Information Theory and Large Sensor Networks

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Is There A Role For Information Theory

Information theory is the theory of optimal communication systems,

where "optimal" means: regardless of coding complexity and delay.

This definitely looks like a meaningless criterion for sensor networks.

So Let's Just Go Ahead...

Let the sensor apply some simple quantization to the sensed values.
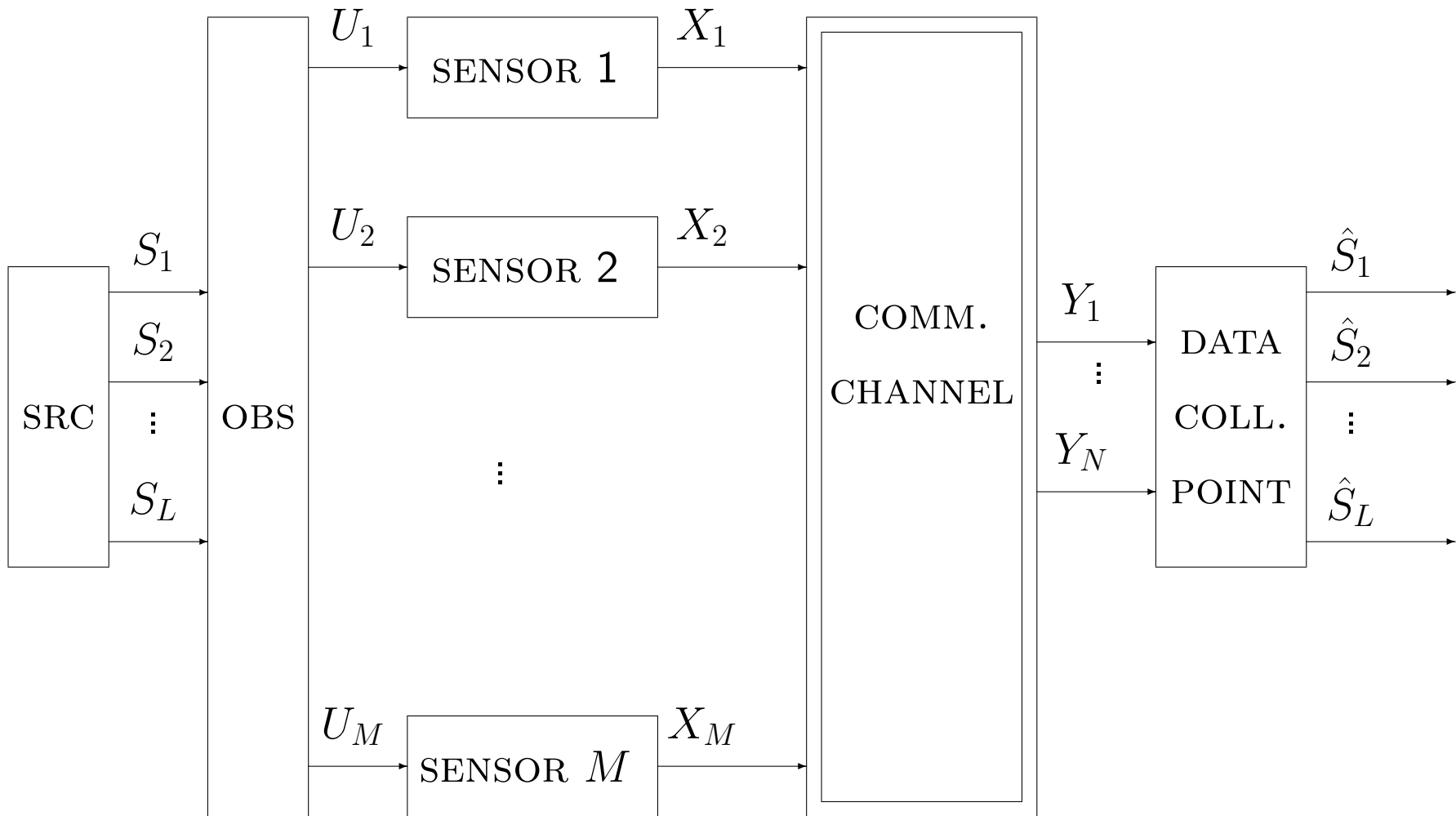
Use some error-correcting codes on the channel to best communicate the bit streams resulting from quantization.

