a 25% probability of getting burglarized. In desperation, Luqman decided to buy a home alarm system.

Exercise 4. Luqman bought his first alarm from a street vendor on Telegraph avenue. He checked the circuitry of the device and discovered that it was programmed to always have 1/3 probability of going off every night independent of whether there is burglary in progress or not. Use this information to fill out the following chart: (Let A be the event that alarm is going off and B be the event that there is a burglar in the house, and $\neg A$ represent the complement of A, i.e. alarm is not going off)

Sample Point	Probability
$A \cap B$	
$A \cap \neg B$	
$\neg A \cap B$	
$\neg A \cap \neg B$	

Exercise 5. After purchasing his second alarm from Craigslist, Luqman stayed up for 72 nights straight, observed his alarm, and gathered the following data:

Date: April 3, 2008.

What happened?	Happened for how many nights?
Alarm sounded and found burglar	4
Alarm sounded and no burglar	20
Alarm didn't sound and found burglar	14
Alarm didn't sound and no burglar	34

Use these data to approximate the probabilities of the following joint distribution:

Sample Point	Probability
$A \cap B$	
$A \cap \neg B$	
$\neg A \cap B$	
$\neg A \cap \neg B$	

Exercise 6. Assuming that the probability Luqman derived are accurate, use the information to calculate the $Pr[A]$ and $Pr[A|B]$. Note that this example shows that knowing $Pr[A]$ and $Pr[B]$ is not enough to specify the joint distribution. Is Luqman's second alarm a wise purchase?

Exercise 7. Take a break from Luqman's adventure and prove that for any two events A, B of any probability space, $Pr[A|B] + Pr[\neg A|B] = 1$. Interprete this result intuitively. Prove also that X, Y are disjoint events, then $Pr[X \cup Y|B] = Pr[X|B] + Pr[Y|B]$. Notice that setting $Pr_B[\omega] = Pr[\omega|B]$, where $\omega \in \Omega$, defines a new probability space on the same sample space!

Exercise 8. Continuing on from Luqman's catastrophic second alarm. Luqman thought to himself, "Well, since my alarm is more likely to be SILENT when there is a burglar than on average, then it should also be more likely to be silent when there is burglar than when there is not." Prove that Luqman's conjecture is correct by showing that for any probability space and any events A, B, $Pr[A|B] > Pr[A]$ implies that $Pr[A|\neg B] < Pr[A]$. What happens when $Pr[A|B] = Pr[A|\neg B]$? Again, be sure to justify these results intuitively to yourself and not just memorize them as esoteric results of some random algebraic manipulations.

Answer:

Exercise 9. Rather than buying a new alarm system, Luqman contemplated, "because my Craiglist alarm is more likely to be silent when there is a burglar than when there is not, I can just use my alarm in the reverse role and call the police whenever the alarm doesn't sound!" Use conditional probability to show Luqman that this is not a good idea. (hint: calculate $Pr[B|\neg A]$)

Exercise 10. Supposing that the following statistics is correct, use the same reasoning from the previous exercise to explain why it is that introverted people are much more likely to be a salesman than a librarian:

Type	Number
Extrovert and Salesman	20 million
Introvert and Salesman	2 million
Extrovert and Librarian	100
Introvert and Librarian	50000

Exercise 11. Finally, Luqman went to Radioshack and saw an alarm system that is guaranteed to have 99% chance of catching a burglar, i.e. $Pr[A|B] = 99/100$. Should Luqman buy the alarm based on just that information?

Exercise 12. Take another break from Luqman's misadventure and prove that for any two events A, B of any probability space, if event A's occurrence makes event B more likely, i.e. $Pr[B|A] > Pr[B]$, then event B's occurrence also makes event A more likely, i.e. $Pr[A|B] > Pr[A]$.

Exercise 13. After more inquiring at Radioshack, Luqman discovered that $Pr[A|\neg B] = 1/3$ and decided to buy the system. After installing the alarm, Luqman suddenly heard the alarm going off one night. What is the probability that he will catch a burglar?

Answer:

Exercise 14. Shortly after Luqman purchased the Radioshack alarm, the Berkeley city police increased the patrol around Luqman's area and the probability that the thief will visit Luqman at any night is now reduced to 1%. Suppose $Pr[A|B]$ stays constant because of the circuitry inside the alarm, will $Pr[A]$ increase or decrease? What about $Pr[B|A]$?

Exercise 15. After a year, the Berkeley police department made some secret patrol changes. Now, Luqman observes that on any given night, his alarm has 2/3 chance of going off. Did the Berkelely PD increase or decrease the patrol around Luqman's apartment? What is the new probability that Luqman will get burglarized on a night?