FUZZY MODELING AND CONTROL OF CHAOTIC SYSTEMS

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ABSTRACT

In this paper we explore the interaction between fuzzy control systems and chaos. First we show that fuzzy modeling techniques can be used to model chaotic dynamical systems. Then we apply some of the newly developed fuzzy control design techniques to the control of chaotic systems. The design procedure is conceptually simple, natural and computationally efficient. Therefore the proposed fuzzy methodology represents a systematic and effective framework for modeling and control of chaotic systems. The method is illustrated by application to Chua's circuits.

1. INTRODUCTION

Recently, significant attention has been focused on developing techniques for the control of chaotic dynamical systems (see the reviews [1, 2, 3, 4]). Many of the techniques discussed in the literature are effective to certain extent. At the same time one realizes that the control of chaos can have different interpretations. Some approach the problem by employing linear or nonlinear feedback to stabilize nominal equilibrium points or periodic orbits embedded in chaotic attractors. Others exploit the intrinsic nature of chaos and its associated dynamics to control it. For example, [5] demonstrated the viability of controlling chaos by controlling associated bifurcations.

While chaos has become one of the most focusing research topics in the literature, we have witnessed rapidly growing interest in making the control systems more intelligent. Among intelligent control approaches, fuzzy control has enjoyed remarkable success in various applications [6]. Moreover, recent advances in fuzzy control have laid the foundation for intelligent control of various nonlinear processes, including chaotic systems.

In this paper we explore the interaction between fuzzy control systems and chaos. First, we show that fuzzy modeling techniques can be used to model chaotic dynamical systems, which also implies that fuzzy system can be chaotic. This is not surprising given the fact that fuzzy systems are essentially nonlinear. The particular fuzzy modeling framework employed here is the so-called Takagi-Sugeno model [7]. In this type of fuzzy model, local dynamics in different state space regions are represented by linear models. The overall model of the system is achieved by fuzzy "blending" of these linear models.

Once the fuzzy model representation of a chaotic system is obtained, we can apply some of the newly developed fuzzy control design techniques to the control of the chaotic system. The control design is carried out based on the fuzzy model via a so-called parallel distributed compensation scheme. The idea is that for each local linear model, a linear feedback control is designed. The resulting overall controller, which is nonlinear in general, is a fuzzy blending of each individual linear controller.

The design procedure aims at rendering (globally or semi-globally) stable fuzzy controllers. The design procedure is conceptually simple and natural. Moreover, the analysis and design of fuzzy control systems have been assisted by extremely efficient convex programming techniques involving linear matrix inequalities (LMIs). Therefore, the proposed fuzzy methodology represents a systematic framework for modeling and control of chaotic systems.

Throughout the paper, two versions of Chua's Circuit are used for illustration.

2. FUZZY MODELING OF CHAOTIC SYSTEMS

In this section we present fuzzy modeling of chaotic systems. To begin with we review the so-called Takagi-Sugeno fuzzy model. Then we demonstrate how the TS fuzzy model can be used to represent chaotic systems by application to two versions of Chua's Circuit.

2.1. Takagi-Sugeno Fuzzy Model

We represent a given nonlinear, possibly chaotic, system by the so-called Takagi-Sugeno fuzzy model [7]. This control oriented fuzzy modeling method is simple and natural. The system dynamics is captured by a set of fuzzy implications which characterize local relations in the state space. The main feature of a Takagi-Sugeno fuzzy model is to express the local dynamics of each fuzzy implication (rule) by a linear system model. The overall fuzzy model of the system is achieved by fuzzy "blending" of the linear system models.

