An LMI-based Stable Fuzzy Control of Nonlinear Systems and its Application to Control of Chaos

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Abstract

We present a systematic framework for the stability and design of nonlinear fuzzy control systems. First we represent a nonlinear plant with a Takagi-Sugeno fuzzy model. Then a model-based fuzzy controller design utilizing the concept of so-called "parallel distributed compensation" is employed. The main idea of the controller design is to derive each control rule so as to compensate each rule of a fuzzy system. The design procedure is conceptually simple and natural. Moreover, the stability analysis and control design problems can be reduced to linear matrix inequality (LMI) problems. Therefore they can be solved efficiently in practice by convex programming techniques for LMIs. The design methodology is illustrated by application to the problem of modeling and control of a chaotic system - Chua's circuit.

1. Introduction

We have witnessed rapidly growing interest in fuzzy control in recent years. There has been many successful applications. Despite the success it has been aware that many basic issues remain to be further addressed. Stability analysis and systematic design are certainly among the most important issues for fuzzy control systems. Recently, there have been some great efforts on these issues [1]-[8]. This paper attempts to present a systematic framework for the stability and design of nonlinear fuzzy control systems.

We consider a *nonlocal* approach which is conceptually simple and straightforward. First the nonlinear plant is represented by a Takagi-Sugeno type fuzzy model [9]. In this type of fuzzy model, local dynamics in different state space regions are represented by linear models. The overall model of the system is achieved by fuzzy "blending" of these linear models. This is a multiple model approach that can handle uncertain and time-varying situations.

Once the fuzzy model is obtained, the control design is carried out based on the fuzzy model via the so-called parallel distributed compensation (PDC) scheme. The idea is that for each local linear model, there is an associated linear feedback control. The resulting overall controller, which is nonlinear in general, is the fuzzy blending of each individual linear controller. Hence the PDC approach employs multiple controllers, w.r.t. the multiple models, with automatic switching via fuzzy logic rules.

The design procedure aims at rendering (globally or semi-globally) stable fuzzy controllers. The design procedure is conceptually simple and natural. More significantly, in the proposed framework, the stability analysis and control design problems are reduced to linear matrix inequality (LMI) problems [10]. Numerically the LMI problems can be solved very efficiently by means of some of the most powerful tools available to date in the mathematical programming literature. Therefore recasting the stability analysis and control design problems as LMI problems is equivalent to finding solutions to the original problems. The recasting of stability analysis and design of fuzzy control systems to LMI problems was first made in [7].

For illustration the design methodology is applied to the modeling and control of a representative chaotic system - Chua's circuit.

2. Stability Analysis Using LMIs

To begin with we review the Takagi-Sugeno fuzzy model followed by its stability analysis.

2.1. Takagi-Sugeno Fuzzy Model

In the proposed design procedure, we represent a given nonlinear system by the so-called Takagi-Sugeno fuzzy model [9]. The system dynamics is captured by a set of fuzzy implications which characterize local relations in the

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state space. The main feature of a Takagi-Sugeno fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model. The overall fuzzy model of the system is achieved by fuzzy "blending" of the linear system models. This control oriented fuzzy modeling method is simple and natural in that the validity (or accuracy) of the different linear models obviously depends upon the region in the state space where the system trajectories lie. The TS fuzzy modeling method is a multiple model approach that handles uncertain and time-varying situations.

Specifically, the Takagi-Sugeno fuzzy system is of the following form:

Rule i: IF $x_1(t)$ is M_{i1} $\cdots$ and $x_n(t)$ is M_{in}
THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)$,

where

$$\begin{aligned} \mathbf{x}^T(t) &= [x_1(t), x_2(t), \cdots, x_n(t)], \\ \mathbf{u}^T(t) &= [u_1(t), u_2(t), \cdots, u_m(t)], \end{aligned}$$

$i = 1, 2, \cdots, r$ and r is the number of IF-THEN rules. M_{ij} are fuzzy sets, and $\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)$ is the output from the i-th IF-THEN rule. Given a pair of $(\mathbf{x}(t), \mathbf{u}(t))$, the final output of the fuzzy system is inferred as follows

$$\dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^{r} w_i(t)\{\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)\}}{\sum_{i=1}^{r} w_i(t)}, \qquad (1)$$

where

$$w_i(t) = \prod_{j=1}^{n} M_{ij}(x_j(t)).$$

$M_{ij}(x_j(t))$ is the grade of membership of $x_j(t)$ in M_{ij}.

