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MASTER-SLAVE SYNCHRONIZATION USING DYNAMIC OUTPUT FEEDBACK

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A method of linear dynamic output feedback for master-slave synchronization of two identical Lur'e systems is introduced. In this scheme, synchronization is obtained using one or at least fewer measurement signals and control signals than the number of state variables of the Lur'e system. A sufficient condition for global asymptotic stability of the error system is derived from a quadratic Lyapunov function and is expressed as a matrix inequality. The dynamic controller is designed by solving a constrained nonlinear optimization problem. The method is demonstrated on Chua's circuit and a hyperchaotic circuit consisting of 2-double scroll cells.

1. Introduction

Present master-slave synchronization schemes for general classes of nonlinear systems, such as Lur'e systems, often assume full static state feedback, using the difference between the full state vectors as an error signal to synchronize the slave to the master system [Wu & Chua, 1994; Curran & Chua, 1997; Suykens & Vandewalle, 1997]. Only in more specific cases, such as Chua's circuit, has static state feedback been applied using fewer state variables [Kapitaniak *et al.*, 1994]. A natural question one may raise then is how synchronization could be obtained for a general class of nonlinear systems based on e.g. a single or fewer measurement signals and fewer control signals. In many practical situations, measuring all the state variables of the system is inconvenient or even impossible. A motivating application is in secure communication, where for implementational reasons the use of one single transmission signal is preferable over transmission of the full state vector of the system [Hasler, 1994; Wu & Chua, 1994].

It is well-known that there exists a close relationship between synchronization and control

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problems [Wu & Chua, 1994]. Therefore, in order to obtain synchronization based on fewer signals, we will adopt the idea of *dynamic output feedback* from control theory. When one is confronted with the problem of controlling some plant for which not all the state variables are measurable (lack of full state information), one either constructs an observer system to estimate the state of the plant, used in combination with state feedback, or one directly applies a dynamic output feedback law, from the measurable outputs to the actuator inputs. In *linear* control theory, these ideas are particularly well developed [Boyd & Barratt, 1991; Maciejowski, 1989]. An example is the LQG (Linear Quadratic Gaussian) regulator problem, which considers the optimal control of a linear system corrupted by process and measurement noise. The optimal solution by means of output feedback is given by a linear dynamic controller. For this controller the so-called separation principle holds, which means that one can split the controller into two parts: A dynamical observer system and a static state feedback applied to the observer estimated state [Boyd & Barratt, 1991; Maciejowski, 1989].

Although we are dealing in this paper with nonlinear rather than linear systems, we will investigate the application of a *linear* dynamic output feedback mechanism in order to synchronize two identical nonlinear systems. We consider a class of nonlinear systems which can be represented in Lur'e form [Khalil, 1992; Vidyasagar, 1993]. Examples of Lur'e systems are Chua's circuit [Chua *et al.*, 1986; Chua, 1994; Madan, 1993] and piecewise-linear versions of n-double scroll circuits [Suykens & Vandewalle, 1993; Arena *et al.*, 1996]. Arrays which consist of such circuits as cells with linear coupling between the cells can also be represented as Lur'e systems. Examples are the double-double scroll attractor [Kapitaniak & Chua, 1994] and the n-double scroll hypercube CNN [Suykens & Chua, 1997]. Furthermore, a Lur'e representation for generalized cellular neural networks has been derived in [Guzelis & Chua, 1993]. In this paper we derive a sufficient condition for synchronization of Lur'e systems based on a quadratic Lyapunov function. The condition is expressed as a matrix inequality (see [Boyd *et al.*, 1994] for an overview of linear matrix inequalities in systems and control theory) on which the design of the output feedback controller is based by solving a constrained nonlinear optimization problem. The controller design is illustrated both on a chaotic system (Chua's circuit) and a hyperchaotic system (coupled 2-double scroll cells). The dynamic control laws for achieving synchronization obtained are fairly simple and are of first and second order respectively.

This paper is organized as follows. In Sec. 2 we introduce the master-slave synchronization scheme with dynamic output feedback. In Sec. 3 we derive the error system and a sufficient condition for global asymptotic stability based on a quadratic Lyapunov function. In Sec. 4, design of the dynamic output feedback controller, based on the matrix inequality, is explained. Finally, in Sec. 5 the method is illustrated on a chaotic and a hyperchaotic Lur'e system, respectively Chua's circuit and a coupled system with 2-double scroll cells.

