CS 70 SPRING 2008 — DISCUSSION #2

LUQMAN HODGKINSON, AARON KLEINMAN, MIN XU

1. Administrivia

- Office Hours have been decided. Please go if you have any questions about the material covered, trouble about the homework, or if you want to talk about other topics or life in general. Feel free to email the course staff to arrange alternate office hours if none of the ones provided fit your schedule.

2. Simple Inductions

Exercise 1. Prove that where $x \neq 1$

$$\sum_{i=0}^{n} x^i = \frac{x^{n+1} - 1}{x - 1}$$

Exercise 2. Prove that

$$\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

3. Some Trickier Inductions

Exercise 3. Prove that if a number n comprises of just 3^k digits of "1"s (i.e. $n = \underbrace{111...111}_{3^k}$) where $k \in \mathbb{N}$ and $k \geq 1$, then n is divsible by 3^k. For example, 111 is divisible by 3, 111,111,111 is divisible by 9, and so forth. (**hint:** Try dividing 111,111,111 by 111 and remember that a number is divisble by 3 iff the sum of all of the digits of the number is divisible by 3.)

4. Variations on Induction

Exercise 4. Suppose you know that $P(1)$ is true, and that $\forall k \geq 1, P(k) \Rightarrow P(2k)$. Use induction to show that $P(n)$ is true whenever n is a power of 2.

Exercise 5. The sequence of Fibonacci numbers is defined by: $F_1 = F_2 = 1$ and $F_{n+1} = F_n + F_{n-1}$ if $n \geq 2$. Prove that any natural number can be represented as the sum of several different Fibonacci numbers.

Exercise 6. Prove that $2^{m+n-2} \geq mn$ where m, n are positive integers.

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