The open-loop system of (1) is

$$\dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^{r} w_i(t)\mathbf{A}_i\mathbf{x}(t)}{\sum_{i=1}^{r} w_i(t)} \qquad (2)$$

where it is assumed that

$$\begin{aligned} \sum_{i=1}^{r} w_i(t) &> 0, \\ w_i(t) &\geq 0 \qquad i = 1, 2, \cdots, r. \end{aligned}$$

Each linear component $\mathbf{A}_i\mathbf{x}(t)$ is called a *subsystem.*

2.2. Stability Analysis Using LMIs

A sufficient condition for ensuring stability of (2) is given as follows.

Theorem 1 [1] *The equilibrium of a fuzzy system (2) is asymptotically stable in the large if there exists a common positive definite matrix* $\mathbf{P}$ *such that*

$$\mathbf{A}_i^T\mathbf{P} + \mathbf{P}\mathbf{A}_i < \mathbf{0}, \qquad i = 1, 2, \cdots, r, \qquad (3)$$

i.e., a common $\mathbf{P}$ *has to exist for all* $\mathbf{A}_i$*'s.*

This theorem reduces to the Lyapunov stability theorem for linear systems when $r = 1$.

To check the stability of a fuzzy system, it has long been considered difficult to find a common positive definite matrix $\mathbf{P}$. Most of the time a trial-and-error type of procedure is used [1]. In [7], it was pointed out that the common $\mathbf{P}$ problem can be solved numerically. To do this a very important observation is that the stability condition of Theorem 1 is expressed in linear matrix inequalities (LMIs) [10]. To check stability we need to find $\mathbf{P}$ satisfying the LMI

$$\mathbf{P} > 0, \qquad \mathbf{A}_i^T\mathbf{P} + \mathbf{P}\mathbf{A}_i < \mathbf{0}, \qquad i = 1, 2, \cdots, r,$$

or determine that no such $\mathbf{P}$ exists. This is a *convex* feasibility problem. Numerically this feasibility problem can be solved very efficiently in practice by means of the most powerful tools available to date in the mathematical programming literature, e.g., the recently developed interior-point methods [11].

3. Fuzzy Control Design Using LMIs

We employ the concept of *parallel distributed compensation* (PDC) [2, 7] to synthesize fuzzy control laws for the stabilization of nonlinear systems represented by fuzzy model (1).

3.1. Parallel Distributed Compensation

The idea of PDC is to associate a compensator for each rule of the fuzzy model. The resulting overall fuzzy controller is a fuzzy blending of each individual linear controller. The fuzzy controller shares the same fuzzy sets with the fuzzy system (1).

Rule i: IF $x_1(t)$ is M_{1i} $\cdots$ and $x_n(t)$ is M_{ni}
THEN $\mathbf{u}(t) = -\mathbf{F}_i\mathbf{x}(t)$,

where $i = 1, 2, \cdots, r$. Hence the fuzzy controller is

$$\mathbf{u}(t) = \frac{-\sum_{i=1}^{r} w_i(t)\mathbf{F}_i\mathbf{x}(t)}{\sum_{i=1}^{r} w_i(t)} \qquad (4)$$

Note that the controller (4) is *nonlinear* in general. It is easy to see that the PDC method, as the counterpart of the multi-model TS fuzzy model, employs multiple controllelrs with automatic switching via fuzzy rules.

Substituting (4) into (1) we obtain

$$\dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(t)w_j(t)\{\mathbf{A}_i - \mathbf{B}_i\mathbf{F}_j\}\mathbf{x}(t)}{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(t)w_j(t)} \tag{5}$$

Rewrite system (5) as

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \frac{1}{W}[\sum_{i=1}^{r} w_i(t)w_i(t)\{\mathbf{A}_i - \mathbf{B}_i\mathbf{F}_i\}\mathbf{x}(t) \\ &\quad +2\sum_{i<j} w_i(t)w_j(t)\mathbf{G}_{ij}\mathbf{x}(t)]\end{aligned} \tag{6}$$

where

$$\mathbf{G}_{ij} = \frac{\{\mathbf{A}_i - \mathbf{B}_i\mathbf{F}_j\} + \{\mathbf{A}_j - \mathbf{B}_j\mathbf{F}_i\}}{2} \quad i<j$$

$$W = \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(t)w_j(t).$$

Apply Theorem 1 we have the following sufficient condition for stability.