2. Synchronization Scheme

Let us introduce the following master-slave synchronization scheme

$$\mathcal{M}: \begin{cases} \dot{x} = Ax + B\sigma(Cx) \\ p = Hx \end{cases}$$

$$\mathcal{S}: \begin{cases} \dot{z} = Az + B\sigma(Cz) + Du \\ q = Hz \end{cases} \tag{1}$$

$$\mathcal{C}: \begin{cases} \dot{\rho} = E\rho + G(p-q) \\ u = M\rho + N(p-q) \end{cases}$$

which consists of a master system $\mathcal{M}$, a slave system $\mathcal{S}$ and a controller $\mathcal{C}$ (Fig. 1). The master system is a Lur'e system with state vector $x \in \mathbb{R}^n$ and matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n_h}$, $C \in \mathbb{R}^{n_h\times n}$. A Lur'e system is a linear dynamical system, feedback interconnected to a static nonlinearity $\sigma(\cdot)$ that satisfies a sector condition [Khalil, 1992; Vidyasagar, 1993] (here it has been represented as a recurrent neural network with one hidden layer, activation function $\sigma(\cdot)$ and n_h hidden units [Suykens *et al.*, 1996]). We assume that $\sigma(\cdot)$: $\mathbb{R}^{n_h} \mapsto \mathbb{R}^{n_h}$ is a diagonal nonlinearity with $\sigma_i(\cdot)$ belonging to sector $[0, k]$ for $i = 1, \ldots, n_h$. The output vector of the master system is $p \in \mathbb{R}^l$, with $l \le n$. The slave system consists of an identical Lur'e system with state vector $z \in \mathbb{R}^n$, but is controlled by means of the control vector $u \in \mathbb{R}^m$ through the matrix $D \in \mathbb{R}^{n\times m}$. The signal u is the output of a linear dynamic output feedback controller. The input of this controller is the difference between the outputs of the master and the slave system, i.e. p and q.

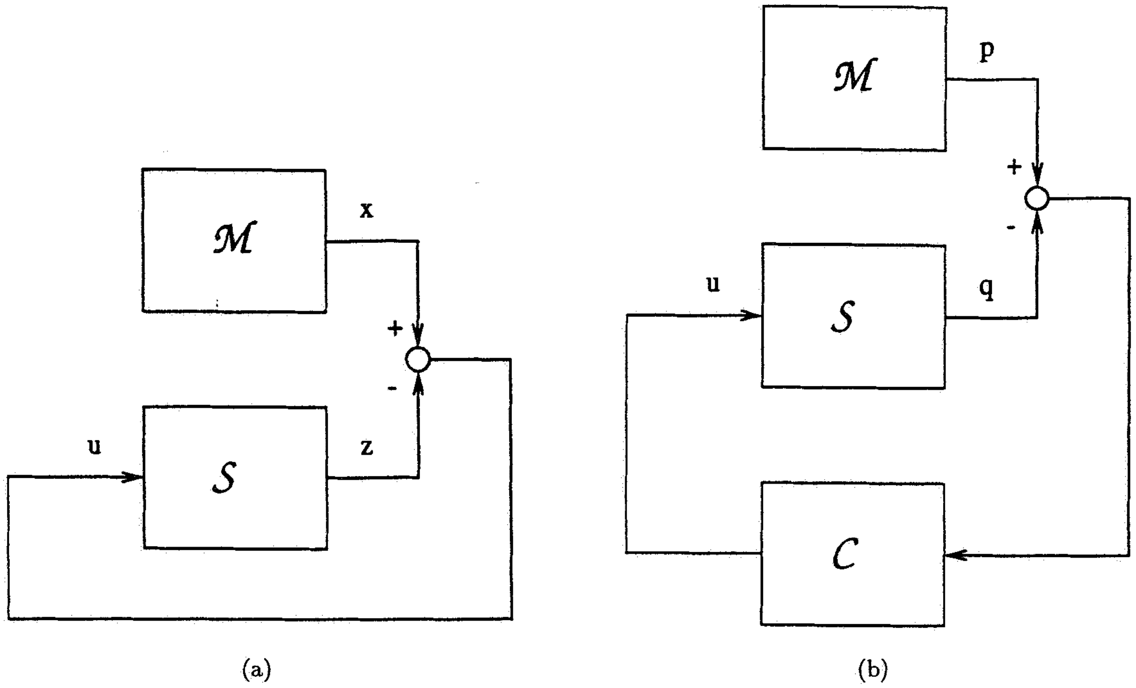


Fig. 1. This figure illustrates the difference between full static state feedback (a), and dynamic output feedback (b), for master-slave synchronization of Lur'e systems: (a) the slave system $\mathcal{S}$ is synchronized to the master system $\mathcal{M}$ by taking the difference between the state vectors x, $z \in \mathbb{R}^n$ and applying a feedback matrix. (b) output variables p, $q \in \mathbb{R}^l$ are defined with $l \leq n$. A linear dynamic output feedback controller is taken instead of a static feedback controller. For the control vector $u \in \mathbb{R}^m$ one has $m \leq n$.

