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EXPERIMENTAL DEMONSTRATION OF CHAOTIC SYNCHRONIZATION IN THE MODIFIED CHUA'S OSCILLATORS

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In this paper we demonstrate experimentally the properties of bifurcation, chaos, synchronization and the secure communication idea by making use of the modified Chua's oscillators.

1. Introduction

Although Chua's circuit is the simplest chaotic circuit, it has various complex chaotic dynamic properties. Its theoretical analyses, numerical simulations and experimental results can coincide with each other very well. Therefore, it has been studied extensively and deeply. Since two chaotic systems in a master-slave configuration can be made to control and synchronize [Pecora & Carroll, 1990, 1991; Carroll & Pecora, 1991], chaos control and synchronization built the foundation for chaotic secure communication and have become a very active research topic in chaos field. The secure communication via chaotic synchronization has been demonstrated experimentally in the research of Kocarev *et al.* [1992], Parlitz *et al.* [1992], Wu & Chua [1993], Halle *et al.* [1993], Dedieu *et al.* [1993] and Dmitriev *et al.* [1995], Chua's circuit or its decomposition has been used. The chaotic signal is used as a masking signal. Due to the synchronization, the "encrypted" analog or digital information signal can be recovered. Cuomo *et al.* [1993a, 1993b, 1993c] used Lorenz-based chaotic analog circuit to achieve secure communication by the masking signal technique. Frey [1994] and Carroll [1995] introduced the filter technique into the chaotic secure communication.

Recently, we have put forward a new type of modified Chua's circuit in which a RC parallel circuit is added into the L arm of Chua's circuit [Yin, 1996]. This modified Chua's circuit has two additional components more than Chua's circuit and makes the circuit equations become four-dimensional instead of three-dimensional, but it has three advantages in comparison with Chua's circuit: First, when we keep the parameters of Chua's circuit constant and only change R or C in the RC parallel circuit, the various phenomena of bifurcation and chaos can be obtained. It makes Chua's circuit more flexible. Although in the region of parameters that we have investigated up to now, the modified Chua's circuit has shown the same characteristics of bifurcation and chaos as Chua's circuit, but the modified Chua's circuit perhaps is the simplest chaotic circuit with 4 dimensions that is the lowest dimension for hyperchaos. The hyperchaos which has been experimentally demonstrated in coupled Chua's circuits [Kapitaniak *et al.*, 1994a] has much higher dimensions. Second, it is easier to achieve synchronization for the modified Chua's circuits than for Chua's circuits. It means the modified Chua's circuits have better robustness than Chua's circuits. Finally, the inductor L always has inner resistance R_0 (in general, any added resistor in series with L can be included in R_0). Chua's circuit with R_0 is called Chua's oscillator [Chua *et al.*, 1993]. When R_0 becomes large, it is difficult to produce chaos for Chua's oscillator. But for the modified

Chua's oscillator, even though R_0 is quite large (in our experiment, $R_0 > 100\ \Omega$), it can produce chaos by adjusting RC parallal circuit. So we think the modified Chua's oscillator is a good tool to study chaotic dynamics and secure communication.

It should be pointed out that up to now there have been a lot of modifications done to Chua's circuit. For example, the piecewise linear element (Chua's diode) is replaced by a cubic nonlinear element of the form $I(V) = aV + bV^3 (a < 0, b > 0)$ [Murali & Lakshmanan, 1993a]; one inductor and one or two voltage sources are added into Chua's circuit [Murali & Lakshmanan, 1991, 1992, 1993b], these modified Chua's circuits become four-dimensional and autonomous; Chua's circuit is driven by a voltage source [Halle *et al.*, 1992] or by a current source [Itoh *et al.*, 1994] and so on.

