Application of method of integral equations for analysis of complex periodic behaviors in Chua's circuits

Sergey Volovodov
Dept. of ship automated control systems
Marine Technical University
3 Lotsmanskaya Str.
St. Petersburg 190008, Russia

Bernhard Lampe
Dept. of automated control systems
Rostock University
Rostock 18051 BRD

Yephim Rozenvasser
Dept. of ship automated control systems
Marine Technical University
3 Lotsmanskay Str.
St.Petersburg190008, Russia

Abstract

Chua's circuit is a third order electrical circuit resented in Fig.1a, where the nonlinear resistor R2 is described by a cubic characteristic. The circuit can also be presented by a third order linear link L(s) with a cubical nonlinear feedback f(y) [1,2], Fig.1.b.

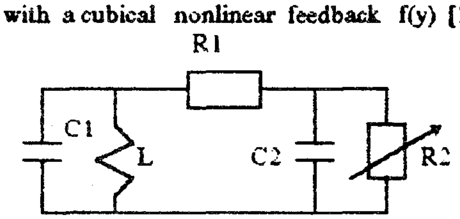

Figure 1a: Electrical scheme of a Chua's circuit

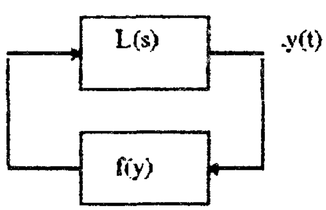

Figure 1b: Circuit block - diagram

It is known [1,2] that the following autonomous behaviors are possible in such circuit:

- stable periodic oscillations with non zero bias (short period);
- stable points of equilibrium;
- unstable motions;
- stable periodic oscillations with zero bias (long period);
- limited complex motion (chaos).

Usually analysis of such systems involves the determination of regions on parametrical plane (parameters of the linear link) corresponding to the above behaviors of different kinds of bifurcations. There are many research methods for studing such nonlinear circuits. A technique for analyzing Chua's circuit was presented in [1,2]. This method involves:

1. Determination of possible behaviors by the harmonic balance method and estimation of their stability.
2. Rough estimation of possible distortions for determined regimes.

In [1,2] some conditions were formulated for determining of possible bifurcations depending on the values of these distortions. In some cases the values of distortions can be determined using simplest physical considerations. Control design in this case can be reduced to the construction of feedback controller that makes the distortions of desirable behavior bounded.

An exact method for estimating of distortions was presented in [3]. Application of this method to a system with a cubical nonlinearity is given below.

1. An application of the integral equations method

1.1 Determination of possible behaviors

Suppose that the Galerkin's method gives the following periodical behavior

$$y_T(t) = a_0 + \sum_{k=1}^{N} a_k \sin(k\omega_T t) + \sum_{k=2}^{N} b_k \cos(k\omega_T t)\,, \quad (1)$$

where $y_T(t)$ satisfies the integral equation [3,4]:

$$y_T(t) = \int_0^{T_T} \varphi_T(t-\tau)\, f[y_T(\tau)]\, d\tau \qquad (2)$$

and

$$\varphi_T(t-\tau) = 1/T_T \sum_{k=-N}^{N} L(ki\omega_T) \exp(ki\omega_T(t-\tau)). \quad (3)$$

Equation (2) can be presented as the following system of equations:

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$$a_0 = 1/T_r \ L(0) \int_0^{T_r} f[y_r(\tau)]\, d\tau \ , \quad (4)$$

$$a_k = 2/T_r \int_0^{T_r} [P_k \cos(k\omega\tau) + Q_k \sin(k\omega\tau)]\, f[y(\tau)]\, d\tau \ ,$$

$$b_k = 2/T_r \int_0^{T_r} [P_k \sin(k\omega\tau) - Q_k \cos(k\omega\tau)]\, f[y(\tau)]\, d\tau \ ,$$

and $P_k = \mathrm{Re}\, L(ki\omega_r)$, $Q_k = \mathrm{Im}\, L(ki\omega_r)$.
It is assumed, that the phase of the first harmonic of autonomous periodic behavior equals zero. Thence, the frequence of oscillation $\omega_r = 2\pi/T_r$ is determined by equation