The linear dynamic controller has the state vector $\rho \in \mathbb{R}^{n_c}$ and consists of the matrices $E \in \mathbb{R}^{n_c \times n_c}$, $G \in \mathbb{R}^{n_c \times l}$, $M \in \mathbb{R}^{m \times n_c}$, $N \in \mathbb{R}^{m \times l}$. In previous work on master-slave synchronization, only full static state error feedback $u = F_s(x - z)$ or static output feedback $u = F_o(p - q)$ have been considered with $F_s \in \mathbb{R}^{n \times n}$, $F_o \in \mathbb{R}^{m \times l}$ [Wu & Chua, 1994; Kapitaniak *et al.*, 1994; Curran & Chua, 1997; Suykens & Vandewalle, 1997].

3. Error System and Global Asymptotic Stability

We define the error signal $e = x - z$, which yields the error system:

$$\begin{bmatrix} \dot{e} \\ \dot{\rho} \end{bmatrix} = \begin{bmatrix} A - DNH & -DM \\ GH & E \end{bmatrix} \begin{bmatrix} e \\ \rho \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \eta(Ce;\, z) \tag{2}$$

with nonlinearity $\eta(Ce;\, z) = \sigma(Ce + Cz) - \sigma(Cz)$. Assume that the nonlinearity $\eta(Ce;\, z)$ belongs to sector $[0,\, k]$ [Curran & Chua, 1997; Suykens & Vandewalle, 1997]:

$$0 \leq \frac{\eta_i(c_i^T e;\, z)}{c_i^T e} = \frac{\sigma_i(c_i^T e + c_i^T z) - \sigma(c_i^T z)}{c_i^T e} \leq k\,, \quad \forall e,\, z; \quad i = 1, \ldots, n_h \quad (c_i^T e \neq 0)\,. \tag{3}$$

The following inequality then holds [Boyd *et al.*, 1994; Khalil, 1992; Vidyasagar, 1993]:

$$\eta_i(c_i^T e;\, z)[\eta_i(c_i^T e;\, z) - k c_i^T e] \leq 0\,, \quad \forall e,\, z; \quad i = 1, \ldots, n_h\,. \tag{4}$$

It follows from the mean value theorem that for differentiable $\sigma(\cdot)$ the sector condition $[0,\, k]$ on $\eta(\cdot)$ corresponds to [Curran & Chua, 1997]:

$$0 \leq \frac{d}{d\rho}\sigma_i(\rho;\, z) \leq k, \quad \forall \rho,\, z; \quad i = 1, \ldots, n_h\,. \tag{5}$$

We are interested to see under what condition the error system is globally asymptotically stable, which means that whatever the choice of the initial states $x(0)$, $z(0)$, $\rho(0)$, the error $\|[x - z;\, \rho]\|_2 \to 0$ as $t \to \infty$. Defining the state vector $\xi = [e;\, \rho]$ and taking a positive definite quadratic Lyapunov function (which is radially unbounded)

$$V(\xi) = \xi^T P \xi = [e^T \rho^T] \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} \begin{bmatrix} e \\ \rho \end{bmatrix}, \quad P = P^T > 0, \tag{6}$$

a sufficient condition for global asymptotic stability of Eq. (2) is derived in the following Theorem.

Theorem. *Let $\Lambda = \mathrm{diag}\{\lambda_i\}$ be a diagonal matrix with $\lambda_i \geq 0$ for $i = 1, \ldots, n_h$. Then a sufficient condition for global asymptotic stability of the error system of Eq. (2) is given by the matrix inequality*

$$Y = Y^T = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ \cdot & Y_{22} & Y_{23} \\ \cdot & \cdot & Y_{33} \end{bmatrix} < 0 \tag{7}$$

where

$$\begin{aligned} Y_{11} &= (A - DNH)^T P_{11} + P_{11}(A - DNH) \\ &\quad + H^T G^T P_{21} + P_{12} GH \\ Y_{12} &= (A - DNH)^T P_{12} + H^T G^T P_{22} \\ &\quad - P_{11} DM + P_{12} E \\ Y_{13} &= P_{11} B + k C^T \Lambda \\ Y_{22} &= E^T P_{22} + P_{22} E - M^T D^T P_{12} - P_{21} DM \\ Y_{23} &= P_{21} B \\ Y_{33} &= -2\Lambda . \end{aligned}$$

Proof. Taking the time-derivative of the Lyapunov function, applying the S-procedure [Boyd *et al.*, 1994] and using the inequalities of Eq. (4), one obtains:

$$\begin{aligned} \dot{V} &= \dot{\xi}^T P \xi + \xi^T P \dot{\xi} \\ &\leq \dot{\xi}^T P \xi + \xi^T P \dot{\xi} - \sum_i 2\lambda_i \eta_i (\eta_i - k c_i^T e) \\ &\leq [(A - DNH)e - DM\rho + B\eta]^T (P_{11} e + P_{12}\rho) \\ &\quad + (E\rho + GHe)^T (P_{21} e + P_{22}\rho) \\ &\quad + (e^T P_{11} + \rho^T P_{21})[(A - DNH)e - DM\rho + B\eta] \\ &\quad + (e^T P_{12} + \rho^T P_{22})(E\rho + GHe) \\ &\quad - 2\eta^T \Lambda (\eta - kCe) \\ &\leq \zeta^T Y \zeta < 0 \end{aligned}$$

where $\zeta = [e; \rho; \eta]$. The latter expression is negative for all non-zero ζ provided Y is negative definite. ■