The theoretical analysis and numerical simulation of the modified Chua's circuit have been published in our previous paper [Yin, 1996], here we only present the experimental results of the modified Chua's oscillator. This paper is organized as follows: In Sec. 2, we demonstrate experimentally the bifurcation and chaos of the modified Chua's oscillator. In Sec. 3, we demonstrate experimentally the properties of chaotic control and synchronization. In Sec. 4, on the basis of the results presented in Secs. 2 and 3, the chaotic secure communication by using two modified Chua's oscillators has been successfully demonstrated at lest in principle. Finally, a brief conclusion is given in Sec. 5.

2. Bifurcation and Chaos of the Modified Chua's Oscillator

The modified Chua's oscillator is shown in Fig. 1. It is different from Chua's oscillator only in that a RC parallel circuit which consists of R_3 and C_3 is added into the L-arm of Chua's oscillator. In Fig. 1, N_R is a nonlinear resistor with three-segment piecewise-linear v–i characteristic of negative slops, called Chua's diode [Kennedy, 1992]. In our experimetal setup, the v–i characteristic of the nonlinear resistor is given by

$$g(V) = G_b V + 0.5(G_a - G_b)[|V + B_p| - |V - B_p|] \quad (1)$$

where $G_a = -0.78$ mS, $G_b = -0.42$ mS and $B_p = 1.0$ V. In addition, the following parameters are fixed: $C_1 = 9.59$ nF, $C_2 = 99.8$ nF, $C_3 = 0.615\ \mu$F, $L = 43.9$ mH, $R_0 = 105.3\ \Omega$, $R = 1.64$ kΩ.

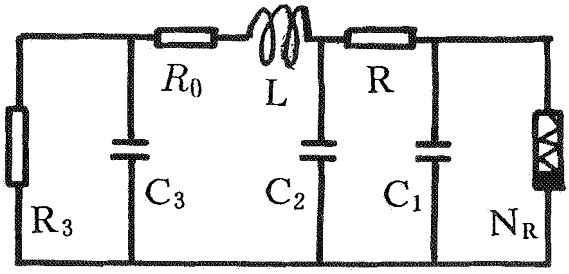


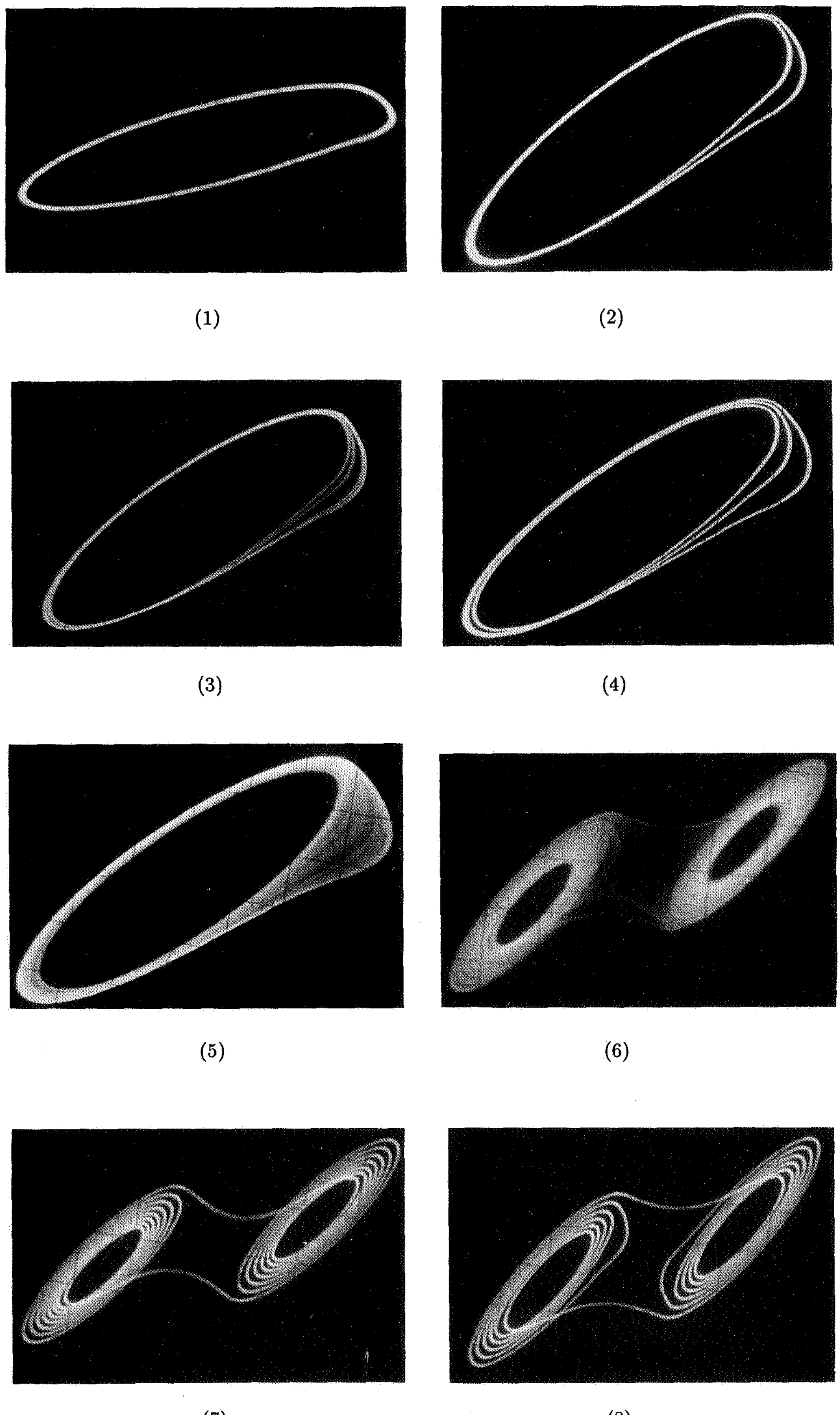
Fig. 1. The modified Chua's oscillator.

