CS 70 Spring 2008 — Discrete Mathematics for CS — David Wagner — HW 9

Due Thursday, April 3rd

1. (8 pts.) De Méré's problem

The birth of the probability theory has been partially attributed (by some) to two problems that the Chevalier de Méré posed to Pascal and Fermat in the 17th century, thereby triggering Pascal and Fermat to study this field systematically. Here is the first of de Méré's two problems.

(a) What is the probability of getting at least one double six in 24 throws of a pair of fair dice?

(b) What is the probability of getting at least one six in 4 throws of a fair die?

(c) Which is larger?

Historical side note, for the curious: When Pascal told M. de Méré that the answer to part (a) is different from part (b), de Méré initially refused to believe it, insisting that $24/36 = 4/6$ and therefore the two cases must be equivalent.

2. (12 pts.) Poisoned pancakes

You've been hired as an actuary by IHOP corporate headquarters, and have been handed a report from corporate intelligence that indicates that a covert team of ninjas hired by Denny's will sneak into some IHOP, and will have time to poison 5 of the pancakes being prepared (they can't stay any longer to avoid being discovered by Pancake Security). Given that an IHOP kitchen has 50 pancakes being prepared, and there are 10 patrons, each ordering 5 pancakes (which are chosen uniformly at random from the pancakes in the kitchen), calculate the probability that a particular patron at the targeted IHOP:

(a) will not receive any poisoned pancakes

(b) will receive exactly 1 poisoned pancake

(c) will receive at least one poisoned pancake

3. (7 pts.) Monty Hall revisited

In this variant of the Monty Hall problem, after the contestant has chosen a door, Monty asks another contestant to open one of the other two doors. That contestant, who also has no idea where the prize is, opens one of those two remaining doors at random, and (as it happens) you both see that there is no prize there. Monty now asks you if you wish to switch or stick with your original choice. What is your best strategy? Why? What is the probability you win if you stick, given that the other contestant's door didn't contain the prize? What is the probability you win if you switch, given that the other contestant's door didn't contain the prize?

4. (5 pts.) Coins most unfair

Your gambling buddy found a website online where he could buy trick coins that are heads or tails on both sides. He puts three coins into a bag: one coin that is heads on both sides, one coin that is tails on both sides, and one that is heads on one side and tails on the other side. You shake the bag, draw out a coin at random, put it on the table without looking at it, then look at the side that is showing. Suppose you notice that the side that is showing is heads. What is the probability that the other side is heads? Show your work.

5. (8 pts.) Money bags

I have a bag containing either a \$1 or \$5 bill (with probability $1/2$ for each of these two possibilities). I then add a \$1 bill to the bag, so it now contains two bills. The bag is shaken, and you randomly draw a bill from the bag (without looking). Suppose it turns out to be a \$1 bill. If a second student draws the remaining bill from the bag, what is the chance that it too is a \$1 bill? Show your calculations.

6. (10 pts.) Discreet math

A non-profit wants to poll a sample of people to ask them whether they have ever had an extramarital affair. This being an extremely sensitive subject, one obvious problem is that if the surveyers ask this question straight-out, respondents may lie to avoid revealing personal information about their private lives.

The surveyers come up with the following clever scheme. They will ask the respondent to secretly roll a fair die. If the die comes up 1, 2, 3, or 4, the respondent is supposed to answer truthfully. If the die comes up 5 or 6, the respondent is supposed to answer the *opposite* of the truthful answer. The respondent is cautioned not to reveal what number came up on the die. Notice that if the respondent answers "Yes," this answer is not necessarily incriminating: for all the surveyer knows, this particular respondent might have rolled a 5 or 6 and might have never had an affair in his/her life.

Let p be the probability that, if we select a person at random, then they will have had an extramarital affair. (Of course, the surveyers do not know p; that is what they want to estimate.) Let q denote the probability that, if we select a person at random and have them to follow the scheme above, then they will answer "Yes."

(a) Calculate a simple formula for q, as a function of p.

(b) Next, suppose the surveyers have estimated q. Now they want to solve for p. Find a simple formula for p, as a function of q.