$$b_1 = q(T_r) = 2/T_r \int_0^{T_r} \xi_s(\omega_r, \tau)\, f[y_r(\tau)]\, d\tau = 0 \ , \quad (5)$$

where $\xi_s(\omega_r,\tau) = P(\omega) \sin(\omega t) - Q(\omega) \cos(\omega t)$.
With account for (5), integral equation for periodic behavior (2) may be rewritten in the form

$$y_r(t) = \int_0^{T_r} \varphi_{1r}(t-\tau)\, f[y(\tau)]\, d\tau \ , \quad (6)$$

where

$$\varphi_{1r}(t-\tau) = \varphi_r(t-\tau) - 2/T_r \ \xi_s(\omega,\tau) \sin(\omega t) \ . \quad (7)$$

Solving of the equations (5,6) allows to determine approximately periodic behavior with zero phase.

1.2 Estimation of the distortions

The exact expression for the periodical behavior has form

$$y(t) = \int_0^T \varphi(t-\tau)\, f[y(\tau)]\, d\tau \quad (8)$$

or taking in account that phase of the first harmonic equals zero

$$y(t) = \int_0^T \varphi_1(t-\tau)\, f[y(\tau)]\, d\tau \ , \quad (9)$$

$$\varphi_1(t-\tau) = \varphi(t-\tau) - 2/T\, \xi_s(\omega,\tau) \sin(\omega t) \ , \quad (10)$$

$$\varphi(t-\tau) = 1/T \ \Sigma \ L(ki\omega(t-\tau) \exp(ki\omega(t-\tau))$$

and

$$b_1 = q = 2/T \int_0^T \xi_s(\omega,\tau)\, f[y(\tau)]\, d\tau = 0. \quad (11)$$

Let $y(t) = y_r(t) + z(t)$, where $z(t)$ is a distortion. Thence the integral equation (9) can be rewritten

$$y_r(t) + z(t) = \int_0^T \varphi(t-\tau) f[y_r(\tau) + z(\tau)]\, d\tau \ . \quad (12)$$

For a cubical nonlinearity $f[y]$ we have

$$z(t) = \int_0^T \varphi 1(t-\tau) 3 y_r^2(\tau) z(\tau) d\tau + \int_0^T \varphi 1(t-\tau) 3 y(\tau) z^2(\tau) d\tau + \int_0^T \varphi(t-\tau)\, z^3(\tau) d\tau + y_v(t) \ , \quad (13)$$

where

$$y_v(t) = \int_0^T \varphi 1(t-\tau)\, y_r^3(\tau)\, d\tau - y_r(t). \quad (14)$$

$y_v(t)$ can be determined by analitical way with using (10) and the analitical form for $\varphi(t-\tau)$ [3,4].
In the case when $L(s) = b(s)/d(s)$ and equation $d(s)=0$ have n simple roots s_k, analitical form for $\varphi(t-\tau)$ appears as

$$\varphi(t-\tau) = \begin{cases} \sum_{k=1}^{n} b(s_k)/d'(s_k) \ \exp(s_k(t-\tau))/(1-\exp(s_k T)), & 0 < t-\tau < T, \\ \sum_{k=1}^{n} b(s_k)/d'(s_k) \exp(s_k(t-\tau+T))/(1-\exp(s_k T)), & -T < t-\tau < 0. \end{cases} \quad (15)$$

In many cases equation (13) can be solved by simple iteration. Also, we can use Fredgolm's apparate (resolvent method). In this case expression (13) can be rewritten as [3,4]

$$z(t) = \int_0^T l(t,\tau) 3 y_r(\tau) z^2(\tau) d\tau + \int_0^T l(t,\tau) z^3(\tau) d\tau + y_v(t). \quad (16)$$

Linear part equation (13) respectly z was excluden in 16, that provide ssuccesful decision the equation by simple iterations or estimation z(t) with majorant equation method [3,4]. The function $l(t,\tau)$ connected with resolvent $r(t,\tau)$ of the kerner $k(t,\tau) = \varphi 1(t-\tau)\, 3y(\tau)$ by the relation

$$r(t,\tau) = l(t,\tau)\, 3 y_r^2(\tau) = l(t,\tau)\, f[y_r(\tau)] \quad (17)$$

Hence, $l(t,\tau)$ can be found solving of the equation [3]

$$l(t,\tau) = \int_0^T \varphi 1(t-\nu) 3 y_r^2(\nu)\, l(\nu,\tau)\, d\nu + \varphi 1(t,\tau). \quad (18)$$

Below we use the majorant equation method for estimating the error of Galerkin's method (distortion).