Table 1. The route to chaos via period doubling of the modified Chua's circuit as a function of R_3.

Status	$R_3(\Omega)$	No. of Photograph (Fig. 2)
equilibrium point	> 131.3	
period-1 limit cycle	129.5	1
period-2 limit cycle	126.2	2
period-3 limit cycle	124.0	3
period-4 limit cycle	123.7	4
spiral Chua's attractor	120.74	5
double scroll	113.4	6
Chua's attractor		
7-7 window	107.9	7
6-6 window	107.3	8
4-6 window	104.5	9
4-4 window	99.2	10
3-3 window	93.5	11
3-2 window	89.5	12
2-1 window	87.8	13
outer peroidic attractor	< 86.4	14

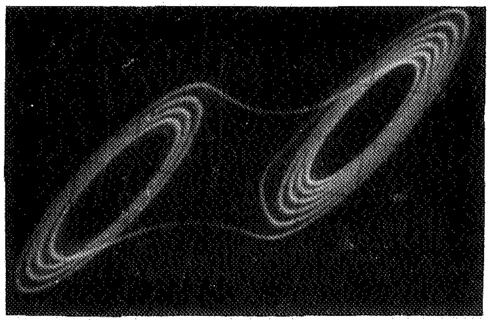
We change R_3 and keep the other parameters constant. From the screen of oscilloscope we take the phase portraits of V_{C_2} and V_{C_1} which are the voltages on the capacitors C_2 and C_1, respectively. The typical results are presented in Table 1 and Fig. 2. From them we can clearly see the route to chaos via period doubling.

Near the period-3 limit cycle, we find the intermittency which is the phenomenon where the signal is virtually periodic except for some irregular (unpredictable) bursts [Chua *et al.*, 1993]. When R_3 increases, the stable period-3 limit cycle becomes unstable but the period-3 limit cycle still remains. Increasing R_3 further, the Chua's spiral attractor appears. This is an another route to approach to



(1) (2) (3) (4) (5) (6) (7) (8)

Fig. 2. The route to chaos via period doubling (horizontal axis: V_{C_1}, vertical axis: V_{C_2}).