Theorem 2 [2, 7] *The equilibrium of a fuzzy control system (5) is asymptotically stable in the large if there exists a common positive definite matrix* $\mathbf{P}$ *such that the following two conditions are satisfied:*

$$\{\mathbf{A}_i - \mathbf{B}_i\mathbf{F}_i\}^T\mathbf{P} + \mathbf{P}\{\mathbf{A}_i - \mathbf{B}_i\mathbf{F}_i\} < \mathbf{0}, \quad i = 1, \ldots, r \tag{7}$$

$$\mathbf{G}_{ij}^T\mathbf{P} + \mathbf{P}\mathbf{G}_{ij} < \mathbf{0}, \qquad i < j \leq r. \tag{8}$$

The control design problem is to select $\mathbf{F}_i$ $(i = 1, 2, \cdots, r)$ such that conditions (7) and (8) are satisfied. One way to utilize these conditions is through an iterative design process. First for each rule a controller is designed based on consideration of local performance only. Then an LMI based stability analysis is carried out to check whether the stability conditions are satisfied. In the case that the stability conditions are not satisfied, the controller for each rule will be redesigned. The iterative design procedure has proven to be very effective (see, e.g., [2, 7]). From the standpoint of control design, however, it is more desirable to be able to directly design a control that ensures the stability of the closed-loop system. Due to the limited space, we only give a representative result in the next subsection. Details on LMI-based fuzzy control design with guaranteed stability and performance will be presented in the full version of this paper. Some preliminary results are also contained in [8].

3.2. PDC Design Using LMIs

Conditions (7) and (8) are neither linear or not jointly convex in $\mathbf{F}_i$'s and $\mathbf{P}$. To cast these conditions into LMIs, we define $\mathbf{Q} = \mathbf{P}^{-1}$. From (7) and (8) with $\mathbf{P} > \mathbf{0}$ it is easy to obtain the following equivalent stability conditions:

$$\mathbf{Q}\{\mathbf{A}_i - \mathbf{B}_i\mathbf{F}_i\}^T + \{\mathbf{A}_i - \mathbf{B}_i\mathbf{F}_i\}\mathbf{Q} < \mathbf{0}, \quad i = 1, \ldots, r \tag{9}$$

$$\mathbf{Q}\mathbf{G}_{ij}^T + \mathbf{G}_{ij}\mathbf{Q} < \mathbf{0}, \qquad i < j \leq r. \tag{10}$$

with $\mathbf{Q} > \mathbf{0}$.

Further define $\mathbf{W}_i = \mathbf{F}_i\mathbf{Q}$, $i = 1, 2, \ldots, r$ so that for $\mathbf{Q} > \mathbf{0}$ we have $\mathbf{F}_i = \mathbf{W}_i\mathbf{Q}^{-1}$. Substituting into (9) and (10) yields the following LMI conditions.

Theorem 3 *The fuzzy control system (5) is stabilizable in the large via PDC if there exist a* $\mathbf{Q} < \mathbf{0}$ *and* $\mathbf{W}_i$, $i = 1, 2, \ldots, r$ *such that the following LMI conditions hold:*

$$\mathbf{Q}\mathbf{A}_i^T + \mathbf{A}_i\mathbf{Q} - \mathbf{B}_i\mathbf{W}_i - \mathbf{W}_i^T\mathbf{B}_i^T < \mathbf{0}, \quad i = 1, \ldots, r \tag{11}$$

$$\begin{aligned}&\mathbf{Q}\mathbf{A}_i^T + \mathbf{A}_i\mathbf{Q} + \mathbf{Q}\mathbf{A}_j^T + \mathbf{A}_j\mathbf{Q} - \mathbf{B}_i\mathbf{W}_j - \mathbf{W}_j^T\mathbf{B}_i^T \\ &\quad -\mathbf{B}_j\mathbf{W}_i - \mathbf{W}_i^T\mathbf{B}_j^T < \mathbf{0}, \quad i < j \leq r.\end{aligned} \tag{12}$$

Recasting the control design problem in terms of LMI conditions (11) and (12) constitutes a (numerical) solution to the original problem. Moreover, the proposed framework also facilitates incorporation of performance consideration as well as development of multi-objective control design. Details will be presented elsewhere.

