CS 70 Spring 2008 — Discrete Mathematics for CS — David Wagner — HW 1

Due January 31

Administrative details of preparing and submitting homework submissions: In CS 70 this semester, you will be submitting all homework solutions online. (You will probably recognize the submission system from CS 61ABC.) To get ready for online submission, get an EECS instructional computer account if you don't have one already, and register with the grading system. See the CS 70 home page for instructions on how to do this.

You should typeset your solution using the LaTeX document processing system. Submit both the LaTeX source files (e.g., `hw1.tex`) as well as the resulting PDF document (e.g., `hw1.pdf`). You can produce a PDF file from your LaTeX input with the command

```
pdflatex hw1.tex
```

Before submitting, make sure you check carefully that the PDF comes out correctly and correct any errors. We suggest using

```
acroread hw1.pdf
```

Each homework assignment will be accompanied with a LaTeX template for you to fill in your answers. The templates will be accessible from the CS 70 home page.

Your submission needs to start with the following information:

Your full name
Your login name
The name of the homework assignment (e.g. hw1)
Your section number
Your list of partners for this homework, or "none" if you had no partners

To submit your answers to this homework assignment, create a directory named `hw1`, copy your solution files to that directory, `cd` to that directory, and then give the command

```
submit hw1
```

You are welcome to form small groups (up to four people) to work through the homework, but you **must** write up all your solutions on your own.

1. (4 pts.) Getting started

What is David Wagner's favorite number? The answer is found on the course newsgroup, `ucb.class.cs70`. Look for the post from David Wagner titled "The answer to question 1," and write down the answer you find there. Instructions on how to access the newsgroup may be found on the course web page.

(Why are we having you do this? The class newsgroup is your best source for recent announcements, clarifications on homeworks, and related matters, and we want you to be familiar with how to read the newsgroup.)

2. (5 pts.) Exclusive OR

The "exclusive OR" connective (written as XOR or $\oplus$) is just what it sounds like: $P \oplus Q$ is true when exactly one of P, Q is true (but not both). Write down the truth table for $P \oplus Q$ and for $(P \wedge \neg Q) \vee (\neg P \wedge Q)$, and hence show that $P \oplus Q$ is logically equivalent to $(P \wedge \neg Q) \vee (\neg P \wedge Q)$.

3. (8 pts.) Implications

Which of the following implications is true?

1. If 30 is divisible by 10 then 40 is divisible by 10.
2. If 30 is divisible by 9 then 40 is divisible by 10.
3. If 30 is divisible by 10 then 40 is divisible by 9.
4. If 30 is divisible by 9 then 40 is divisible by 9.

4. (15 pts.) Practice with quantifiers

Which of the following propositions is true? In part 5, $Q(k)$ denotes the proposition "$1+2+\cdots+k = k(k+1)/2$".

1. $(\forall x \in \mathbb{N} \,.\, x^2 < 5) \Longrightarrow (\forall x \in \mathbb{N} \,.\, x^2 < 4)$.
2. $(\forall x \in \mathbb{N} \,.\, x^2 < 4) \Longrightarrow (\forall x \in \mathbb{N} \,.\, x^2 < 5)$.
3. $\forall x \in \mathbb{N} \,.\, (x^2 < 5 \Longrightarrow x^2 < 4)$.
4. $\forall x \in \mathbb{N} \,.\, (x^2 < 4 \Longrightarrow x^2 < 5)$.
5. $\forall n \in \mathbb{N} \,.\, Q(n) \Longrightarrow Q(n+1)$.

5. (10 pts.) Working with quantifiers

Recall that $\mathbb{N} = \{0, 1, 2, \ldots\}$ denotes the set of natural numbers.

1. For any natural number n, let $P(n)$ denote the proposition

   $$P(n) = \forall i \in \mathbb{N} \,.\, (i < n \Longrightarrow (\forall j \in \mathbb{N} \,.\, (j = n \vee n \neq ij))).$$

   Concisely, for which numbers $n \in \mathbb{N}$ is $P(n)$ true?

2. Rewrite the following quantified proposition in an equivalent form with all negations ("$\neg$", "$\neq$") removed.

   $$\neg\forall i \in \mathbb{N} \,.\, \neg\exists j \in \mathbb{N} \,.\, \exists k \in \mathbb{N} \,.\, \neg\forall \ell \in \mathbb{N} \,.\, f(i,j) \neq g(k,\ell).$$

6. (8 pts.) You be the grader

Assign a grade of A (correct) or F (failure) to the following proof. If you give a F, please explain exactly everything that is wrong with the structure or the reasoning in the *proof*. Justify your answer (saying that the claim is false is *not* a justification).

Theorem 0.1: $\forall n \in \mathbb{N} \,.\, n^2 \leq n \Longrightarrow (n+1)^2 \leq n+1$.

Proof: Suppose that $n \in \mathbb{N}$ and $n^2 \leq n$. (Otherwise, there is nothing to prove.) We need to show that

$$(n+1)^2 \leq n+1.$$

Working backwards we see that:

$$\begin{aligned} (n+1)^2 &\leq n+1 \\ n^2+2n+1 &\leq n+1 \\ n^2+2n &\leq n \\ n^2 &\leq n \end{aligned}$$

So we get back to our original hypothesis which was assumed to be true. Hence, for every $n \in \mathbb{N}$ we know that if $n^2 \leq n$, then $(n+1)^2 \leq n+1$. $\square$