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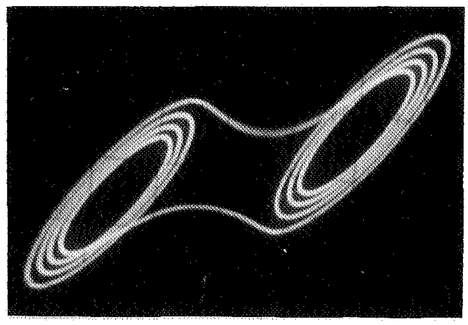

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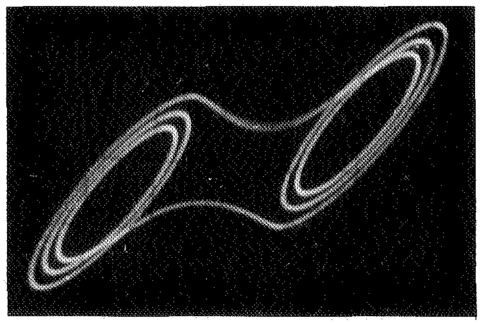

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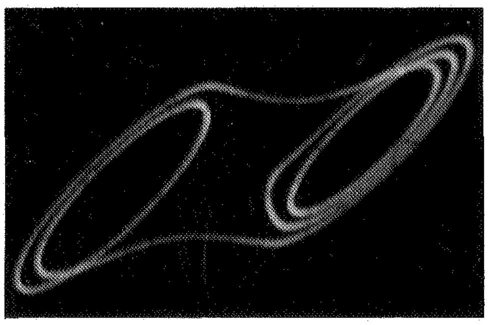

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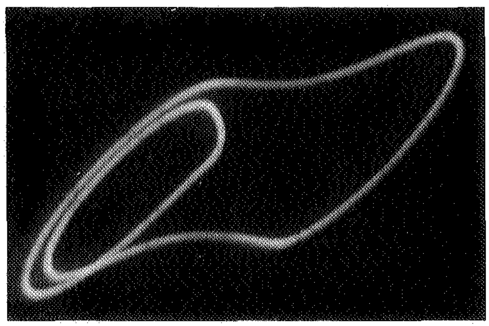

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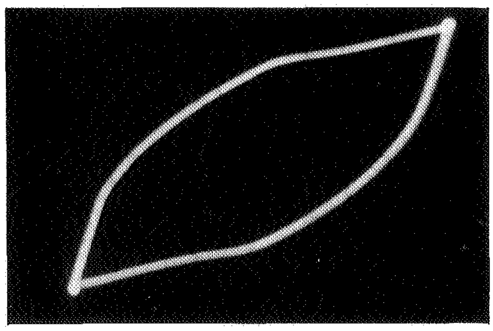


(14)

Fig. 2. (*Continued*)

chaos called type I intermittency which is different from the period doubling as pointed out by Chua *et al.* [1993]. The wave forms ($V_{C_1} \sim t$) of the stable period-3 limit cycle and intermittency and its phase portrait of V_{C_2} and V_{C_1} are shown in Figs. 3(a)–3(c), respectively.

When R_3 is larger than a certain value, the phase portrait becomes a large outer period attractor [Fig. 2(14)]. This is due to the v–i characteristic of the real Chua's diode has two outer regions with positive slopes in addition to the three regions with negative slopes as given by Eq. (1). When V_{C_1} is large and enters these outer passive regions, the large outer period attractor appears. In the computer simulation which does not take this into account, the trajectory will simply diverge to infinity [Chua *et al.*, 1993].