Next we apply the proposed fuzzy modeling and control framework to the control of chaos.

4. Fuzzy Modeling and Control of Chaotic Systems

Chaotic behavior of a physical system can either be desirable or undesirable, depending on the application. It can be beneficial in many circumstances, such as enhanced mixing of chemical reactants. Chaos can, on the other hand, entail large amplitude motions and oscillations that might lead to system failure. Clearly the ability to control chaos is of much practical importance. Recently, significant attention has been focused on developing techniques for the control of chaotic dynamical systems (see the reviews [12, 13, 14]). Many of the techniques discussed in the literature are effective to certain extent. At the same time one realizes that the control of chaos can have different interpretations. Some approach the problem by employing linear or nonlinear feedback to stabilize nominal equilibrium points or periodic orbits embedded in chaotic attractors. Others exploit the intrinsic nature of chaos and its associated dynamics to control it. For instance, [15] employs a small amplitude control law in a restricted region of the state space, thereby stabilizing a pre-existing equilibrium or periodic orbit. In another example, [16] demonstrated the viability of controlling chaos by controlling associated bifurcations.

In this section we demonstrate that the proposed fuzzy modeling and control framework can be effectively applied to chaotic systems. A representative chaotic system - Chua's circuit is used as a vehicle for illustration.

4.1. Fuzzy Modeling of Chua's Circuit

The well known Chua's circuit is a simple electronic system, which consists of one inductor (L), two capacitors (C_1, C_2), one linear resistor (R) and one piecewise-linear or nonlinear resistor (g). It has been shown to possess very rich nonlinear dynamics such as bifurcations and chaos [17].

The dynamical behavior of Chua's circuit is described by

$$\dot{v}_{C_1} = \frac{1}{C_1}\left(\frac{1}{R}(v_{C_2} - v_{C_1}) - g(v_{C_1})\right) \quad (13)$$

$$\dot{v}_{C_2} = \frac{1}{C_2}\left(\frac{1}{R}(v_{C_1} - v_{C_2}) + i_L\right) \quad (14)$$

$$\dot{i}_L = \frac{1}{L}(-v_{C_2} - R_0 i_L) \quad (15)$$

where v_{C_1}, v_{C_2}, and i_L are the state variables. The characteristic of the nonlinear resistor $g(v_{C_1})$ is taken as the well known piecewise-linear characteristic (see Fig. 1)

$$g(v_{C_1}) = G_b v_{C_1} + \frac{1}{2}(G_a - G_b)(|v_{C_1} + E| - |v_{C_1} - E|) \quad (16)$$

where $G_a, G_b < 0$.

Our objective is to otain a fuzzy model in the form (2) for Chua's circuit with characteristic (16). Assuming $v_{C_1} \in [-d\ d]$, $d > E > 0$, we obtain the following sector to bound $g(v_{C_1})$ (Fig. 1):

$$g_1(v_{C_1}) = G_a v_{C_1}, \quad (17)$$

$$g_2(v_{C_1}) = (G_b + \frac{(G_a - G_b)E}{d})v_{C_1} = G v_{C_1} \quad (18)$$

where $G \triangleq G_b + \frac{(G_a-G_b)E}{d}$.