This probably gives reasonable performance, and with the coding complexity available to the sensor nodes and the affordable delays, it's probably not possible to do substantially better.

Let's Look At An Example
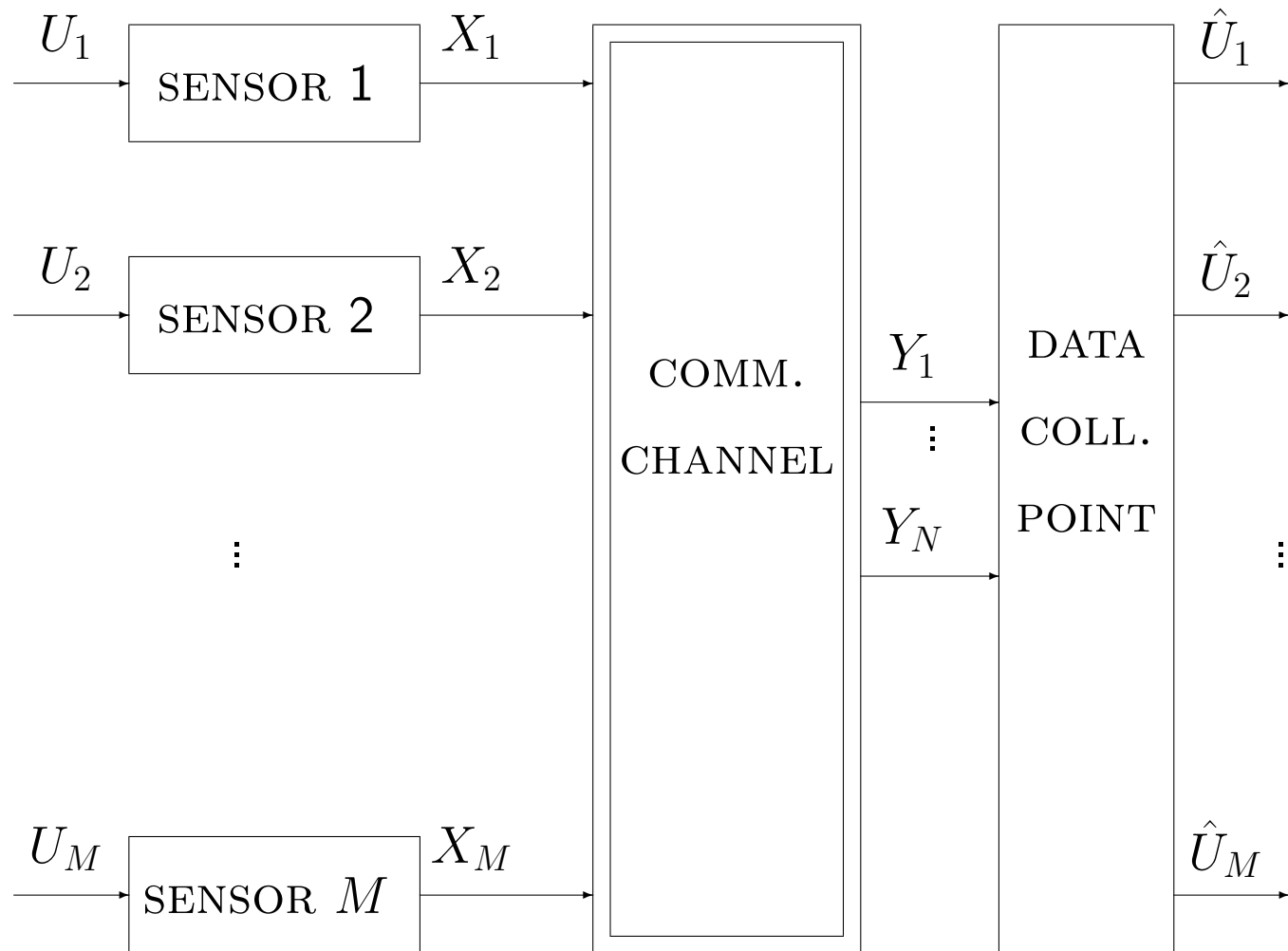
Let's Look At An Example