4. Controller Design

The dynamic output feedback controller $\mathcal{C}$ can be designed on the basis of matrix inequality Eq. (7) by solving the following nonlinear optimization problem:

$$\min_{E,G,M,N,P,\Lambda} \lambda_{\max}[Y(E, G, M, N, P, \Lambda)], \quad \text{such that} \quad \begin{cases} P = P^T > 0 \\ \Lambda \geq 0 \ \text{ and diagonal} \end{cases} \tag{8}$$

where $\lambda_{\max}[.]$ denotes the maximal eigenvalue of a symmetric matrix. When the maximal eigenvalue of Y is negative, a feasible point to the matrix inequality is obtained, resulting in a controller which synchronizes the Lur'e systems. However, the optimization problem of Eq. (8) is non-convex. It may also become non-differentiable when the two largest eigenvalues of the matrix Y coincide. Convergent algorithms for this kind of non-differentiable optimization problem have been described e.g. in [Polak & Wardi, 1982]. The constraint $P > 0$ can be eliminated by considering the parameterization $P = Q^T Q$. A similar idea applies to Λ. The optimization problem becomes then

$$\min_{E,G,M,N,Q,\Lambda} \lambda_{\max}[Y(E, G, M, N, Q, \Lambda)]. \tag{9}$$

5. Examples

5.1. *Chua's circuit*

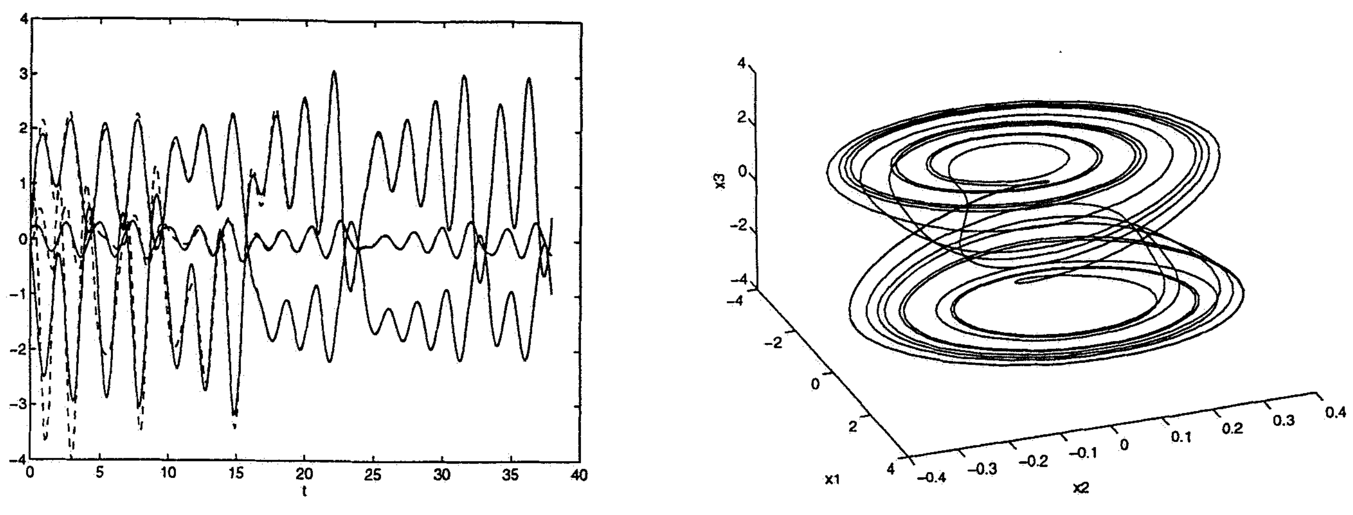
In this example we consider master-slave synchronization of two identical Chua's circuits using linear dynamic output feedback. We take the following representation of Chua's circuit:

$$\begin{cases} \dot{x}_1 = \alpha[x_2 - h(x_1)] \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = -\beta x_2 \end{cases} \tag{10}$$