Rewrite (16) as

$$g(v_{C_1}) = \begin{cases} G_b v_{C_1} + (G_a - G_b)E & v_{C_1} \geq E \\ G_a v_{C_1} & -E < v_{C_1} < E \\ G_b v_{C_1} - (G_a - G_b)E & v_{C_1} \leq -E \end{cases}$$

Only when $G_a \neq G_b$ is of interest (otherwise Chua's circuit becomes a simple linear system). With $G_a \neq G_b$,

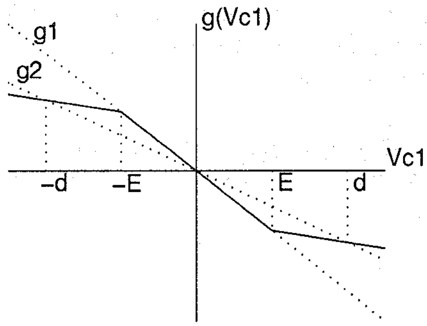

Figure 1. Resistor characteristic of Chua's circuit

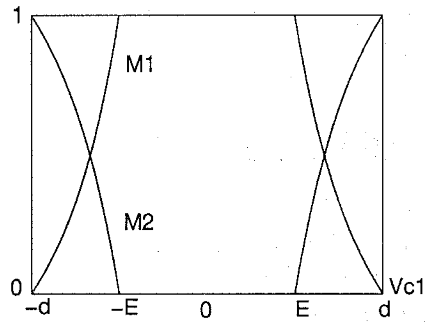

Figure 2. Membership functions for Chua's circuit

we arrive at the following membership functions (Fig. 2):

$$M_1(v_{C_1}) = \begin{cases} \frac{-\frac{E}{d}v_{C_1}+E}{(1-\frac{E}{d})v_{C_1}} & v_{C_1} \geq E \\ 1 & -E < v_{C_1} < E \\ \frac{-\frac{E}{d}v_{C_1}-E}{(1-\frac{E}{d})v_{C_1}} & v_{C_1} \leq -E \end{cases}$$

and

$$M_2(v_{C_1}) = 1 - M_1(v_{C_1}).$$

Denote $\mathbf{x} = [v_{C_1}, v_{C_2}, i_L]^T$. Chua's circuit with characteristic (16) can be represented exactly for $v_{C_1} \in [-d\ d]$ by the following fuzzy model:

Rule 1: IF v_{C_1} is $M_1(v_{C_1})$ (near 0)
THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_1\mathbf{x}(t)$
Rule 2: IF v_{C_1} is $M_2(v_{C_1})$ (near $\pm d$)
THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_2\mathbf{x}(t)$

where

$$\mathbf{A}_1 = \begin{bmatrix} -\frac{1}{C_1R} - \frac{G_a}{C_1} & \frac{1}{C_1R} & 0 \\ -\frac{1}{C_2R} & -\frac{1}{C_2R} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & -\frac{R_0}{L} \end{bmatrix}$$

and

$$\mathbf{A}_2 = \begin{bmatrix} -\frac{1}{C_1 R} - \frac{G}{C_1} & \frac{1}{C_1 R} & 0 \\ -\frac{1}{C_2 R} & -\frac{1}{C_2 R} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & -\frac{R_0}{L} \end{bmatrix}.$$

For any region of interest, Chua's circuit can be modeled exactly by the fuzzy system with properly chosen d (including $d \to \infty$).

4.2. Fuzzy Control of Chua's Circuits

Consider Chua's circuit with control inputs

$$\dot{v}_{C_1} = \frac{1}{C_1}\left(\frac{1}{R}(v_{C_2} - v_{C_1}) - g(v_{C_1})\right) + u_1 \quad (19)$$

$$\dot{v}_{C_2} = \frac{1}{C_2}\left(\frac{1}{R}(v_{C_1} - v_{C_2}) + i_L\right) + u_2 \quad (20)$$

$$\dot{i}_L = \frac{1}{L}(-v_{C_2} - R_0 i_L) + u_3 \quad (21)$$

It can be represented by the following fuzzy model.

Rule 1: IF v_{C_1} is $M_1(v_{C_1})$
THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_1\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$
Rule 2: IF v_{C_1} is $M_2(v_{C_1})$
THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_2\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$

where $M_i(v_{C_1})$'s, $\mathbf{A}_i$'s are defined as in the last section and $\mathbf{B}$ is a 3×3 identity matrix.

The control objective is to steer any chaotic and/or oscillatory trajectory to the origin. Applying the PDC design, we arrive at the fuzzy controller as follows.

Rule 1: IF v_{C_1} is $M_1(v_{C_1})$
THEN $\mathbf{u}(t) = -\mathbf{F}_1\mathbf{x}(t)$,
Rule 2: IF v_{C_1} is $M_2(v_{C_1})$
THEN $\mathbf{u}(t) = -\mathbf{F}_2\mathbf{x}(t)$.