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where

each sensor observes a certain linear combination of the sources, plus additive white Gaussian (measurement) noise,

and the communication channel is wireless, i.e., it adds up scaled copies of what the sensors transmit, again subject to additive white Gaussian noise.

The sensors share a total power P_{tot}.

Each sensor quantizes its observation...

Suppose the total bit rate available to the sensors is R_{tot},

and the sources need to be reconstructed to within the smallest mean-squared error possible.

Now, let's apply the most sophisticated quantization possible.

Even in the full information-theoretic sense, this cannot give a distortion smaller than

$$D(M, R_{tot}, L) = \frac{\sigma_S^2 \sigma_W^2}{M\sigma_S^2 + \sigma_W^2} + \frac{\sigma_S^4}{\sigma_S^2 + \sigma_W^2/M} \frac{1}{1 + \frac{\sigma_S^2}{L\sigma_W^2}R_{tot}}$$

$$\sim \frac{c_1}{M} + \frac{c_2}{R_{tot}}$$

Sensors use error-correcting codes...

The best rate that any error-correcting code can achieve is smaller than

$$C(M, P_{tot}, K, N) \;=\; KN \log_2 \left(1 + \frac{M P_{tot}}{KN\sigma_Z^2} \right) \text{ Bits.}$$

Combining this with the quantization step, this intuitive approach gives at best a distortion (fidelity) of

$$
\begin{aligned}
D(&M, P_{tot}, L, K, N) \\
&= \frac{\sigma_S^2 \sigma_W^2}{M\sigma_S^2 + \sigma_W^2} + \frac{\sigma_S^4}{\sigma_S^2 + \sigma_W^2/M} \frac{1}{1 + \frac{KN\sigma_S^2}{L\sigma_W^2} \log_2 \left(1 + \frac{M P_{tot}}{KN\sigma_Z^2} \right)} \\
&\sim \frac{c_1}{M} + \frac{c_2}{\log_2(M P_{tot})}
\end{aligned}
$$

Instead: Information-theoretic Optimum

For our example, it is possible to determine the information-theoretic optimum:

$$D_{lower}(M, P_{tot}, L, K, N)$$

$$= \frac{\sigma_S^2 \sigma_W^2}{M \alpha_0^2 \sigma_S^2 + \sigma_W^2} + \frac{\alpha_0^2 \sigma_S^4}{\alpha_0^2 \sigma_S^2 + \sigma_W^2/M} \left(\frac{1}{1 + M P_{tot} \beta_0^2/(K N \sigma_Z^2)} \right)^{KN/L}$$

$$\sim \frac{d_1}{M} + \frac{d_2}{M P_{tot}} \qquad \text{versus} \qquad \frac{c_1}{M} + \frac{c_2}{\log_2(M P_{tot})}$$

For a number of examples, it is easy to verify that simple forwarding strategies approach this optimum, as M becomes large.

So?

Is this just some odd coincidence that occurs with probability zero?

We can show that this is not the case.

Its general extension is currently under investigation.

Is There A Role For Information Theory

Information Theory can furnish *intuition* to build communication systems.

"Old Intuition" (Point-to-point)

    Information is "abstract" and takes the same shape always: Bits.

    Optimal communication is: quantization plus clever transmit protocols (error-correction)

"New Intuition" (Networks)

    Information has a concrete shape

    Optimal communication is: ?