Specifically, the Takagi-Sugeno fuzzy system is of the following form:

Rule i: IF $x_1(t)$ is M_{i1} $\cdots$ and $x_n(t)$ is M_{in}
THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)$,

where

$$\begin{aligned}\mathbf{x}^T(t) &= [x_1(t), x_2(t), \cdots, x_n(t)],\\ \mathbf{u}^T(t) &= [u_1(t), u_2(t), \cdots, u_m(t)],\end{aligned}$$

$i = 1, 2, \cdots, r$ and r is the number of IF-THEN rules. M_{ij} are fuzzy sets, and $\dot{\mathbf{x}}(t) = \mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)$ is the output from the i-th IF-THEN rule. Given a pair of $(\mathbf{x}(t), \mathbf{u}(t))$, the final output of the fuzzy system is inferred as follows

$$\dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^{r} w_i(t)\{\mathbf{A}_i\mathbf{x}(t) + \mathbf{B}_i\mathbf{u}(t)\}}{\sum_{i=1}^{r} w_i(t)}, \qquad (1)$$

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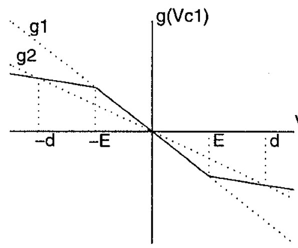

Fig. 1. Resistor characteristic in Chua's circuit (Case 1)

where

$$w_i(t) = \prod_{j=1}^{n} M_{ij}(x_j(t)).$$

$M_{ij}(x_j(t))$ is the grade of membership of $x_j(t)$ in M_{ij}.

The open-loop system of (1) is

$$\dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^{r} w_i(t)\mathbf{A}_i\mathbf{x}(t)}{\sum_{i=1}^{r} w_i(t)} \tag{2}$$

where it is assumed that

$$\begin{aligned} \sum_{i=1}^{r} w_i(t) &> 0, \\ w_i(t) &\geq 0 \qquad i = 1, 2, \cdots, r. \end{aligned}$$

Next we show the application of the fuzzy model (2) to Chua's circuits.

2.2. Fuzzy Modeling of Chua's Circuits

The well known Chua's circuit is a simple electronic system, which consists of one inductor (L), two capacitors (C_1, C_2), one linear resistor (R) and one piecewise-linear or nonlinear resistor (g). It has been shown to possess very rich nonlinear dynamics such as bifurcations and chaos [8].

The dynamical behavior of Chua's circuit is described by

$$\dot{v}_{C_1} = \frac{1}{C_1}\left(\frac{1}{R}(v_{C_2} - v_{C_1}) - g(v_{C_1})\right) \tag{3}$$

$$\dot{v}_{C_2} = \frac{1}{C_2}\left(\frac{1}{R}(v_{C_1} - v_{C_2}) + i_L\right) \tag{4}$$

$$\dot{i}_L = \frac{1}{L}(-v_{C_2} - R_0 i_L) \tag{5}$$

where v_{C_1}, v_{C_2}, and i_L are the state variables.

Let us consider two types of characteristic of the nonlinear resistor $g(v_{C_1})$. One is the well known piecewise-linear characteristic and the other a cubic one.

Case 1. $g(v_{C_1})$ is piecewise linear:

$$g(v_{C_1}) = G_b v_{C_1} + \frac{1}{2}(G_a - G_b)(|v_{C_1} + E| - |v_{C_1} - E|) \tag{6}$$

where $G_a, G_b < 0$.

Our objective is to otain a fuzzy model in the form (2) for Chua's circuit with characteristic (6). Assuming $v_{C_1} \in [-d\ d]$, $d > E > 0$, we obtain the following sector to bound $g(v_{C_1})$ (Fig. 1):

$$g_1(v_{C_1}) = G_a v_{C_1}, \tag{7}$$

$$g_2(v_{C_1}) = (G_b + \frac{(G_a - G_b)E}{d})v_{C_1} = G v_{C_1} \tag{8}$$

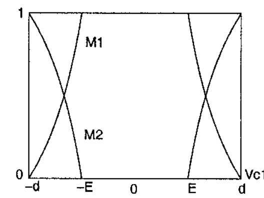

Fig. 2. Membership functions (Case 1)

where $G \triangleq G_b + \frac{(G_a - G_b)E}{d}$.