1:3 Majorant equations method

Assume that for $T_r \in T1,T2$ there exists $y_r(t)$ such that

$$|y_r(t)| < \mathrm{const}$$

and for the kernel equation (14) the function $l(t,\tau)$ is determined. Then, we costruct the values

$$u = \max_t |z(t)|,$$

$$h_0 = \max_t |y_v(t)|, \qquad (19)$$

$$h_2 = \max_t \int_0^T |l(t,\tau)\, 3y_r(\tau)|\, d\tau$$

$$h_3 = \max_t \int_0^T |l(t,\tau)|\, d\tau .$$

Then from (13) with account for (19) we can obtain the following majorant equation [3,4]

$$u = h_0 + h_2 u^2 + h_3 u^3 \qquad (20)$$

This majorant equation depends on a parameter T. Suppose that for $T \in [T_1, T_2]$ there exists a majorant equation with the least pozitive root $u^*(T)$. As was shown in [3] the following conditions:

$$q(T_1)\, q(T_2) < 0 \qquad (21)$$

are satisfied and

$$|q(T_i)| > 4/\pi\ L(2\pi/T_i) N\, u^*(T_i),\ i=1,2, \qquad (22)$$

where N - constant such that the inequality

$$|f(y_r + z) - f(y_r)| < N z \quad \text{holds.} \qquad (23)$$

then the exist error or distortion of Galerkin's method satisfies the estimate

$$|\ y(t) - y_r(t,T)\ | < \max u^*(T) = \delta \ . \qquad (24)$$

2. Analysis of complex periodical behaviors

Consider a Chua's circuit with the linear part

$$L(s) = \frac{as^2+as+ab}{s^3+(1+a)s^2+bs+ab} \qquad (25)$$

and the cubical nonlinearity

$$f(y) = \frac{8}{7}\, y - \frac{4}{63}\, y^3 . \qquad (26)$$

The periodical behaviors were described approximately by bias and first harmonic

$$y_r(t) = a_0 + a_1 \sin(\omega_r t) \qquad (27)$$

and were estimated by

$$\delta = \max_{T \in [T_1, T_2]} u^*(T)$$

The received results were verifie by simulation of differential equations according to the system under consideration:

$$y_1' = y_2 \qquad (28)$$

$$y_2' = y_3$$

$$y_3' = -\left(1 - \frac{a}{7} + \frac{12a}{63} y_1^2\right) y_3 - \left(b - \frac{8}{7} a + \frac{12}{63} a y_1^2\right) y_2 - \left(\frac{24}{63} a y_2^2 - \frac{1}{7} a b\right) y_1 - \frac{4}{63} a b y_1^3$$

The results of the estimation of z(t) for different kinds of behaviors are shown in Table 1

Table 1

N	1	2	3	4	5
a	6	2	4	6	2
b	12	3	5	7	1
a_0	1,5	1,5	1,21	0	0
a_1	0,11	0	0,36	1,81	5,21
T_r	4,45	-	5,1	15,38	12,3
δ	0,005	0,0001	0,124	2,73	0,18

In Table 1 the following notation is used:

1 - harmonical oscillations with non-zero bias and low distortions;
2 - a point of equilibrium;
3 - oscillations with non-zero bias and high distortions;
4 - chaos m otion with zero bias and very high distortions
5 – harmonical oscillations with zero bias and low distort.

The forms of oscillations of $y_1(t)$ and phase diagram on the plane (y_1, y_2) for cases 1,4,5 are shown in Fig 3- 5 respectively.