When the series resistor R_0 of the inductor L or the resistor R increases, the chaotic characteristics of Chua's oscillator will disappear. For the modified Chua's oscillator, we can decrease the resistor R_3 to compensate for this effect and retain its chaotic characteristics. The experimental results are shown in Figs. 4 and 5. At the beginning we increases, this double scroll Chua's attractor disappears, but we can correspondingly decrease R_3 to get this double scroll Chua's attractor back. Repeating this procedure we can plot Fig. 5. In Figs. 4 and 5 the other parameters are kept constant as mentioned above. As a matter of fact, in the experiment it is impossible to get the exactly same period-2 limit cycle and double scroll Chua's attractor back when the parameters are changed, here "get back" means basically similar. Therefore, in Figs. 4 and 5 the measured values scatter around the smooth curves. We think this compensation effect of R_3 can be done for other parameters, for example, the capacitor C_1 and C_2 or the inductor L. Figures 4 and 5 are two examples to show how R_3 play a role of compensation to R and R_0 for the period-2 limit cycle and double scroll Chua's attractor. For different attractors the compensation curve will be different, but the compensation effect still exists.

In our theoretical study [Yin, 1996], we change C_3 and keep other parameters constant, but in experiment to change a resistor continuously is much easier than to change a capacitor, so in this experimental paper we change R_3. When R_3 is changed in computer simulation, we get similar results as C_3 is changed. So we think the similar experimental results should be obtained when C_3 is changed.

3. Chaos Control and Synchronization

In this paper, we use two modified Chua's oscillators to do experiments on the chaos control and synchronization. We apply the continuous chaos controlling method developed by Pyragas [1992, 1993]. It has been demonstrated experimentally that two identical chaotic Chua's oscillators can be synchronized by applying this method [Kapitaniak *et al.*, 1994b; Celka, 1994]. The setup for synchronization of two modified Chua's oscillators is shown in Fig. 6 when switch S_1 is turned off. We call the first modified Chua's oscillator the drive system and the second one the response system. These two oscillators are connected through an OpAmp (voltage buffer) which sets up the unidirectional couple from the drive system to the response system. These two modified Chua's oscillators have the same components as in Fig. 1. Here, "same" means that only the values printed on the components are the same. In general, their measured values can differ by $\pm(1 \sim 10)\%$.

Table 2. Synchronization experiment of the modified Chua's circuit.

Status of the Driving System	$R_3^{(1)}(\Omega)$	$R_c(\Omega)$
period-1 limit cycle	130.8	< 3.4
period-2 limit cycle	127.3	< 15.1
spiral Chua's attractor	121.2	< 34.0
double scroll Chua's attractor	115.6	< 74.0

The typical experimental results are summarized in Table 2. As an example, when $R_3^{(1)} = 130.8\ \Omega$, according to Table 2, we can see it produces a period-1 limit cycle. Only if the couple resistor R_C is less than the critical value 3.4 Ω, no matter what value (from 0 to ∞) $R_3^{(2)}$ is, the response system has the same phase portrait as the drive system does. Of course, when $R_3^{(2)} = R_3^{(1)}$, the acuracy of synchronization is the highest. But from Table 1, we can see that when $R_3^{(2)}$ changes from 0 to ∞, the phase portrait of the response system can be anything from a fixed point through period doubling to chaos and the large outer period attractor. It means the modified Chua's oscillators are achieve chaos control and synchronization very easily. We emphasize that control and synchronization can be done in both directions. In one direction, the drive system is in the periodic status and the response system is in the chaotic status. Due to the chaotic control the response system is synchronized to the periodic status of the drive system. In the opposite direction, the drive system is in the chaotic status and the response system is in the periodic status. Due to the chaotic control the response system is synchronized to the chaotic status of the drive system.

When two systems synchronize, $V_{C_1}^{(2)} = V_{C_1}^{(1)}$, on the screen of oscilloscope the relation between $V_{C_1}^{(2)}$ and $V_{C_1}^{(1)}$ is a 45° line through the origin as shown in Fig. 7(a). When the two systems do not synchronize, the relation between $V_{C_1}^{(2)}$ and $V_{C_1}^{(1)}$ deviates from this line. The greater the couple resistor R_C is larger than its critical value, the greater the deviation; that is, the more the two systems do not synchronize.