with nonlinear characteristic

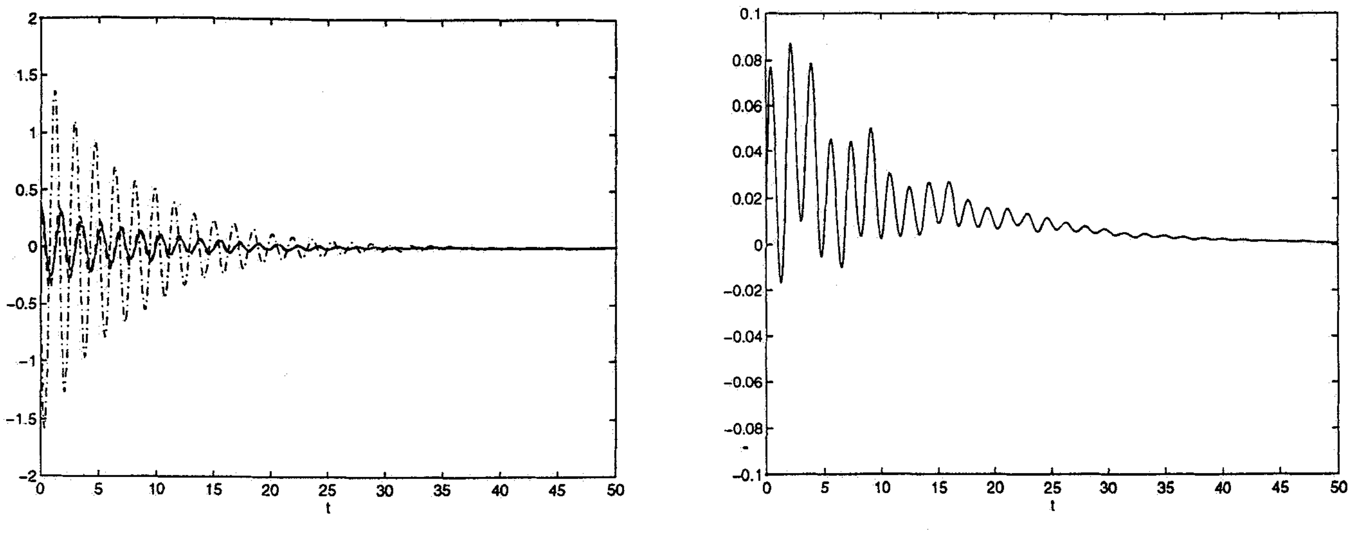
$$h(x_1) = m_1 x_1 + \frac{1}{2}(m_0 - m_1)(|x_1 + c| - |x_1 - c|) \tag{11}$$

and parameters $\alpha = 9$, $\beta = 14.286$, $m_0 = -1/7$, $m_1 = 2/7$ in order to obtain the double scroll attractor [Chua *et al.*, 1986; Chua, 1994; Madan, 1993]. The nonlinearity $\varphi(x_1) = \frac{1}{2}(|x_1 + c| - |x_1 - c|)$ (linear characteristic with saturation) belongs to



(a) (b)

Fig. 2. This figure shows master-slave synchronization of two identical Chua's circuits using a first order linear dynamic output feedback controller. One single output and control signal have been used. (a) Shown are the state variables of the Chua's circuits with respect to time for a randomly chosen initial condition: (-) master system, (- -) slave system. (b) three-dimensional view on the double scroll attractor generated at the master system.



(a) (b)

Fig. 3. Chua's circuit (continued). (a) error signal $x - z$ with respect to time: (-) $x_1 - z_1$, (- -) $x_2 - z_2$, (-.) $x_3 - z_3$. (b) state variable ρ of the controller with respect to time.

sector [0, 1]. Hence Chua's circuit can be interpreted as the Lur'e system $\dot{x} = Ax + B\varphi(Cx)$ where

$$A = \begin{bmatrix} -\alpha m_1 & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix}, \quad B = \begin{bmatrix} -\alpha(m_0 - m_1) \\ 0 \\ 0 \end{bmatrix},$$

$$C = [1 \quad 0 \quad 0]. \tag{12}$$

Suppose now that we measure the first state variables x_1 and z_1 only in order to synchronize the circuits and that we take just one control signal to control the slave system. This corresponds to the choice $H = [1\ 0\ 0]$, $D = [1; 0; 0]$ ($l = m = 1$). Controllers of order 1, 2 and 3 have been designed based on the optimization problem of Eq. (9). We report the results here for the simplest controller with

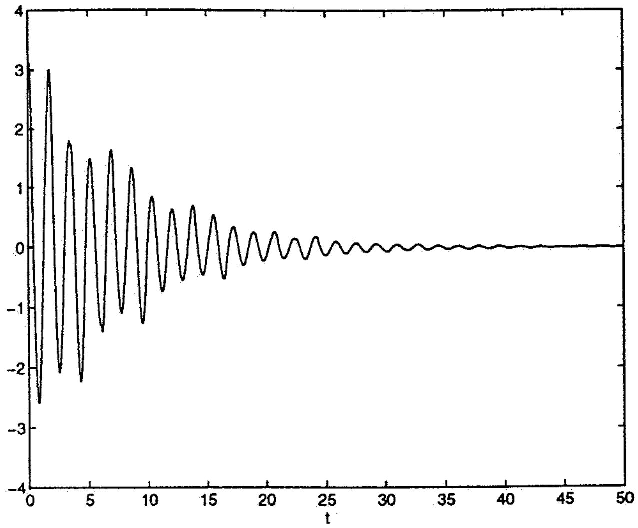


Fig. 4. Chua's circuit (continued). Control signal u with respect to time, applied to the slave system using the first order linear dynamic output feedback controller.