Rewrite (6) as

$$g(v_{C_1}) = \begin{cases} G_b v_{C_1} + (G_a - G_b)E & v_{C_1} \geq E \\ G_a v_{C_1} & -E < v_{C_1} < E \\ G_b v_{C_1} - (G_a - G_b)E & v_{C_1} \leq -E \end{cases}$$

Only when $G_a \neq G_b$ is of interest (otherwise Chua's circuit becomes a simple linear system). With $G_a \neq G_b$, we arrive at the following membership functions (Fig. 2):

$$M_1(v_{C_1}) = \begin{cases} \frac{-\frac{E}{d}v_{C_1} + E}{(1 - \frac{E}{d})v_{C_1}} & v_{C_1} \geq E \\ 1 & -E < v_{C_1} < E \\ \frac{-\frac{E}{d}v_{C_1} - E}{(1 - \frac{E}{d})v_{C_1}} & v_{C_1} \leq -E \end{cases}$$

and

$$M_2(v_{C_1}) = 1 - M_1(v_{C_1}).$$

Denote $\mathbf{x} = [v_{C_1}, v_{C_2}, i_L]^T$. Chua's circuit with characteristic (6) can be represented exactly for $v_{C_1} \in [-d\ d]$ by the following fuzzy model:

Rule 1: IF v_{C_1} is $M_1(v_{C_1})$ (near 0)
THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_1\mathbf{x}(t)$

Rule 2: IF v_{C_1} is $M_2(v_{C_1})$ (near $\pm d$)
THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_2\mathbf{x}(t)$

where

$$\mathbf{A}_1 = \begin{bmatrix} -\frac{1}{C_1R} - \frac{G_a}{C_1} & \frac{1}{C_1R} & 0 \\ -\frac{1}{C_2R} & -\frac{1}{C_2R} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & -\frac{R_0}{L} \end{bmatrix},$$

and

$$\mathbf{A}_2 = \begin{bmatrix} -\frac{1}{C_1R} - \frac{G}{C_1} & \frac{1}{C_1R} & 0 \\ -\frac{1}{C_2R} & -\frac{1}{C_2R} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & -\frac{R_0}{L} \end{bmatrix}.$$

For any region of interest, Chua's circuit can be modeled exactly by the fuzzy system with properly chosen d.

Case 2. $g(v_{C_1})$ is cubic:

$$g(v_{C_1}) = a v_{C_1} + c v_{C_1}^3, \tag{9}$$

where $a < 0, c > 0$.

Similarly as in Case 1, assuming $v_{C_1} \in [-d\ d]$, $d > 0$, we obtain the following sector to bound $g(v_{C_1})$:

$$g_1(v_{C_1}) = a v_{C_1}, \tag{10}$$

$$g_2(v_{C_1}) = (a + cd^2)v_{C_1} = G_c v_{C_1} \tag{11}$$

where $G_c \triangleq a + cd^2$.

The membership functions are derived as:

$$\begin{aligned} M_1(v_{C_1}) &= 1 - (\tfrac{v_{C_1}}{d})^2, \\ M_2(v_{C_1}) &= 1 - M_1(v_{C_1}) = (\tfrac{v_{C_1}}{d})^2. \end{aligned}$$

The fuzzy model for Chua's circuit with characteristic (9) is hence obtained as the following.

Rule 1: IF v_{C_1} is $M_1(v_{C_1})$ (near 0)
THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_1\mathbf{x}(t)$

Rule 2: IF v_{C_1} is $M_2(v_{C_1})$ (near $\pm d$)
THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_2\mathbf{x}(t)$

where

$$\mathbf{A}_1 = \begin{bmatrix} -\frac{1}{C_1R} - \frac{a}{C_1} & \frac{1}{C_1R} & 0 \\ -\frac{1}{C_2R} & -\frac{1}{C_2R} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & -\frac{R_0}{L} \end{bmatrix},$$

and

$$\mathbf{A}_2 = \begin{bmatrix} -\frac{1}{C_1R} - \frac{G_a}{C_1} & \frac{1}{C_1R} & 0 \\ -\frac{1}{C_2R} & -\frac{1}{C_2R} & \frac{1}{C_2} \\ 0 & -\frac{1}{L} & -\frac{R_0}{L} \end{bmatrix}.$$