When R_C is large enough, the two systems do not synchronize and the relation between $V_{C_1}^{(2)}$ and $V_{C_1}^{(1)}$ becomes chaotic as shown in Fig. 7(b). This chaotic status is called hyperchaos which occurs in the four or higher dimensional system (in the present

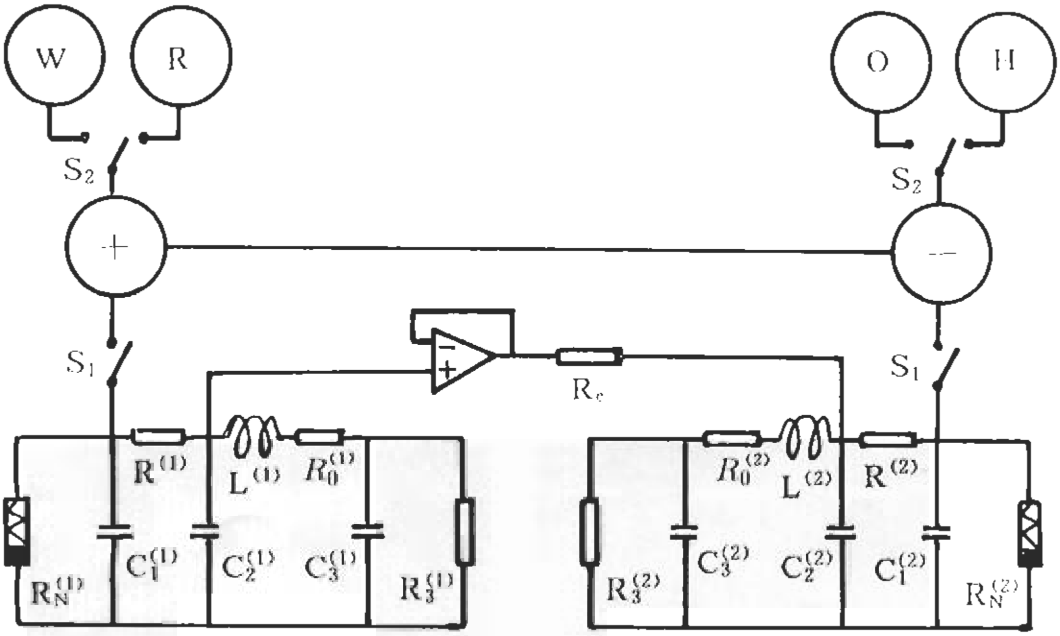


Fig. 6. The experiment setup for chaos control, synchronization and secure communication. W–signal generator of rectangular wave, R–radio, + –additor, O–oscilloscope, H–horn, – –reductor, S_1 and S_2–switches for selection.

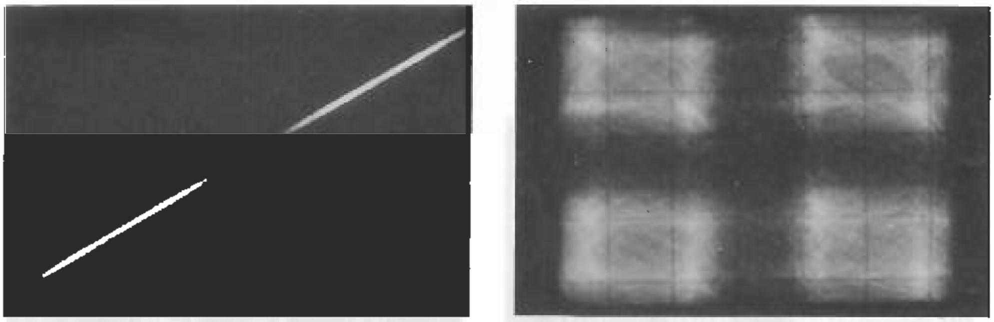


Fig. 7. The relation between $V_{C_1}^{(2)}$ and $V_{C_1}^{(1)}$: (a) synchronization, (b) nonsynchronization.