$n_c = 1$. A 2-norm constraint on the controller parameter vector $[E(:); G(:); M(:); N(:)]$ (< 10) has been used for Eq. (9), where " : " denotes a columnwise scanning of a matrix. Sequential quadratic programming [Fletcher, 1987] has been applied in order to optimize Eq. (9) using Matlab's optimization toolbox (function *constr*). As starting point for the iterative procedure, a random controller parameter vector has been chosen according to a normal distribution with zero mean and variance 0.1. For the matrix Q, a square random matrix was chosen according to the same distribution but with variance equal to 3. The matrix Λ has been initialized as $100\,I$. Simulation results for master-slave synchronization of the Chua's circuits are shown in Figs. 2–4 for the resulting controller

$$\left[\begin{array}{c|c} E & G \\ \hline M & N \end{array}\right] = \left[\begin{array}{c|c} -0.1403 & 0.6492 \\ \hline -0.8758 & 9.9394 \end{array}\right].$$

The systems have been simulated using a Runge–Kutta integration rule with adaptive step size (*ode23* in Matlab) [Parker & Chua, 1989].

5.2. *Hyperchaotic system with 2-double scroll cells*

In this example we consider master-slave synchronization for a special case of the n-double scroll hypercube CNN, which consist of two 2-double scroll cells with unidirectional coupling between the cells [Suykens & Chua, 1997]. The system is described by

$$\begin{cases} \dot{x}_1 = \alpha[x_2 - h(x_1)] \\ \dot{x}_2 = x_1 - x_2 + x_3 \\ \dot{x}_3 = -\beta x_2 \\ \dot{x}_4 = \alpha[x_5 - h(x_4)] + K(x_4 - x_1) \\ \dot{x}_5 = x_4 - x_5 + x_6 \\ \dot{x}_6 = -\beta x_5 \end{cases} \tag{13}$$

with, as nonlinear function, the piecewise linear characteristic:

$$h(x_1) = m_{2n-1}x_1 + \frac{1}{2}\sum_{i=1}^{2n-1}(m_{i-1} - m_i) \times (|x_1 + c_i| - |x_1 - c_i|)\,, \tag{14}$$

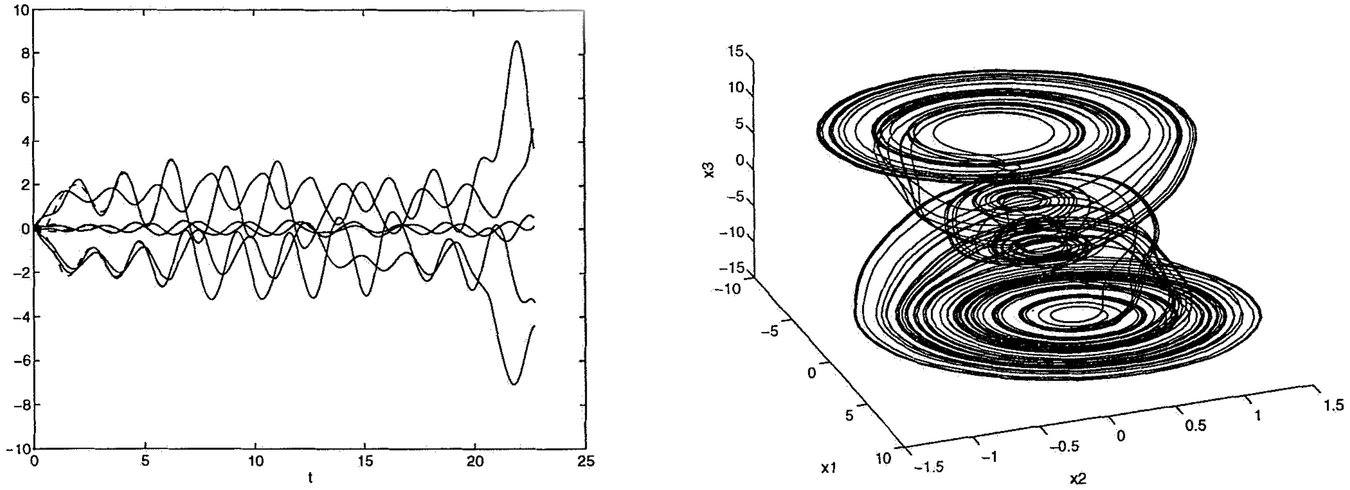
consisting of $2(2n-1)$ breakpoints, where n is a natural number which determines the number of scrolls obtained. An isolated cell behaves as a 2-double scroll for $\alpha = 9$, $\beta = 14.286$, $m_0 = -1/7$, $m_1 = 2/7$, $m_2 = -4/7$, $m_3 = m_1$, $c_1 = 1$, $c_2 = 2.15$, $c_3 = 3.6$. For weak coupling between the two cells, hyperchaos is obtained with two positive Lyapunov exponents, as shown in [Suykens & Chua, 1997]. We choose $K = 0.01$ for the unidirectional coupling constant in Eq. (13).