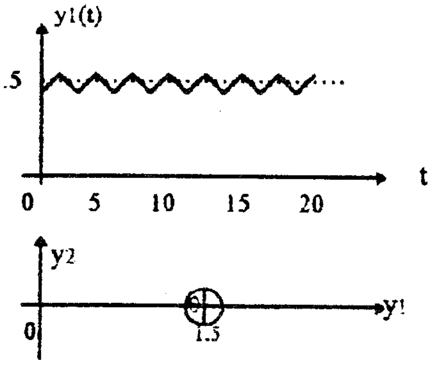

Fig.3

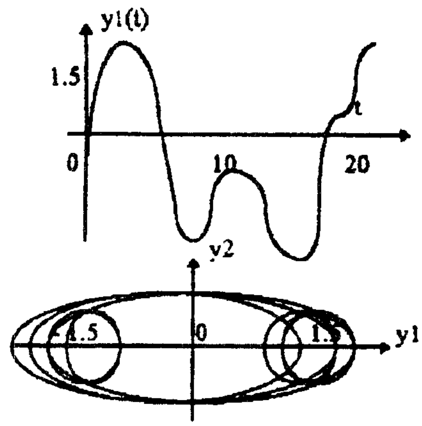

Fig.4
a=6, b=7, chaos motion

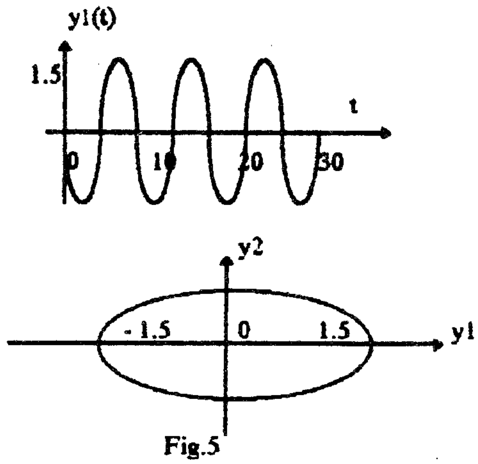

Fig.5
a=2, b=1, harmonic motion with zero bias

The borders on a-b plane, which separate the regions of possible behaviors, are shown on Fig.6.

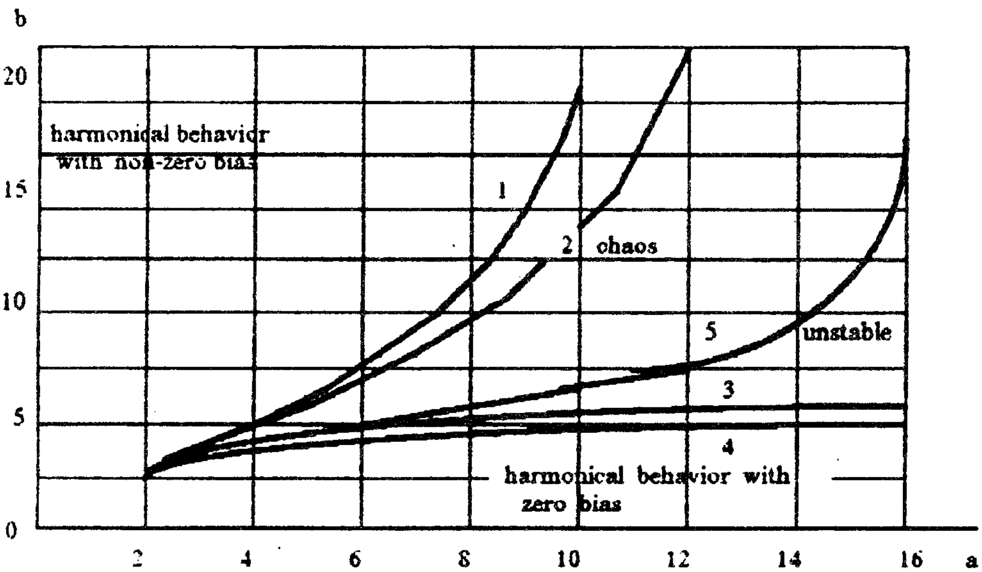

Fig.6

The areas bounded by curves 1,2 and 3,4 are regions where $\delta \in [0.1.0.2]$. The area bounded by curves 1,2 is the region of a passage from periodical behavior with non-zero bias to chaos.

The area bounded curves 3,5 is the region of of a passage from periodical regime with zero-bias to chaos. The area bounded curves 3,5 is the region of unstable behavior.

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