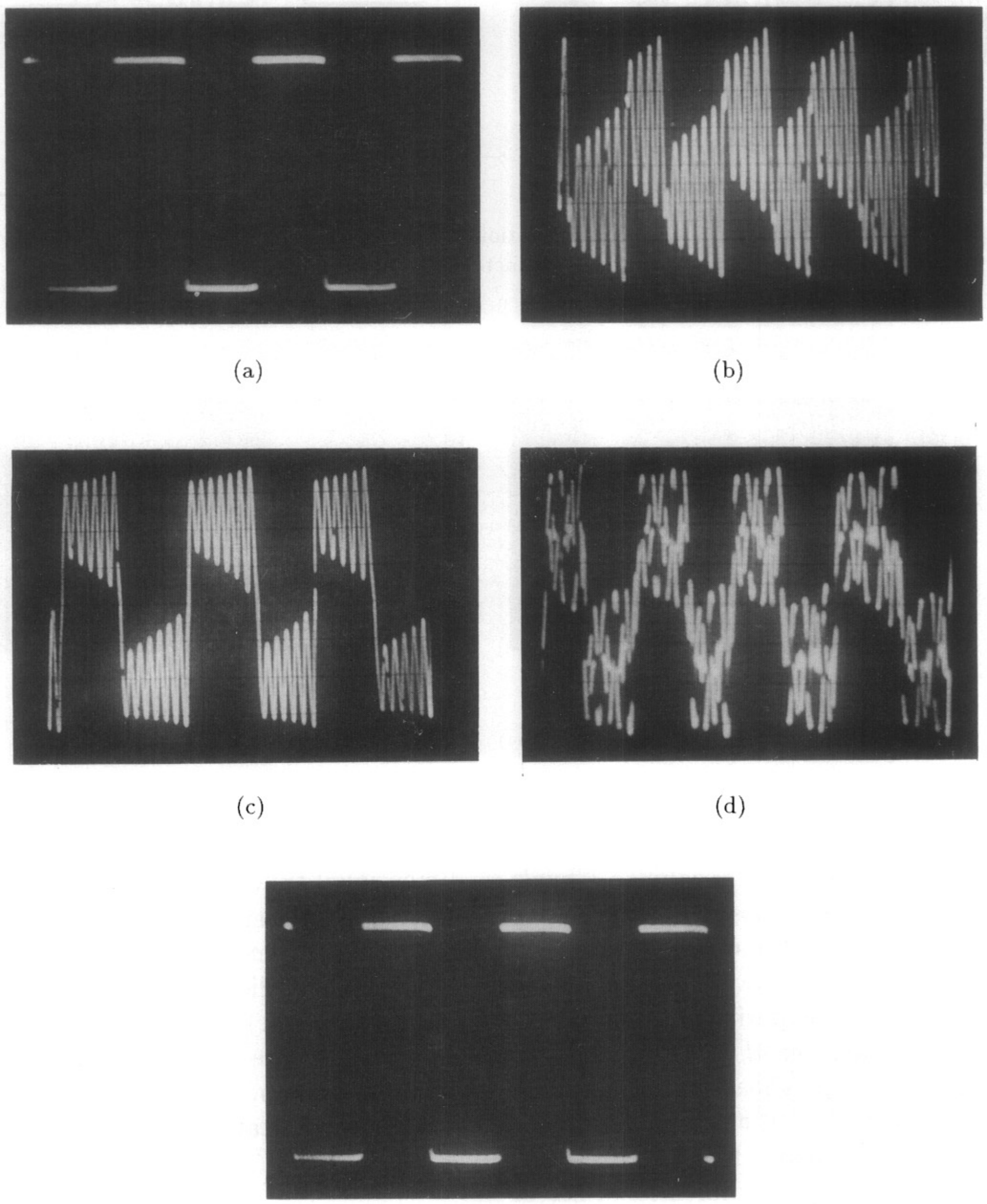
case, the whole system has 8 dimensions). The hyperchaos in two unidirectionally-coupled Chua's circuits and both open and closed chains of five coupled Chua's circuits which have more than one positive Lyapunov exponents have been studied by computer simulation and experiment [Kapitaniak & Chua, 1994; Kapitaniak *et al.*, 1994a]. Now we are trying to find the hyperchoas in the modified Chua's oscillator which has the lowest 4 dimensions necessary for hyperchaos. Rossler [1979] has found the hyperchaos in a four-dimensional set of ordinary differential equations. Kapitaniak *et al.* [1996] have suggested the use of higher-dimensional chaotic systems with more than one positive Lyapunov exponent in secure communication, as there are no effective methods to analyze the higher-dimensional time series.