3. FUZZY CONTROL OF CHAOTIC SYSTEMS

We employ the concept of *parallel distributed compensation* (PDC) [11, 12] to synthesize fuzzy control laws for the stabilization of nonlinear systems (including chaotic systems) represented by fuzzy model (1). To begin with, we need the following results on stability analysis.

3.1. Stability Analysis

A sufficient condition for ensuring stability of (2) is given as follows.

Theorem 1 [10] *The equilibrium of a fuzzy system (2) is asymptotically stable in the large if there exists a common positive definite matrix* $\mathbf{P}$ *such that*

$$\mathbf{A}_i^T\mathbf{P} + \mathbf{P}\mathbf{A}_i < \mathbf{0}, \qquad i = 1, 2, \cdots, r, \tag{12}$$

i.e., a common $\mathbf{P}$ *has to exist for all* $\mathbf{A}_i$*'s.*

This theorem reduces to the Lyapunov stability theorem for linear systems when $r = 1$.

To check the stability of a fuzzy system, it has long been considered difficult to find a common positive definite matrix $\mathbf{P}$. Most of the time a trial-and-error type of procedure is used [10]. In [12, 13], it was pointed out that the common $\mathbf{P}$ problem can be solved numerically. To do this a very important observation is that the stability condition of Theorem 1 is expressed in linear matrix inequalities (LMIs) [14]. To check stability we need to find $\mathbf{P}$ satisfying the LMI

$$\mathbf{P} > \mathbf{0}, \qquad \mathbf{A}_i^T\mathbf{P} + \mathbf{P}\mathbf{A}_i < \mathbf{0}, \qquad i = 1, 2, \cdots, r,$$

or determine that no such $\mathbf{P}$ exists. This is a *convex* feasibility problem. Numerically this feasibility problem can be solved very efficiently in practice by means of the most powerful tools available to date in the mathematical programming literature, e.g., the recently developed interior-point methods [15].

3.2. Parallel Distributed Compensation

The idea of PDC is to design a compensator for each rule of the fuzzy model. For each rule, we can use linear control design techniques. The resulting overall fuzzy controller is a fuzzy blending of each individual linear controller. The fuzzy controller shares the same fuzzy sets with the fuzzy system (1).

Rule i: IF $x_1(t)$ is M_{1i} $\cdots$ and $x_n(t)$ is M_{ni}
THEN $\mathbf{u}(t) = -\mathbf{F}_i\mathbf{x}(t)$,

where $i = 1, 2, \cdots, r$. Hence the fuzzy controller is

$$\mathbf{u}(t) = \frac{-\sum_{i=1}^{r} w_i(t)\mathbf{F}_i\mathbf{x}(t)}{\sum_{i=1}^{r} w_i(t)} \tag{13}$$

Note that the controller (13) is *nonlinear* in general.

Substituting (13) into (1) we obtain

$$\dot{\mathbf{x}}(t) = \frac{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(t)w_j(t)\{\mathbf{A}_i - \mathbf{B}_i\mathbf{F}_j\}\mathbf{x}(t)}{\sum_{i=1}^{r}\sum_{j=1}^{r} w_i(t)w_j(t)} \tag{14}$$