A Lur'e representation $\dot{x} = Ax + B\varphi(Cx)$ for the system Eq. (13) is given by:

$$A = \left[\begin{array}{ccc|ccc} -\alpha m_1 & \alpha & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & -\beta & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & -\alpha m_1 & \alpha & 0 \\ 0 & -K & 0 & 1 & -1+K & 1 \\ 0 & 0 & 0 & 0 & -\beta & 0 \end{array}\right],$$

$$B = \left[\begin{array}{ccc|ccc} b_{11} & b_{12} & b_{13} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & b_{11} & b_{12} & b_{13} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$C = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{array}\right] \tag{15}$$



(a) (b)

Fig. 5. This figure shows master-slave synchronization between two identical hyperchaotic systems, that consist of two unidirectionally coupled 2-double scroll cells, using a second order dynamic output feedback controller. (a) Shown are the state variables of the 6-dimensional circuit with respect to time for a randomly chosen initial condition: (-) master system, (- -) slave system. (b) three-dimensional view on the 2-double scroll attractor generated at the first cell.

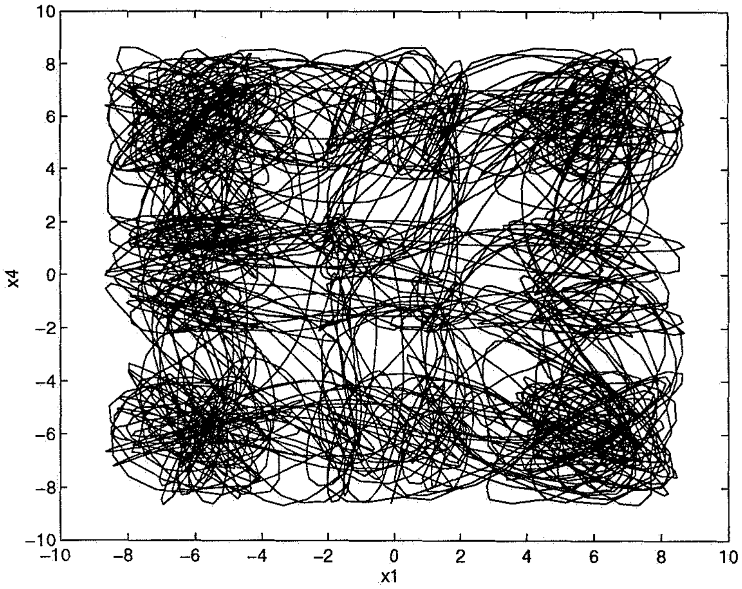


Fig. 6. 2-double scroll cells (continued). For weak coupling between the two 2-double scroll cells, a 2-double scroll square is obtained in the common cell space (x_1, x_4) as a special case of the n-double scroll hypercube CNN [Suykens & Chua, 1997].

with $b_{11} = -\alpha(m_0 - m_1)$, $b_{12} = -\alpha(m_1 - m_2)$, $b_{13} = -\alpha(m_2 - m_3)$. The nonlinearity $\varphi(\cdot)$: $\mathbb{R}^6 \mapsto \mathbb{R}^6$ belongs to sector [0, 1] with $\varphi_i(x_1) = \frac{1}{2}(|x_1 + c_i| - |x_1 - c_i|)$ ($i = 1, 2, 3$) and $\varphi_i(x_4) = \frac{1}{2}(|x_4 + c_{i-3}| - |x_4 - c_{i-3}|)$ ($i = 4, 5, 6$).

We consider two identical Lur'e systems Eq. (15) for the master and slave system. In order to synchronize the systems by means of linear dynamic output feedback, we define two outputs and two control signals ($l = m = 2$) as:

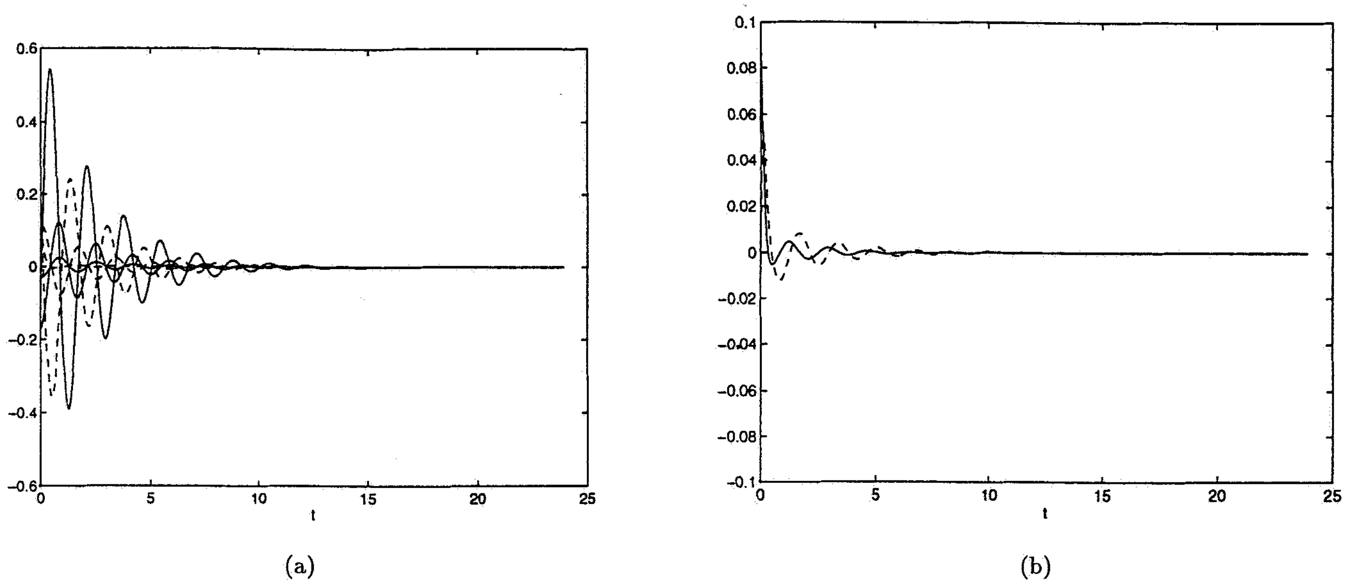


Fig. 7. 2-double scroll cells (continued). (a) error signal $x - z$ with respect to time: (-) $x_i - z_i$ for $i = 1, 2, 3$, (- -) $x_i - z_i$ for $i = 4, 5, 6$. (b) state variables ρ of the controller with respect to time.

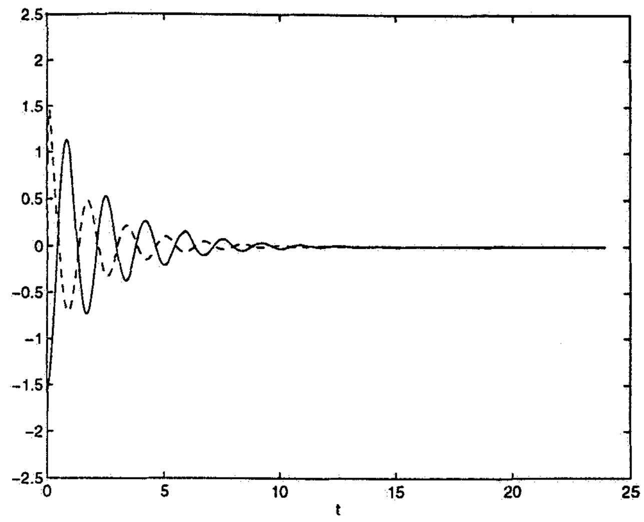


Fig. 8. 2-double scroll cells (continued). Control signals u with respect to time, applied to the slave system using the linear dynamic output feedback controller.

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{16}$$

We demonstrate the method for a second order controller ($n_c = 2$). A 2-norm constraint on the controller parameter vector $[E(:); G(:); M(:); N(:)]$ (< 80) has been taken into account for Eq. (9). As starting point for sequential quadratic programming in order to optimize Eq. (9), a random controller parameter vector has been chosen according to a normal distribution with zero mean and variance 0.1. For the matrix Q a square random matrix was chosen according to the same distribution but with variance equal to 3. The matrix Λ has been initialized as $1000\,I$. Simulation results are shown in Figs. 5–8 for the resulting controller

$$\left[\begin{array}{c|c} E & G \\ \hline M & N \end{array}\right] = \left[\begin{array}{cc|cc} 0.3729 & -4.7355 & -2.3533 & -4.1974 \\ 8.4883 & -19.5393 & -8.6672 & 6.3477 \\ \hline -7.2920 & -2.3984 & 45.4291 & 5.2582 \\ -38.8729 & 7.9657 & -16.7269 & 42.0817 \end{array}\right].$$

6. Conclusion

In this paper a linear dynamic output feedback mechanism has been introduced for master-slave synchronization of Lur'e systems. Rather than using the error between the full state vectors to synchronize the systems, the scheme enables one to use fewer measurement signals and fewer control signals than the number of state variables of the Lur'e systems. A sufficient condition for global asymptotic stability of the error system has been derived based on a quadratic Lyapunov function. The controller design is achieved by solving a constrained nonlinear optimization problem, based on the derived matrix inequality. The method has been illustrated on Chua's circuit and a hyperchaotic system that consists of coupled 2-double scroll cells.

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