We should emphasize that here "synchronization" does not mean two different statuses (for example, a periodic limit cycle and a double scroll Chua's attractor) synchronize each other, it means that because the response system is controlled by the drive system, the status of the response system turns into the status of the drive system.

4. Chaotic Secure Communication

Because the chaotic systems have the property of control and synchronization, it becomes possible to use the chaotic signal to achieve secure communication. On the basis of the results of control and synchronization for the two modified Chua's oscillators, we do experiments on the chaotic secure communication. Its circuit is shown in Fig. 6 (when the switches S_1 and S_2 are all turned on).

The input signal is a rectangular wave or a signal output from a radio. First, this input signal is added with the chaotic signal $V_{C_1}^{(1)}$ of the drive system (modulation), then this modulated signal and the chaotic signal $V_{C_2}^{(1)}$ are sent to the response system. Because these two systems synchronize, the response system produces a chaotic signal $V_{C_1}^{(2)}$ which is the same as $V_{C_1}^{(1)}$. Therefore, after the modulated signal reduces $V_{C_1}^{(2)}$ (demodulation), the input signal is recovered. Taking the rectangular wave as an example, the input signal, the chaotic signal $V_{C_1}^{(1)}$ and $V_{C_2}^{(1)}$ of the drive system, the modulated signal and the recovered signal in the response system are shown in Figs. 8(a)–8(e). Because chaotic signals are different at any monent, the photographs



(a) (b) (c) (d) (e)

Fig. 8. Chaotic secure communication. (a) input signal of rectangular wave, (b) chaotic signal of $V_{C_1}^{(1)}$, (c) chaotic signal of $V_{C_2}^{(1)}$, (d) modulated signal, (e) recovered signal.

of transient wave forms in Fig. 8 are all taken by Tektronix 7834 storage oscilloscope.

For the input signal from a radio, we can tune the radio to any broadcast station, no matter whether it is speech or music, from the horn of the response system we can hear the input signal clearly. But from the modulated signal which is intercepted during its transmission we can hear the sound as noise only. So we have successfully done the principle experiment on the chaotic secure communication.

In order to make chaotic secure commnication practicable, there are a lot of technical problems to be solved. For example, in this experiment we transmit two chaotic signals $V_{C_1}^{(1)}$ and $V_{C_2}^{(1)}$ from the drive system to the response system. Although in this way the message signal can be large enough to increase the signal-to-noise ratio, this is not convenient in practice. If we transmit only one chaotic signal, the message signal which is added to the chaotic signal should be at a very low power, then it decreases the signal-to-noise ratio and degrade the systems. So we had better find a more effective method of modulation and demodulation, which only needs to transmit one chaotic signal but can still get good results for secure communication, as here when we transmit two chaotic signals. In addition, we need to find out how to develop from wire transmission to wireless propagation; what kind of effects the spectrum of chaotic signal will have; how the interference affects the fidelity of secure communication; how to solve the contradiction between the robustness and the security; how the same circuits can be used for secure communication of both analog and digital signal and so on.

5. Conclusion

On the basis of the theoretical analysis and computer simulation of the bifurcation, we have demonstrated experimentally the chaotic properties, the chaotic control and synchronization of the modified Chua's circuit [Yin, 1996]. The experimental results show that the modified Chua's oscillator has good chaotic control and synchronization. And we have successfully carried out the principle experiment on the chaotic secure communication.

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