Apply Theorem 1 we have the following sufficient condition for stability

Theorem 2 [11, 12] *The equilibrium of a fuzzy control system (14) is asymptotically stable in the large if there exists a common positive definite matrix* $\mathbf{P}$ *such that*

$$\{\mathbf{A}_i - \mathbf{B}_i\mathbf{F}_j\}^T\mathbf{P} + \mathbf{P}\{\mathbf{A}_i - \mathbf{B}_i\mathbf{F}_j\} < \mathbf{0}, \tag{15}$$

for $M_i \cdot M_j \neq 0, \quad i, j = 1, 2, \cdots, r.$

Note that system (14) can be also written as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \frac{1}{W}[\sum_{i=1}^{r} w_i(t)w_i(t)\{\mathbf{A}_i - \mathbf{B}_i\mathbf{F}_i\}\mathbf{x}(t) \\ &\quad + 2\sum_{i<j} w_i(t)w_j(t)\mathbf{G}_{ij}\mathbf{x}(t)] \end{aligned} \tag{16}$$

where

$$\mathbf{G}_{ij} = \frac{\{\mathbf{A}_i - \mathbf{B}_i\mathbf{F}_j\} + \{\mathbf{A}_j - \mathbf{B}_j\mathbf{F}_i\}}{2} \quad i < j$$

$$W = \sum_{i=1}^{r}\sum_{j=1}^{r} w_i(t)w_j(t).$$

Therefore we have the following sufficient condition.

Theorem 3 [11, 12] *The equilibrium of a fuzzy control system (14) is asymptotically stable in the large if there exists a common positive definite matrix* $\mathbf{P}$ *such that the following two conditions are satisfied:*

$$\{\mathbf{A}_i - \mathbf{B}_i\mathbf{F}_i\}^T\mathbf{P} + \mathbf{P}\{\mathbf{A}_i - \mathbf{B}_i\mathbf{F}_i\} < \mathbf{0}, \quad i = 1, 2, \ldots, r \tag{17}$$

$$\mathbf{G}_{ij}^T\mathbf{P} + \mathbf{P}\mathbf{G}_{ij} < \mathbf{0}, \qquad i < j \leq r. \tag{18}$$

Remark The conditions of Theorem 3 are more relaxed than those of Theorem 2.

The control design problem is to select $\mathbf{F}_i$ $(i = 1, 2, \cdots, r)$ such that conditions (17) and (18) are satisfied. This is an iterative process. For each rule a controller is designed based on consideration of local performance only. Then an LMI based stability analysis is carried out to check whether the stability conditions are satisfied. In the case that the stability conditions are not satisfied, the controller for each rule will be redesigned. The iterative design procedure has been very effective in our experience. From the standpoint of control design, however, it is more desirable to be able to directly design a control that ensures the stability of the closed-loop system. Details on directly solving the control design problem using LMIs can be found in [13].

Next we apply the PDC fuzzy design to Chua's circuits.

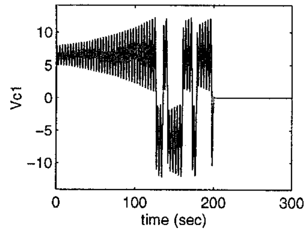

Fig. 3. Response of Chua's circuit (Case 1)

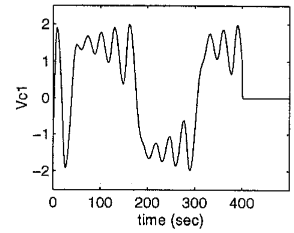

Fig. 4. Response of Chua's circuit (Case 2)

3.3. Fuzzy Control of Chua's Circuits

Consider Chua's circuit with control inputs

$$\dot{v}_{C_1} = \frac{1}{C_1}\left(\frac{1}{R}(v_{C_2} - v_{C_1}) - g(v_{C_1})\right) + u_1 \quad (19)$$

$$\dot{v}_{C_2} = \frac{1}{C_2}\left(\frac{1}{R}(v_{C_1} - v_{C_2}) + i_L\right) + u_2 \quad (20)$$

$$\dot{i}_L = \frac{1}{L}(-v_{C_2} - R_0 i_L) + u_3 \quad (21)$$

It can be represented by the following fuzzy model.