The overall PDC controller is hence

$$\mathbf{u} = -w_1\mathbf{F}_1\mathbf{x} - w_2\mathbf{F}_2\mathbf{x}$$

which is nonlinear. The feedback gains $\mathbf{F}_1$ and $\mathbf{F}_2$ can be obtained by solving the conditions (11) and (12) with $\mathbf{Q} > \mathbf{0}$ of Theorem 3.

Choose $R = 10/7, R_0 = 0, C_1 = 0.1, C_2 = 2, L = 1/7$, $G_b = -0.1$, $G_a = -4$, $E = 1$ and $d = 15$. Using LMI algorithms, we have obtained the following solutions for $\mathbf{Q} > \mathbf{0}$ and $\mathbf{W}_i$, $i = 1, 2$.

$$\mathbf{Q} = \begin{bmatrix} 47.1825 & 0.0000 & 0.0000 \\ 0.0000 & 47.1825 & 0.0000 \\ 0.0000 & 0.0000 & 47.1825 \end{bmatrix},$$

and

$$\mathbf{W}_1 = \begin{bmatrix} 1.5728e+03 & 1.4919e+03 & 8.4744e+01 \\ -1.1451e+03 & -7.8638e-01 & 9.3458e+01 \\ -8.4744e+01 & -4.0014e+02 & 1.5728e+01 \end{bmatrix}.$$

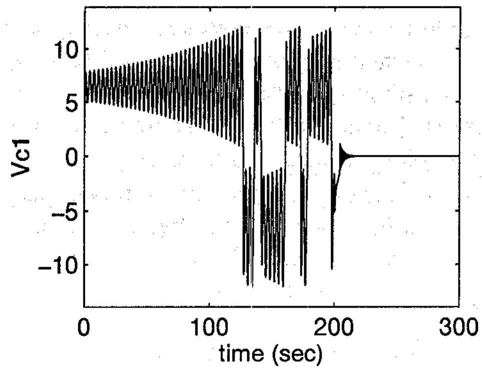

Figure 3. Response of Chua's circuit

$$\mathbf{W}_2 = \begin{bmatrix} -1.4469e+02 & -1.0115e+03 & 1.7532e+02 \\ 1.3583e+03 & -7.8638e-01 & 9.5649e+02 \\ -1.7532e+02 & -1.2632e+03 & 1.5728e+01 \end{bmatrix}.$$

The resultings feedback gains are

$$\mathbf{F}_1 = \mathbf{W}_1\mathbf{Q}^{-1} = \begin{bmatrix} 33.3333 & 31.6202 & 1.7961 \\ -24.2702 & -0.0167 & 1.9808 \\ -1.7961 & -8.4808 & 0.3333 \end{bmatrix},$$

and

$$\mathbf{F}_2 = \mathbf{W}_2\mathbf{Q}^{-1} = \begin{bmatrix} -3.0667 & -21.4379 & 3.7158 \\ 28.7879 & -0.0167 & 20.2722 \\ -3.7158 & -26.7722 & 0.3333 \end{bmatrix}.$$

Figure 3 shows the response of Chua's circuit before and after the control is applied (initial condition $(0, 1, 0)$, control is activated at $t = 200$).

Remark The proposed control laws guarantee the stability of the fuzzy control system consisted of the fuzzy model and the PDC controller. When the fuzzy model is an exact representation of the nonlinear plant, the global stability is achieved. In the application to Chua's circuits, the semi-global stability is achieved, i.e., the control law can achieve any prescribed region of stability by employing proper sectors. This is a very powerful and practical aspect of the proposed framework.

5. Conclusions

A systematic framework for the stability and design of nonlinear fuzzy control systems is presented. The framework is based on Takagi-Sugeno fuzzy model and parallel distributed compensation control design. The design procedure is conceptually simple and natural. Moreover, the stability analysis and control design problems are reduced to linear matrix inequality (LMI) problems. Therefore they can be solved very efficiently in practice by convex programming techniques for LMIs. The design methodology is illustrated by application to the control of a well known chaotic system - Chua's circuit.

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