CS 70 SPRING 2008 — DISCUSSION #12

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1. Warm-Up

Exercise 1. Decide whether the following statements are True or False:

(1) Like expectation, the variance of a random variable can be negative
(2) X has 0 variance if and only if X is constant
(3) Suppose X has uniform distribution over the values $\{0, 1, 2, 3\}$, then X has the highest variance of all random variables over the same range of values
(4) X is a constant random variable if and only if $\mathbf{E}[X^2] = \mathbf{E}[X]^2$
(5) if X, Y are independent random variables with the same uniform distribution, then $X + Y$ has uniform distribution also

2. I Play My Music In The Sun

Exercise 2. Suppose you add a joker to a standard deck of cards and shuffle repeatedly. You then start turning over cards from the top of the deck until the joker appears. What is the expected number of cards you must turn over? What is the variance?

3. Why Don't They Just Lock The Door?

Exercise 3. James Bond is imprisoned in a cell from which there are three possible ways to escape: an air-conditioning duct, a sewer pipe and the door (which is unlocked). The air-conditioning duct leads him on a two-hour trip whereupon he falls through a trap door onto his head, much to the amusement of his captors. The sewer pipe is similar but takes five hours to traverse. Each fall produces temporary amnesia and he is returned to the cell immediately after each fall. Assume that he always immediately chooses one of the three exits from the cell with probability $\frac{1}{3}$. On the average, how long does it take before he realizes that the door is unlocked and escapes?

Hint: if you're doing a lot of algebra, you're taking the wrong approach.

4. String Theory

Exercise 4. Luqman holds n pieces of string in his closed fist so the $2n$ ends are all sticking out. Each second, you randomly and uniformly choose two of the ends and tie them together. When all the ends are tied, Luqman opens his fist and the strings fall to the floor. Let the random variable X denote the number of string bracelets that you have created. Compute $\mathbf{E}[X]$. Find an expression for $\mathrm{Var}(X)$.

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