Rule 1: IF v_{C_1} is $M_1(v_{C_1})$ THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_1\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$
Rule 2: IF v_{C_1} is $M_2(v_{C_1})$ THEN $\dot{\mathbf{x}}(t) = \mathbf{A}_2\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$

where $M_i(v_{C_1})$'s, $\mathbf{A}_i$'s are defined as in the last section and $\mathbf{B}$ is a 3×3 identity matrix.

The control objective is to steer any chaotic and/or oscillatory trajectory to the origin. Applying the PDC design, we arrive at the fuzzy controller as follows.

Rule 1: IF v_{C_1} is $M_1(v_{C_1})$ THEN $\mathbf{u}(t) = -\mathbf{F}_1\mathbf{x}(t)$,
Rule 2: IF v_{C_1} is $M_2(v_{C_1})$ THEN $\mathbf{u}(t) = -\mathbf{F}_2\mathbf{x}(t)$.

The overall PDC controller $\mathbf{u} = -w_1\mathbf{F}_1\mathbf{x} - w_2\mathbf{F}_2\mathbf{x}$ is nonlinear. The feedback gains $\mathbf{F}_1$ and $\mathbf{F}_2$ can be obtained by a number of linear control techniques. Here they are obtained by solving riccati equations from linear optimal control.

Case 1. Choose $R = 10/7$, $R_0 = 0$, $C_1 = 0.1$, $C_2 = 2$, $L = 1/7$, $G_b = -0.1$, $G_a = -4$, $E = 1$ and $d = 15$. We have

$$\mathbf{F}_1 = \begin{bmatrix} 64.1991 & 11.5414 & 0.7025 \\ 11.5414 & 7.2798 & -1.5885 \\ 0.7025 & -1.5885 & 2.3227 \end{bmatrix},$$

$$\mathbf{F}_2 = \begin{bmatrix} 1.2225 & 0.8829 & 0.1750 \\ 0.8829 & 6.0161 & -1.5067 \\ 0.1750 & -1.5067 & 2.4885 \end{bmatrix}.$$

By using LMI algorithms, a common $\mathbf{P}$ that satisfies stability conditions (17),(18) is found to be

$$\mathbf{P} = \begin{bmatrix} 0.0292 & -0.0300 & 0.0130 \\ -0.0300 & 0.1094 & -0.0585 \\ 0.0130 & -0.0585 & 0.1504 \end{bmatrix}.$$

Fig. 3 shows the response of Chua's circuit (initial condition $(0, 1, 0)$, control is activated at $t = 200$).

Case 2. Choose $R = 10/7$, $R_0 = 0$, $C_1 = 1.0$, $C_2 = 19/2$, $L = 19/14$, $a = -4/5$, $c = 2/45$ and $d = 3$. We have

$$\mathbf{F}_1 = \begin{bmatrix} 3.2489 & 0.3890 & 0.0219 \\ 0.3890 & 3.2060 & -0.3066 \\ 0.0219 & -0.3066 & 3.1370 \end{bmatrix},$$

$$\mathbf{F}_2 = \begin{bmatrix} 2.8655 & 0.3468 & 0.0191 \\ 0.3468 & 3.2018 & -0.3069 \\ 0.0191 & -0.3069 & 3.1370 \end{bmatrix}.$$

and a stability guaranteeing common $\mathbf{P}$ is found to be

$$\mathbf{P} = \begin{bmatrix} 0.5219 & 0.0041 & -0.0030 \\ 0.0041 & 0.5038 & -0.0030 \\ -0.0030 & -0.0030 & 0.5252 \end{bmatrix}.$$

Fig. 4 shows the response of Chua's circuit (initial condition $(-1, 0.8, 1)$, control is activated at $t = 400$).

Remark The proposed control laws guarantee the stability of the fuzzy control system consisted of the fuzzy model and the PDC controller. When the fuzzy model is an exact representation of the nonlinear plant, the global stability is achieved. In the application to Chua's circuits, the semi-global stability is achieved, i.e., the control law can achieve any prescribed region of stability by employing proper sectors. This is a very powerful and practical aspect of the proposed framework.

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