CS 70 SPRING 2008 — DISCUSSION #14

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1. Chebyshev's

Exercise 1. Suppose you roll a fair die 60 times. Use Chebyshev's inequality to bound the probability that you will get fewer than 5 sixes.

2. Continuous Distributions

Exercise 2. Let X be a nonnegative random variable with probability density function $f(x) = \frac{1}{(1+x)^2}$.

(1) What is $\Pr[X > 3]$?
(2) Compute $\mathbb{E}[X]$.

3. Central Limit Theorem

Exercise 3. The Cal Foosball Team plays 100 matches over the course of the season. Suppose that the probability that the team wins a given game is 80%, and that the games are independent. Approximate the probability that the team wins more than 90 games.

4. Countability

Exercise 4. Prove that if $(S_1, S_2, ...)$ is a countably large collection of countable and disjoint sets, then $S = S_1 \cup S_2 \cup S_3...$ is also countable.

Exercise 5. Let Σ be a finite set of symbols, e.g. $\{a, b, c\}$ and define a *word* to be a finite sequence of symbols. A *language* L is then defined as a set of words. Prove that every language contains either finite or countably infinite many words.

Exercise 6. Determine whether the following sets are countable or uncountable. You can informally outline your proof.

(1) The set of all natural number polynomials in 3 variables.
(2) The set of all graphs.
(3) The set of all functions of natural numbers (i.e. $f : \mathbb{N} \to \mathbb{N}$)
(4) The set of all real numbers whose decimal representation has only finitely many non-zero digits.
(5) The set of all decimal numbers whose decimal representation whose decimal representation has only finitely many non-zero or non-one digits.

5. Wile E. Coyote

Exercise 7. Wile E. Coyote just bought some new Acme bombs and wants to drop one on the roadrunner. Wile E. Coyote hypothesizes that the roadrunner is at some location p, moving at some velocity v, and accelerating with constant acceleration a, where $p, v, a \in \mathbb{N}$. If this hypothesis is true, devise a strategy by which Wile E. Coyote is guaranteed to drop an Acme bomb on the roadrunner within a finite amount of time.

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